

# Reduction of Power Line Interference by Using Adaptive Filtering Techniques in Electrocardiogram

G Sundeeep, U V Ratna Kumari

**Abstract:** *Electrocardiogram (ECG) signal is affected by many noise interferences. Out of all the noise effects the power line interference is the predominant one. In this paper the implementation of the adaptive algorithm techniques for reduction in this power line interference is shown and a comparison of these techniques is performed. The adaptive filters used have shown a good improvement in the SNR (Signal- to - Noise Ratio). The LMS (Least mean square), NLMS (Normalized LMS), and SLMS (Sign LMS) algorithms are discussed and compared.*

**Keywords:** ECG (Electrocardiogram), LMS (Least mean square), NLMS (Normalized LMS), SLMS (Sign LMS) and MSE (Mean square error)

## I. INTRODUCTION

Adaptive filters play a very important role in the area of biomedical instrumentation, biotelemetry and signal processing applications. Here the Figure 1 shows the block diagram for the adaptive filter method utilized in this work.

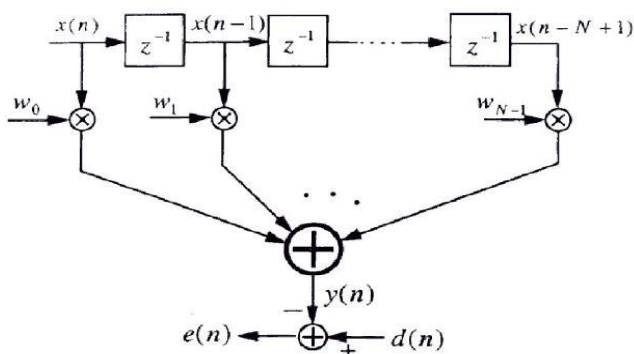


Figure 1: Adaptive Filter Block Diagram

Here  $w$  represents the coefficients of the FIR filter tap weight vector,  $x(n)$  is the input vector samples,  $z^{-1}$  is a delay of one sample period,  $y(n)$  is the desired echoed signal and  $e(n)$  is the estimation error at time  $n$ . The aim of the adaptive filter is to calculate the difference between desired signal adaptive filter outputs,  $e(n)$ . This error signal is fed back into adaptive filter and its coefficients are changed algorithmically in order to

minimize function of this difference, known as cost function. In the case of acoustic echo cancellation, the optimal output of the adaptive filter is equal in value to the unwanted echoed signal. When the adaptive filter output is equal to desired signal the error signal goes to zero. In this situation the echoed signal would be completely cancelled and the far user would not hear any of their original speech returned to them.

This section examines adaptive filters and various algorithms utilized. The various methods used in this paper can be divided into two groups based on their cost functions. The first class are known as Mean Square Error (MSE) adaptive filters, they aim to minimize a cost function equal to the expectation of the square of the difference between the desired signal  $d(n)$ , and the actual output of the adaptive filter  $y(n)$  as shown in equation 1.

$$\xi(n) = E[e^2(n)] = E[(d(n) - y(n))^2] \quad (1)$$

The second class are known as Recursive Least Squares (RLS) adaptive filters and they aim to minimize a cost function equal to the weighted sum of the squares of the difference between the desired and the actual output of the adaptive filter for different time instances. The cost function is recursive in the sense that unlike the MSE cost function, weighted previous values of the estimation error are also considered. The cost function is shown below in equation (2), the parameter  $\lambda$  is in the range of  $0 < \lambda < 1$ . It is known as the forgetting factor as for  $\lambda < 1$  it causes the previous values to have an increasingly negligible effect on updating of the filter tap weights. The value of  $1/(1 - \lambda)$  is a measure of the memory of the algorithm this paper will primarily deal with infinite memory, i.e.  $\lambda = 1$ . The cost function for RLS algorithm,  $\xi(n)$ , is stated in equation (2).

$$\xi(n) = \sum_{k=1}^n \rho_n(k) e_n^2(k) \quad (2)$$

$$\rho_n(k) = \lambda^{n-k}$$

Where  $k=1, 2, 3, \dots, n$ ,  $k=1$  corresponds to the time at which the RLS algorithm commences. Later we will see that in practice not all previous values are considered; rather only the previous  $N$  (corresponding to the filter order) error signals are considered. As stated previously, considering that the number of processes in our ensemble averages is equal to one, the expectation of an input or output value is equal to that actual value at a unique time instance. However, for the purposes of deriving these algorithms, the expectation notation shall still be used.

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## II. INTRODUCTION TO LEAST MEAN SQUARE (LMS)

### ALGORITHM

The Least Mean Square (LMS) algorithm was first developed by Widrow and Hoff in 1959 through their studies of pattern recognition. From there it has become one of the most widely used algorithms in adaptive filtering. The LMS algorithm is a type of adaptive filter known as stochastic gradient-based algorithms as it utilizes the gradient vector of the filter tap weights to converge on the optimal wiener solution. It is well known and widely used due to its computational simplicity. It is this simplicity that has made it the benchmark against which all other adaptive filtering algorithms are judged.

The block diagram of the basic adaptive filter is shown in Figure 2. It is common to all the adaptive filtering algorithms. It looks almost same as figure 1. But the later one is used for noise cancellation.

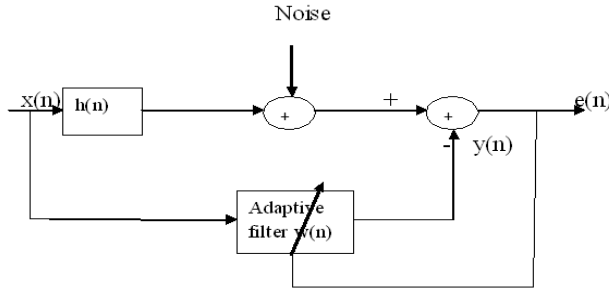


Figure 2: Block Diagram of basic adaptive algorithm

With each iteration of the LMS algorithm, the filter tap weights of the adaptive filter are updated according to the following formula.

$$w(n+1) = w(n) + 2\mu e(n)X(n) \quad (3)$$

Here  $x(n)$  is the input vector of time delayed input values,  $x(n) = [x(n) \ x(n-1) \ x(n-2) \dots x(n-N+1)]^T$ . The vector  $w(n) = [w_0(n) \ w_1(n) \ w_2(n) \dots w_{N-1}(n)]^T$  represents the coefficients of the adaptive FIR filter tap weight vector at time  $n$ . The parameter  $\mu$  is known as the step size parameter and is a small positive constant. This step size parameter controls the influence of the updating factor. Selection of a suitable value for  $\mu$  is imperative to the performance of the LMS algorithm, if the value is too small the time the adaptive filter takes to converge on the optimal solution will be too long; if  $\mu$  is too large the adaptive filter becomes unstable and its output diverges.

### Implementation of the LMS algorithm

Each iteration of the LMS algorithm requires 3 distinct steps in this order:

1. The output of the FIR filter,  $y(n)$  is calculated using equation.

$$Y(n) = \sum_{i=0}^N w(n) x(n-i) = W^T(n) X(n) \quad (4)$$

2. The value of the error estimation is calculated using equation 5.

$$e(n) = d(n) - y(n) \quad (5)$$

3. The tap weights of the FIR vector are updated in preparation for the next iteration, by equation (3).

The main reason for the LMS algorithms popularity in

adaptive filtering is its computational simplicity, making it easier to implement than all other commonly used adaptive algorithms. For each iteration the LMS algorithm requires  $2N$  additions and  $2N+1$  multiplications ( $N$  for calculating the output,  $y(n)$ , one for  $2\mu e(n)$  and an additional  $N$  for the scalar by vector multiplication)

## III. INTRODUCTION TO SIGN LMS (SLMS) ALGORITHM

This algorithm is obtained from conventional LMS Recursion by replacing  $e(n)$  by its sign. This leads to the following recursion:

$$W(n+1) = w(n) + \mu \Phi(n) \text{sgn}\{e(n)\} \quad (6)$$

Where  $\text{sgn}\{e\} = 1+j$  if  $\text{Re}(e) > 0$  and  $\text{Im}(e) > 0$

$1-j$  if  $\text{Re}(e) > 0$  and  $\text{Im}(e) < 0$

$-1+j$  if  $\text{Re}(e) < 0$  and  $\text{Im}(e) > 0$

$-1-j$  if  $\text{Re}(e) < 0$  and  $\text{Im}(e) < 0$

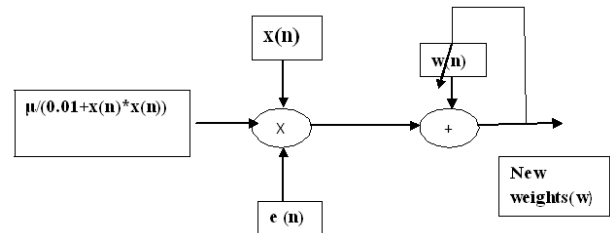


Figure 3: Block Diagram of Sign LMS algorithm

## IV. INTRODUCTION TO NORMALIZED LEAST MEAN SQUARE (NLMS) ALGORITHM

One of the primary disadvantages of the LMS algorithm is having a fixed step size parameter for every iteration. This requires an understanding of the statistics of the input signal prior to commencing the adaptive filtering operation. In practice this is rarely achievable. Even if we assume the only signal to be input to the adaptive echo cancellation system is speech, there are still many factors such as signal input power and amplitude which will affect its performance.

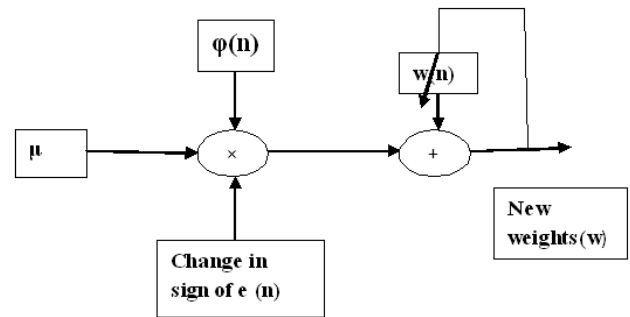


Figure 4: Block Diagram of Normalized LMS Algorithms

The normalized least mean square algorithm (NLMS) is an extension of the LMS algorithm which bypasses this issue by selecting a different step size value,  $\mu(n)$ , for each iteration of the algorithm. This step size is proportional to the inverse of the total expected energy of the instantaneous values of the coefficients of

the input vector  $x(n)$ . This sum of the expected energies of the input samples is also equivalent to the dot product of the input vector with itself, and the trace of input vectors auto-correlation matrix,  $R$

$$\begin{aligned} \text{tr}[R] &= \sum_{i=0}^{N-1} E[x^2(n-i)] \\ &= E \left[ \sum_{i=0}^{N-1} X^2(n-i) \right] \end{aligned} \quad (7)$$

The recursion formula for the NLMS algorithm is stated in equation 8.

### Implementation of the NLMS algorithm

The NLMS algorithm has been implemented in MATLAB. As the step size parameter is chosen based on the current input values, the NLMS algorithm shows far greater stability with unknown signals. This combined with good convergence speed and relative computational simplicity makes the NLMS algorithm ideal for the real time adaptive echo cancellation system. As the NLMS is an extension of the standard LMS algorithm, the NLMS algorithms practical implementation is very similar to that of the LMS algorithm.

Each iteration of the NLMS algorithm requires these steps in the following order.

1. The output of the adaptive filter is calculated.

$$y(n) = \sum_{i=0}^{N-1} W(n) x(n-i) = W^T(n) X(n) \quad (9)$$

2. An error signal is calculated as the difference between the desired signal and the filter output.

$$e(n) = d(n) - y(n) \quad (10)$$

3. The step size value for the input vector is calculated.

$$\mu(n) = \frac{1}{X^T(n)X(n)} \quad (11)$$

4. The filter tap weights are updated in preparation for the next iteration.

$$W(n+1) = W(n) + \mu(n)e(n)X(n) \quad (12)$$

Each iteration of the NLMS algorithm requires  $3N+1$

Multiplications, this is only  $N$  more than the standard LMS algorithm. This is an acceptable increase considering the gains in stability and echo attenuation achieved.

Comparison of Algorithms The verification for refinement of signal by LMS can be done with the removal of power line interference (50 Hz) from the signal.

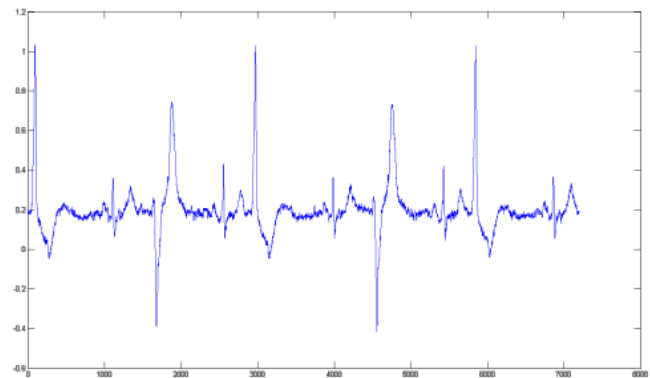


Figure: 5. Original ECG Signal

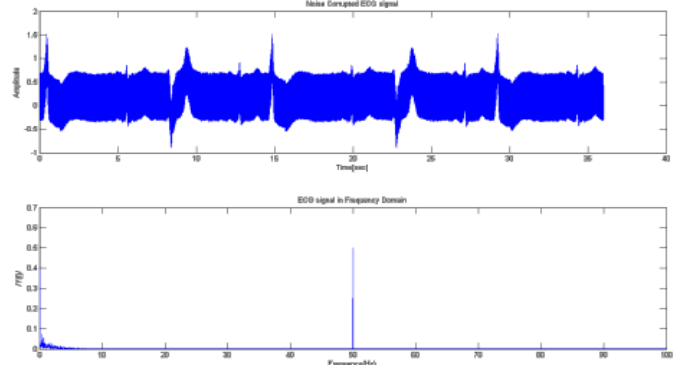


Figure: 6. Noisy signal in time domain and frequency domain

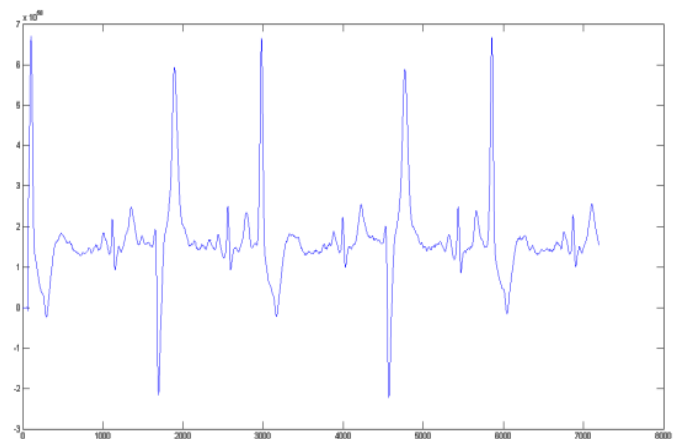


Figure: 7. LMS Algorithm Estimated Output  $y(n)$

$$W(n+1) = W(n) + \frac{1}{X^T(n)X(n)} e(n)X(n) \quad (8)$$

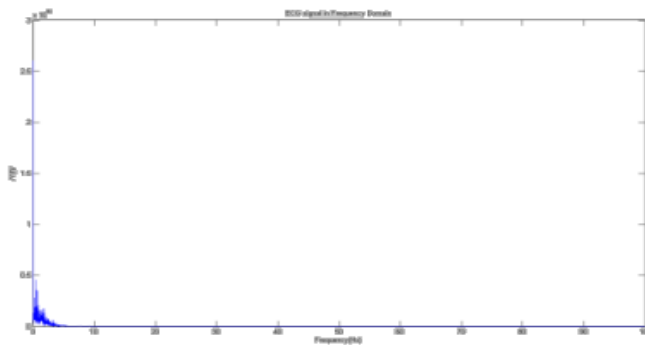


Figure: 8. Removed Noise shown in Frequency Domain

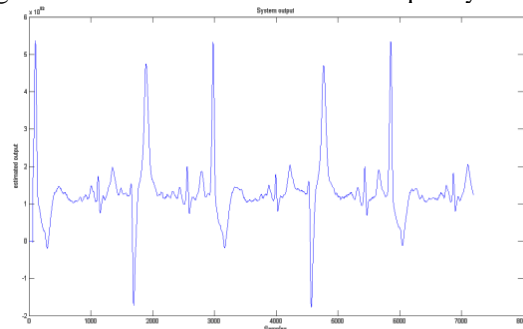


Figure: 9. Sign LMS Algorithm Estimated Output  $y(n)$

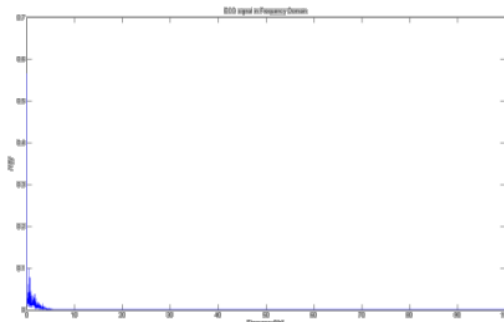


Figure: 10. Removed Noise shown in Frequency Domain

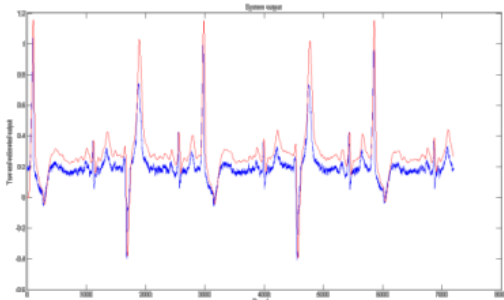


Figure: 11. NLMS Algorithm Outputs Estimated Output  $y(n)$

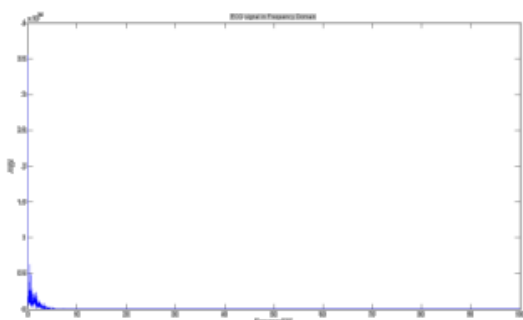


Figure: 12. Removed Noise shown in Frequency Domain

The comparison of different adaptive algorithms with their SNR ratios is shown in below Table.1.

Table 1 : Signal to noise ratio of Adaptive algorithms

ALGORITHMS	SNR (Signal To Noise Ratio)	Total Number of multiplications and additions
LMS	42.13	$4N+2$
NLMS	51.98	$8N+1$
SIGN LMS	46.59	$4N+2$

## III. CONCLUSION

The sole aim of our project is to eliminate the power line interference present in the noisy ECG sample signal. The ECG signal has been filtered using various digital adaptive filters and their performance is evaluated by finding the SNR ratio of the filtered signal. Adaptive filters are to be used when the noise is non-stationary noise. Adaptive filtering with LMS, NLMS and Sign LMS are simulated for noise cancellation in ECG signals and are compared with each other. Finally of all these algorithms, NLMS algorithm eliminates the noise well and the SNR is also high. But Variable Step Normalized LMS is chosen to be best for its performance and good SNR and requires less number of computations than RLS.

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