A Sequential Probability Ratio Test in Assessing Software Quality Using LPETM

D. Haritha, R. Satya Prasad

Abstract— Rapid growth of software usage enforces us to assess the software reliability, a critical task in the development of a software system. In this paper a well known test procedure of statistical science called as Sequential Probability Ratio Test (SPRT) is adopted for Logarithmic Poisson Execution Time Model (LPETM) in assessing the reliability of a developed software. It requires considerably less number of observations when compared with the other existing testing procedures. The model is inspected by using live Data Sets.

I. INTRODUCTION

Software failure data is essential in the analysis of software reliability applications. The failure data is of two types. Time Domain Approach records the failure occurrence times. whereas as interval domain approach deals with number of failures in a given time period. The Time domain approach requires more data collection efforts and highly accurate in parameter estimation than the interval domain approach. The probability equation of the stochastic process representing the failure occurrences is given by a homogeneous Poisson process with the expression

\[ P[N(t) = n] = \frac{e^{-\lambda t}(\lambda t)^n}{n!} \]  \hspace{1cm} (1.1)

Stieber (1997) has identified that classical testing strategies results in misleading of reliability predictions. He pointed that statistical methods can be successfully applied to the failure data. In this paper we consider a popular SRGM known as Logarithmic Poisson Execution Time Model in short LPETM in detecting unreliable software components in order to accept/reject a developed software. The parameter estimation for the given model is shown in section 4. Application of the decision rule to detect unreliable software components is given in Section 5.

II. SEQUENTIAL PROBABILITY RATIO TEST FOR A POISSON PROCESS

A. Wald of Columbia university developed the Sequential Probability Ratio Test. Sequential Tests are well known as they require fewer observations on the average than fixed sample size tests. SPRTs are widely used for statistical quality control in manufacturing and military applications.

An SPRT for homogeneous Poisson processes is described below.

Let \{N(t), t \geq 0\} be a homogeneous Poisson process with rate \('\lambda\}'. Where \(N(t)\) is the number of failures up to time \('t'\) and \('\lambda'\) is the failure rate. To ensure failure free process let us consider two small numbers \('\alpha'\) and \('\beta'\), where \('\alpha'\) is the probability of falsely rejecting the system. That is rejecting the system even if \(\lambda \geq \lambda_0\). \(\beta\) is the probability of falsely accepting the system. That is accepting the system even if \(\lambda \geq \lambda_1\). With specified choices of \(\lambda_0\) and \(\lambda_1\) such that \(0 < \lambda_0 < \lambda_1\), the probability of finding \(N(t)\) failures in the time span \((0,t)\) with \(\lambda_1, \lambda_0\) as the failure rates are respectively given by, \(P_1 = \frac{e^{-\lambda_1 t}[\lambda_1 t]^N(t)}{N(t)!}\) \hspace{1cm} (2.1)

\(P_0 = \frac{e^{-\lambda_0 t}[\lambda_0 t]^N(t)}{N(t)!}\) \hspace{1cm} (2.2)

The ratio \(\frac{P_1}{P_0}\) at any time \('t'\) is considered as a measure of deciding the truth towards \(\lambda_0\) or \(\lambda_1\), given a sequence of Time instants say \(t_1 < t_2 < t_3 < \ldots < t_K\) and the corresponding realizations. \(N(t_1), N(t_2), \ldots, N(t_K)\) of \(N(t)\).

Simplification of \(\frac{P_1}{P_0}\) gives

\(\frac{P_1}{P_0} = \exp(\lambda_0 - \lambda_1)t + \left(\frac{\lambda_1}{\lambda_0}\right)^N(t)\)

The decision rule of SPRT is to decide in favour of \(\lambda_1\), in favour of \(\lambda_0\) or to continue by observing the number of failures at a later time than \('t'\) according as \(\frac{P_1}{P_0}\) is greater than or equal to a constant say \(A\), less than \(A\) or equal to a constant say \(B\) or in between the constants \(A\) and \(B\). We decide the given software product as unreliable, reliable or continue the test process with one more observation in failure data.

\(\frac{P_1}{P_0} \geq A\) \hspace{1cm} (2.3)

\(\frac{P_1}{P_0} \leq B\) \hspace{1cm} (2.4)

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\[ B < \frac{P_1}{P_0} < A \]  
(2.5)

The approximate values of the constants A and B are taken as

\[ A \cong \frac{1-\beta}{\alpha}, \quad B \cong \frac{\beta}{1-\alpha} \]

Where ‘\( \alpha \)’ and ‘\( \beta \)’ are the risk probabilities. A simplified version of the above decision processes is to reject the system as unreliable if \( N(t) \) falls for the first time above the line

\[ N_U(t) = at + b_2 \]  
(2.6)
to accept the system to be reliable if \( N(t) \) falls for the first time below the line

\[ N_L(t) = at - b_1 \]  
(2.7)

To continue the test with one more observation on \( t, N(t) \) as the random graph of \([t, N(t)]\) is between the two linear boundaries given by equations (2.6) and (2.7) where

\[ a = \frac{\lambda_1 - \lambda_0}{\log \left( \frac{\lambda_1}{\lambda_0} \right)} \]  
(2.8)

\[ b_1 = \frac{1-\alpha}{\beta} \log \left( \frac{\lambda_1}{\lambda_0} \right) \]  
(2.9)

\[ b_2 = \frac{1-\beta}{\alpha} \log \left( \frac{\lambda_1}{\lambda_0} \right) \]  
(2.10)

The parameters \( \alpha, \beta, \lambda_0 \) and \( \lambda_1 \) can be chosen in several ways. One way suggested by Stieber (1997) is

\[ \lambda_0 = \frac{\lambda_1 \log(q)}{q-1}, \quad \lambda_1 = q \frac{\lambda_0 \log(q)}{q-1} \]

where \( q = \frac{\lambda_1}{\lambda_0} \)

If \( \lambda_0 \) and \( \lambda_1 \) are chosen in this way, the slope of \( N_U(t) \) and \( N_L(t) \) equals \( \lambda \). The other two ways of choosing \( \lambda_0 \) and \( \lambda_1 \) are from past projects (for a comparison of the projects) and from part of the data to compare the reliability of different functional areas.

### III. SEQUENTIAL TEST FOR SOFTWARE RELIABILITY GROWTH MODEL

We know that for any poisson process, the expected value \( N(t) = \lambda t \) called the average number of failures experienced in time ‘\( t \)’. Consider a Poisson process with a general function \( m(t) \) as its mean value function the probability equation of a such a process is

\[ P \left[ N(t) = Y \right] = \frac{[m(t)]^y}{y!} e^{-m(t)}, \quad y = 0, 1, 2, \ldots \]

Depending on the forms of \( m(t) \) we get various Poisson processes called NHPP for our model the mean value function is \( m(t) = a \log(1+bt) \)

We may write

\[ P_1 = \frac{e^{-m_1(t)} \left[ m_1(t) \right]^{N(t)}}{N(t)!} \]

\[ P_0 = \frac{e^{-m_0(t)} \left[ m_0(t) \right]^{N(t)}}{N(t)!} \]

where \( m_1(t), m_0(t) \) are values of the mean value function at specified sets of its parameters indicating reliable software and unreliable software respectively. For instance the model we have been considering its \( m(t) \) function, contains a pair of parameters \( a, b \) with ‘\( a \)’ as a multiplier. Also \( a, b \) are positive.

Let \( P_0, P_1 \) be values of the NHPP at two specifications of \( b \) say \( b_0, b_1 (b_0 < b_1) \) respectively. It can be shown that for our models \( m(t) \) at \( b_1 \) is greater than that at \( b_0 \). Symbolically \( m_0(t) < m_1(t) \). Then the SPRT procedure is as follows:

Accept the system to be reliable if \( \frac{P_1}{P_0} \leq B \)

i.e., \( \frac{e^{-m_1(t)} \left[ m_1(t) \right]^{N(t)}}{e^{-m_0(t)} \left[ m_0(t) \right]^{N(t)}} \leq B \) \n
i.e., \( N(t) \leq \frac{\log \left( \frac{\beta}{1-\alpha} \right) + m_1(t) - m_0(t)}{\log m_1(t) - \log m_0(t)} \)  
(3.1)

Decide the system to be unreliable and reject if \( \frac{P_1}{P_0} \geq A \)

i.e., \( N(t) \geq \frac{\log \left( \frac{1-\beta}{\alpha} \right) + m_1(t) - m_0(t)}{\log m_1(t) - \log m_0(t)} \)  
(3.2)

Continue the test procedure as long as
The parameters ‘a’ and ‘b’ are estimated using iterative Newton Raphson Method, which is given as

\[ x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \]

\[ m(t) = a \log(1 + bt) \]  \hspace{1cm} (3.8)

Where \( \lambda(t) = \frac{b}{m(t)} \)

\[ \text{LLF} = \log L = \sum_{i=1}^{n} \log \left( \frac{ab}{1 + bs_i} \right) - a \log(1 + bs_n) \]  \hspace{1cm} (3.9)

The unknown parameters a & b of the given LPETM can be obtained as follows:

Firstly, take the partial derivative of eqn.(3.9) w.r.t ‘a’ and equate it to zero we get,

\[ \sum_{i=1}^{n} \frac{b}{1 + bs_i} - \frac{a}{sa} (a \log(1 + bs_n)) = 0 \]

\[ a = \frac{n}{\log(1 + bs_n)} \]  \hspace{1cm} (3.10)

The parameter ‘b’ is estimated by iterative Newton Raphson Method using \( b_{n+1} = b_n - \frac{g(b_n)}{g'(b_n)} \), which is substituted in finding ‘a’. where \( g(b) \) & \( g'(b) \) are expressed as follows.

\[ g(b) = \frac{\partial \log L}{\partial b} = 0 \]  \hspace{1cm} \[ g'(b) = \frac{\partial^2 \log L}{\partial b^2} = 0 \]

\[ g(b) = \sum_{i=1}^{n} \left\{ \frac{s_n}{[1 + bs_n]} - \frac{s_i}{1 + bs_i} \right\} \]  \hspace{1cm} (3.11)

\[ g'(b) = \sum_{i=1}^{n} \left\{ \frac{s_n \log(1 + bs_n)s_n + (1 + bs_n) \log(1 + bs_n)}{[1 + bs_n][1 + bs_n]]^2 \right\} \right. \]

\[ + \frac{s_i^2}{(1 + bs_i)^2} - \frac{1}{b^2} \]  \hspace{1cm} (3.12)

\[ \text{V. SPRT ANALYSIS} \]

In this section we evaluate the decision rules based on the given mean value function for two different data sets. Based on the estimated value of the parameter ‘b’, we have chosen the specifications of \( b_0, b_1 \) that are to be equidistant such that \( b_0 < b < b_1 \). The choices are given in the following table.

<table>
<thead>
<tr>
<th>Data Set</th>
<th>Estimat e of a</th>
<th>Estimat e of b</th>
<th>b0</th>
<th>b1</th>
</tr>
</thead>
<tbody>
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<td>0.000253</td>
<td>0.000223</td>
<td>0.000283</td>
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<td>0.00029</td>
<td>0.00087</td>
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</table>

Using the selected \( b_0, b_1 \) and \( m(t), m_i(t) \) we have calculated the decision rules given by Equations 3.4, 3.5, sequentially at each ‘t’ of the data sets taking the strength (\( \alpha, \beta \)) as (0.05,0.05). These are presented for the model in Tables 5.2.

**Table 5.1: Specifications of b0, b1**
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Table 5.2: SPRT Analysis for LPETM

<table>
<thead>
<tr>
<th>Data Set</th>
<th>T</th>
<th>N(t)</th>
<th>Acceptance Region(≤)</th>
<th>Rejection Region(≥ )</th>
<th>Decision</th>
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</table>

VI. CONCLUSION

The above table shows 2 data sets using SPRT. We are succeeded in proving that sequential tests will give an early conclusion about the reliability or unreliability of a developed software product as par with the existing reliability models.

REFERENCES

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