

An Approach to Find the Stresses Induced in a Flat Belt during Half Rotation of a Driving Pulley

Shrikant A. Thote, M.K. Sonpimple, G.D. Mehta

Abstract: Flat belt drive are the working horses of industries which is also called as loop of flexible material and are used to connect to or more rotating shaft mechanically for massive amount of power transmission. The main purpose of this thesis is to reason out the concept of failure of the belt during continuous operation and to see the stress pattern during half revolution of a pulley in a belt of a belt drive.

Keywords: Belt Drive, Major & Minor Diameter of Pulley, Velocity ratio & Rated Power.

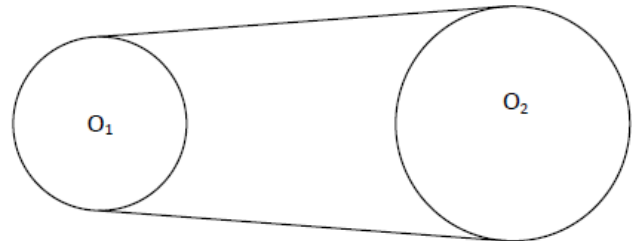


Fig. 1: Schematic of details of belt drive

I. INTRODUCTION

The paper work basically emphasis on the case pertaining to a belt drive, which is used for flour mill operation. The operation of a flour mill is explained as under. A high capacity flour mill demand power approximately 27 Hp for crushing the grain into required form of flour. A bag of grain wheat is poured into the hopper. The wheat grain continuously inserted through cavity to the crushing stone. Generally this crushing stone gets power through belt drive. Due to crushing action of the stone the wheat grain then converted into desired flour. The quality of flour is governed by the gap between stone crusher. During production if belt drive fails then it may turns to production loss. At present this break down in flour mill industries seems to be frequent. The paper finally throws an eye on how to reduce this frequent damage with its significance and effects.

II. ESTIMATION OF TENSION DURING HALF ROTATION OF DRIVING PULLEY

DESIGN OF BELT DRIVE:

Paper uses a case study for designing the belt drive for optimum output. A highly elastic belt is used for belt drive, which is operated by 27 HP electric motor and at the speed of driver & driven pulleys are 1450 rpm & 850 rpm, The diameter of driver (O1) & driven (O2) pulleys are 60 mm & 80 mm. Center to Center distance is 280 mm.

Input Data: -

Diameter of driver pulley	D1 = 60mm
Diameter of driven pulley	D2 = 80 mm
Speed of driver pulley	N1 = 56 mm
Speed of driven pulley	N2 = 40 mm
Velocity ratio (V.R.)	V.R. = 1.33 mm
Center to Center Distance	C = 3.5 x D2 = 280 mm
Rated Power	P _r = 20KW

III. AN APPROACH TO FIND OUT THE STRESSES IN A BELT OF A FLOUR MILL

To estimate the stresses in a belt of belt drive by a FEM approach one need to know the tension at different position of the pulley and the estimation of stresses in a belt of belt drive by an ANSYS approach.

In FEM approach, half rotation of the pulley is considered and half portion of the belt is considered. That half portion of the belt is cut in such a way that, the problem should be laid in symmetrical domain. This cut portion of the belt is then discretized into 50 elements. Applying boundary conditions for each element, material property [D], Jacobean [J], strain displacement matrix [B] and element stiffness matrix [K] are calculated.

Next to this, by obtaining global stiffness matrix & applying boundary conditions, deflections at each node are calculated. These deflections are then useful for calculation of stresses. However conformations of these results are validated by using ANSYS approach.

Design steps:-

A. **DESIGN POWER** = P_d (From the table XV-1)

$$P_d = P_r \times K_1 \times K_\theta$$

Where, P_r is Rated Power

K₁ is Overload service factor = 1.5 (Table XV-2)

K_θ is Capacity coefficient of inclination = 1.0 (Table XV-3)

$$P_d = 20 \times 1.5 \times 1.0 = 23KW$$

Peripheral speed,

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$V_p = 1500$ to 1800 m/min (Table XV-4)

(For max power and economy)

$= 1500$ m/min

$= 28.33$ m/s

Angle of Lap $\theta = \pi \pm \frac{D_2 - D_1}{C}$

$$\theta_1 = \pi - \frac{D_2 - D_1}{C} = \pi - \frac{80 - 60}{280} = 3.07 \text{ rad}$$

$$= 3.07 \times \frac{180}{\pi} = 176 \text{ degree}$$

$$\theta_2 = \pi + \frac{D_2 - D_1}{C} = \pi + \frac{80 - 60}{280} = 3.21 \text{ rad}$$

$$= 3.21 \times \frac{180}{\pi} = 184 \text{ degree}$$

$$T_1/T_2 = e^{\mu \theta} \quad \dots\dots\dots (1)$$

$T_1 - T_2 = P_d/V_p$ (Table XV-1)

$$= 23 \times \frac{10^3}{28.33} = 811.86 \quad \dots\dots\dots (2)$$

From equation 1 and 2,

$T_1 = 1233$ N & $T_2 = 421$ N

Initial Tensions:- $2\sqrt{T_i} = \sqrt{T_1} + \sqrt{T_2}$

$T_i = 773.7$ N

$b_{xt} = T_i / (S_d - S_{cf})$ (Table XV-1)

$$= 1233 / (2.624 - 0.78) \quad \dots\dots\dots (3)$$

Where,

$S_d = \eta \times \text{allowable stress}$

$= 0.82 \times 3.2 \text{ MPa} \dots\dots\dots$ (Table XV-5 & Table XV-6)

$= 2.624$ MPa

$$S_{cf} = \rho V p^2 \times 10^{-6} \quad \dots\dots\dots \text{(Table XV-5)}$$

$$= 0.97 \times 1000 \times 28.33^2 \times 10^{-6}$$

$$= 0.78 \text{ MPa}$$

Therefore from Eq. (4.3)

$$b_{xt} = 684.86 \text{ mm}^2$$

Also,

$$b_{xt} = T_i / S_i \quad \dots\dots\dots (4)$$

where,

$S_i = \text{Stress dur to initial tension} = 1.5 \text{ MPa. (From Data Book)}$

Therefore from Eq. (4)

$$b_{xt} = 773.7 / 1.5$$

$$= 515.8 \text{ mm}^2$$

Selecting larger $b_{xt} = 668.86 \text{ mm}^2$

Thickness of belt (t) = $0.03 D_1 \dots\dots\dots$ (Table XV-4)

$$= 0.03 \times 60$$

$$= 1.8 \text{ mm}$$

$$= 5 \text{ mm}$$

Therefore $b_{xt} = 668.86$

$$b = 668.86 / t$$

$$b = 134 \text{ mm}$$

$$\text{Length of belt (L)} = \frac{\pi}{2} (D_1 + D_2) + 2c + \frac{(D_2 - D_1)^2}{4c}$$

$$= \frac{\pi}{2} + (60 + 80) + 2 \times 280 \frac{(80 - 60)}{4 \times 280} = 780 \text{ mm}$$

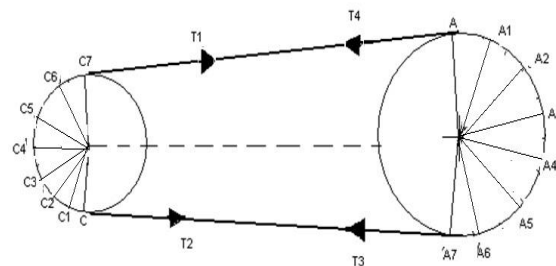
The tensions in the larger pulley are found out suing the same method and the result obtained are as follows:

$$T_3 = 392.2 \text{ N}$$

$$T_4 = 1204 \text{ N}$$

$$T_i = 742.6 \text{ N}$$

B. The Sequential Estimation Of Tension For Each Position



Sr. no.	Angle(Degree)	Tension(N)
1	176	$T_c = 1233$
2	147	$T_{c1} = 1127.58$
3	118	$T_{c2} = 938.77$
4	89	$T_{c3} = 781.57$
5	60	$T_{c4} = 650.7$
6	31	$T_{c5} = 541.74$
7	0	$T_{c7} = 421$

Table 1: Tension at various position of driving pulley (Small pulley)

Sr. no.	Angle(Degree)	Tension(N)
1	0	$T_a = 392.2$
2	30	$T_{a1} = 452.7$
3	60	$T_{a2} = 543.8$
4	90	$T_{a3} = 685.9$
5	120	$T_{a4} = 844.2$
6	150	$T_{a5} = 1039.1$
7	184	$T_{a7} = 1204$

Table 2: Tension at various position of driven pulley (Bigger Pulley)

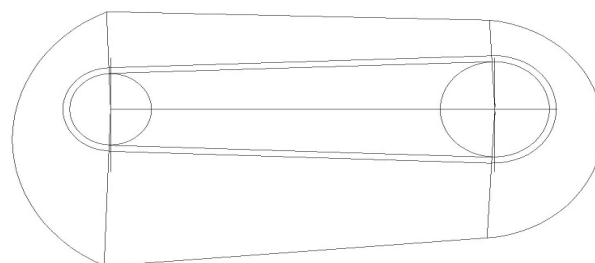


Fig.2: Tension variation diagram during half rotation of driving pulley.

IV. TO ESTIMATE THE STRESSES IN A BELT OF BELT DRIVE BY A FEM APPROACH

A belt of a belt drive in Flour Mill is chosen for the sake of Finite Element Analysis. Fig. 3 shows a pictorial view of a belt drive. If one is looking from a front direction of a belt drive, then a front view can be obtained, which is shown in Fig. 1. It is now required to generate 2-D section of a belt for FEA approach. The section of a belt is as shown in Fig.3. Hence, one can take a half rotation of driver pulley belt section of Fig. 3 for FEA approach. Hence it is now possible to discretize the desired belt section into finite elements.

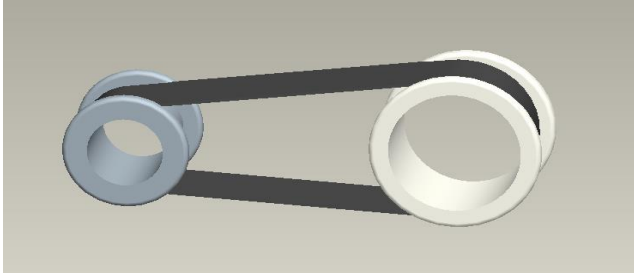


Fig.3: Schematics of belt of a belt drive used in Flour Mill

A. DISCRETIZATION OF 2D SECTION:

The discretization of element is a process which is followed by belt of the belt drive. In the present work belt is discretized into 50 finite elements during half rotation of pulley.

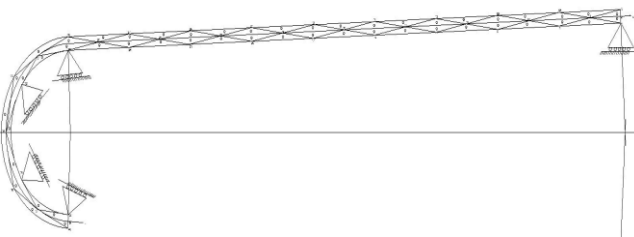


FIG.4: Representation of Discretized Two Dimensional Elements

Every element is numbered by a proper sequence. An example for one such element No. 1 is given below:

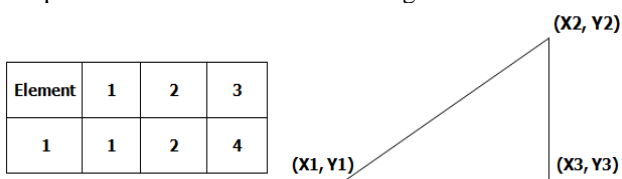
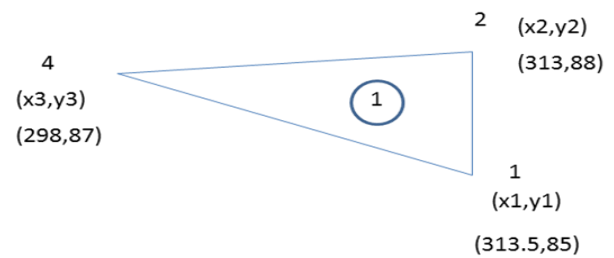


Fig.5: Systematic representation of element number 1 with its Local and Global nodes.

B. Formulation of Material Property Matrix [D], Jacobian Matrix [J], Strain Displacement Matrix [B] & Element Stiffness Matrix [K]:

The next step is to formulate Material Property Matrix [D], Jacobean Matrix [J], Strain Displacement Matrix [B] & Element Stiffness Matrix [K]:

Formulation of matrices for element 1:



Formulation of Material Property Matrix [D]

$$D_1 = \frac{E}{1-\mu^2} \begin{bmatrix} 1 & \mu & 0 \\ \mu & 1 & 0 \\ 0 & 0 & \frac{1-\mu}{2} \end{bmatrix}$$

In the above Matrix:

E = Young's Modulus of Elasticity

μ = Poisson's ratio.

The value of E for (soft rubber) Material is 73×10^3 N/mm² and value of μ is 0.5.

After substituting the above values [D] matrix is converted into

$$D_1 = \frac{73 \times 10^3}{1-0.5^2} \begin{bmatrix} 1 & 0.5 & 0 \\ 0.5 & 1 & 0 \\ 0 & 0 & \frac{1-0.5}{2} \end{bmatrix}$$

$$D_1 = \begin{bmatrix} 9.733 & 4.8667 & 0 \\ 4.8667 & 9.733 & 0 \\ 0 & 0 & 2.433 \end{bmatrix}$$

Formulation of Jacobian Matrix [J]:

$$J_1 = \begin{bmatrix} X_{13} & Y_{13} \\ X_{23} & Y_{23} \end{bmatrix}$$

$$J_1 = \begin{bmatrix} 15.5 & -2 \\ 15 & 1 \end{bmatrix}$$

$$A_1 = |J_1|/2 = 22.75 \text{ mm}^2$$

Formulation of Strain Displacement Matrix [B]:

$$B_1 = 1/J_1 \begin{bmatrix} Y_{23} & 0 & Y_{31} & 0 & Y_{12} & 0 \\ 0 & X_{32} & 0 & X_{13} & 0 & X_{21} \\ X_{32} & Y_{23} & X_{13} & Y_{31} & X_{21} & Y_{12} \end{bmatrix}$$

$$B_1 = \begin{bmatrix} 1 & 0 & 2 & 0 & -3 & 0 \\ 0 & -15 & 0 & 15.5 & 0 & -0.5 \\ -15 & 1 & 15.5 & 2 & -0.5 & -3 \end{bmatrix}$$

$$B_1^T = \begin{bmatrix} 1 & 0 & -15 \\ 0 & -15 & 1 \\ 2 & 0 & 15.5 \\ 0 & 15.5 & 2 \\ -3 & 0 & -0.5 \\ 0 & -0.5 & -3 \end{bmatrix}$$

Formulation of Element Stiffness Matrix [K]_e:

$$[K]_e = t.A.[B]^T[D][B]$$

In the above equation

t = thickness of plate element

A = Area of the element

[B]^T = Transpose of [B] Matrix

For element No. one

$$K^1 = 1 \times 10^6 \begin{bmatrix} 38.91 & -164.85 & -822.43 & 3.66 & -16.49 & 161.19 \\ -164.85 & 3300.7 & -163.02 & -3366.9 & 327.87 & 98.91 \\ -822.43 & -163.02 & 938.74 & 340.69 & -116.31 & -177.67 \\ 3.66 & -3399.6 & 340.69 & 3535.1 & -344.36 & -135.54 \\ -16.49 & 327.8 & -116.31 & -344.36 & 132.8 & 16.49 \\ 161.69 & 98.91 & -177.67 & -135.54 & 16.49 & 36.63 \end{bmatrix}$$

Similarly find stiffness matrices for each element.

C. Estimation of Deflection:

After applying boundary conditions, the global stiffness matrix is reduced from its original size [80 X 80] to [75 X 75]. Then, by using reduced global stiffness matrix and force matrix, one can determine the unknown displacements by using following equation.

$$[F] = [K] \times [Q]$$

Where [F] - force matrix

[K] – reduced global stiffness matrix

[Q] – Nodal displacement matrix

In the present case, reduced global stiffness matrix [K] is of size [75 x 75]. While, force matrix [F] is of [75 x 1]

These matrices are then solved by using Mat-lab software and thus following nodal displacements are obtained (values of displacements are in mm)

q1x'	q1y	q2x	q2y	q3x	q3y	q4x	q4y	q5x	q5y
0.00878	0	0.1043	-0.0019	0.0085	-0.0071	0.0009	0.0111	-0.0071	0.0026

Stresses for each element are calculated using the equation, which is shown below:

$$\sigma_1 = D1 \cdot B1 \cdot Q1 \quad \text{----- (5)}$$

Where

The corresponding stresses for each elements & its calculation can be calculated as above.

D. ESTIMATION OF Von- Mises STRESSES:

The Von Mises (Equivalent) Stresses

$$\sigma_{VN} = (\mathbf{I}_1^2 - 3\mathbf{I}_2)^{1/2} \quad \text{..... (6)}$$

$$\mathbf{I}_1 = \sigma_x + \sigma_y = 0.04966$$

$$\mathbf{I}_2 = \sigma_x \cdot \sigma_y - T_{xy}^2 = -0.0054847$$

From Eq. 6

$$\sigma_{VN} = 0.1375 \text{ Pa}$$

V. TO ESTIMATE THE STRESSES IN A BELT OF BELT DRIVE BY ANSYS APPROACH

A. MATERIAL DATA FOR RUBBER

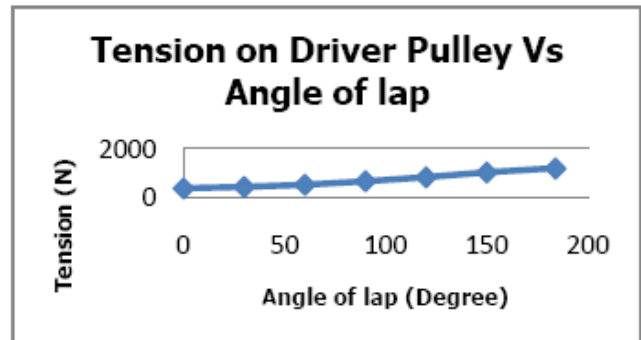
TABLE 1: Rubber > Constants

Structural	
Young's Modulus	7.3e+006 Pa
Poisson's Ratio	0.45
Density	7120. kg/m³
Thermal Expansion	1.2e-005 1/°C
Tensile Yield Strength	2.5e+008 Pa
Compressive Yield Strength	2.5e+008 Pa
Tensile Ultimate Strength	4.6e+008 Pa
Compressive Ultimate Strength	0. Pa
Thermal	
Thermal Conductivity	60.5 W/m.°C
Specific Heat	434. J/kg.°C
Electromagnetics	
Relative Permeability	10000
Resistivity	1.7e-007 Ohm.m

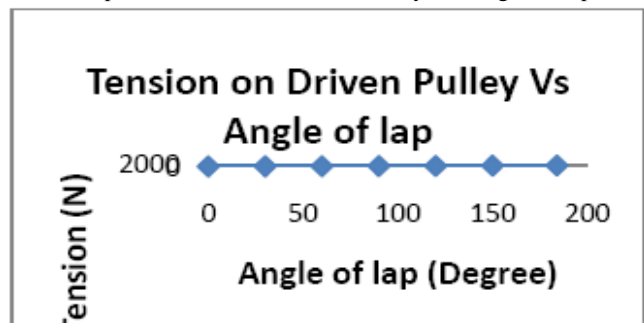
B. VONE MISES STRESSES

VI. RESULT

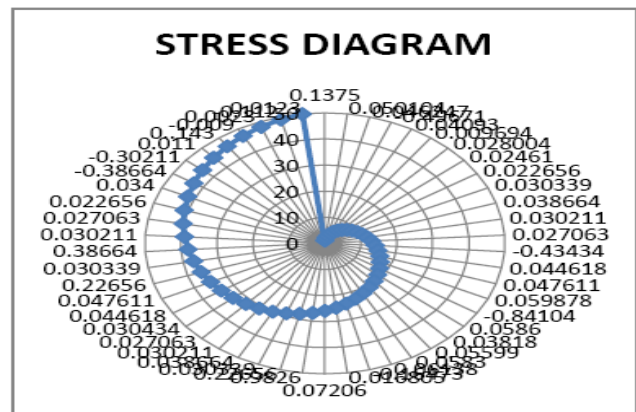
Author cognized by the below stated graphs that In graph 1 tension decrease linearly for Tension Variation in a Belt of a Belt Drive. On the parallel line one can see the value of tension in Graph 2 increases linearly.



Graph: 1: Tension on Driver Pulley vs. Angle of lap



Graph: 2: Tension on Driven Pulley vs. Angle of lap



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