International Journal of Innovative Technology and Exploring Engineering (IJITEE)
ISSN: 2278-3075 (Online), Volume-3 Issue-2, July 2013

Synchronization of Identical Chaotic Systems Using New State Observers Strategy

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Abstract— ***Throughout this paper, the nonlinear observer-based synchronization problem for two coupled Chen chaotic systems is developed. Initially, complete synchronization conditions of coupled chaotic systems, is provided. The active control law developed is based on the use of aggregation techniques for error dynamics stability study and the arrow form matrix for systems description. Afterwards, by the design of an adequate nonlinear state observer, a new synchronization scheme is formulated for two identical chaotic systems. Numerical simulations are carried out to assess the performance and the efficiency of the proposed contributions.***

Index Terms—***Aggregation techniques, Arrow form matrix, Chaotic systems, State observer, Synchronization.***

I. INTRODUCTION

The synchronization phenomenon is an interesting and well-known property of chaotic systems. Since its introduction by Pecora and Carrol in 1990 [1], chaos synchronization has attracted increasing interest in both theory and applications [2-5], as far as several fields are concerned. As a matter of fact, the synchronization of chaotic systems has been successfully applied in secure communication and image encryption, information processing, life science, and so on. Recently, chaos synchronization has been studied from various angles and a variety of different synchronization phenomena have been discovered, such as generalized synchronization [6-7], anti-synchronization [8], hybrid synchronization [9-10], observer-based synchronization [11-13], etc.

Indeed, chaotic systems have many important properties, such as the sensitive dependence on initial conditions and system parameters, pseudorandom property, no periodicity and topological transitivity, etc. In such a way, most properties meet some requirements, such as diffusion and mixing, in the sense of cryptography. Therefore, chaotic cryptosystems have more useful and practical applications.

The layout of this paper is as follows: synchronization behaviour of two identical Chen systems is, firstly, studied. Then, the proposed approach dealing with the nonlinear state observer design viewpoints, relatively to the coupled drive-response Chen chaotic systems, is developed.

Manuscript published on 30 July 2013.
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II. NEW FEEDBACK CONTROL LAW SYNCHRONIZING TWO IDENTICAL COUPLED CHAOTIC CHEN SYSTEMS

The stability study of the dynamical error system is considered, in this part, in order to synchronize two identical chaotic Chen systems [8].

A. Error System Description

The studied chaotic Chen system is described by the following nonlinear differential equations [8]:

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_3(t) \end{bmatrix} = \begin{bmatrix} -\alpha & \alpha & 0 \\ \gamma-\alpha & \gamma & 0 \\ 0 & 0 & -\beta \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} + \begin{bmatrix} 0 \\ -x_1(t)x_3(t) \\ x_1(t)x_2(t) \end{bmatrix} \quad (1)$$

where x_1, x_2 and x_3 are state variables, $x(t)=\left[x_1(t) \quad x_2(t) \quad x_3(t)\right]^T$, α, β and γ are three constant parameters, y the output signal, $y(t)=\left[y_1(t) \quad y_2(t)\right]^T$, and C a constant matrix, such that:

$$y(t)=Cx(t); \quad C=\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (2)$$

It is crucial to denote that the Chen system (1) exhibits a chaotic attractor for the following parameter values: $\alpha=35$, $\beta=3$ and $\gamma=28$.

Consider a master Chen system given by:

$$\begin{cases} \dot{x}_{m1}(t)=\alpha\left(x_{m2}(t)-x_{m1}(t)\right) \\ \dot{x}_{m2}(t)=\left(\gamma-\alpha-x_{m3}(t)\right)x_{m1}(t)+\gamma x_{m2}(t) \\ \dot{x}_{m3}(t)=x_{m1}(t)x_{m2}(t)-\beta x_{m3}(t) \end{cases} \quad (3)$$

which drives a slave Chen system described by:

$$\begin{cases} \dot{x}_{s1}(t)=\alpha\left(x_{s2}(t)-x_{s1}(t)\right) \\ \dot{x}_{s2}(t)=\left(\gamma-\alpha-x_{s3}(t)\right)x_{s1}(t)+\gamma x_{s2}(t)+u_1(t) \\ \dot{x}_{s3}(t)=x_{s1}(t)x_{s2}(t)-\beta x_{s3}(t)+u_2(t) \end{cases} \quad (4)$$

$u_i(t)$, $\forall i=1,2$, are the appropriate control functions to be determined.

Let the dynamical state error vector $e_S(t)$, $e_S(t)=\left[e_{S1}(t) \quad e_{S2}(t) \quad e_{S3}(t)\right]^T$, be such that:

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$$e_{Si}(t) = x_{mi}(t) - x_{si}(t) \quad \forall i = 1,2,3 \tag{5}$$

and leading to the error dynamics equations below:

$$\begin{cases} \dot{e}_{S1}(t) = \alpha\left(e_{S2}(t) - e_{S1}(t)\right) \\ \dot{e}_{S2}(t) = \left(\gamma - \alpha - x_{s3}(t)\right)e_{S1}(t) + \gamma e_{S2}(t) \\ \qquad -x_{m1}(t)e_{S3}(t) + u_1(t) \\ \dot{e}_{S3}(t) = x_{s2}(t)e_{S1}(t) + x_{m1}(t)e_{S2}(t) \\ \qquad -\beta e_{S3}(t) + u_2(t) \end{cases} \tag{6}$$

which can be rewritten in the following form:

$$\begin{bmatrix} \dot{e}_{S1}(t) \\ \dot{e}_{S2}(t) \\ \dot{e}_{S3}(t) \end{bmatrix} = \begin{bmatrix} \alpha\left(e_{S2}(t) - e_{S1}(t)\right) \\ \left(\gamma - \alpha - x_{s3}(t)\right)e_{S1}(t) + \gamma e_{S2}(t) - x_{m1}(t)e_{S3}(t) \\ x_{s2}(t)e_{S1}(t) + x_{m1}(t)e_{S2}(t) - \beta e_{S3}(t) \end{bmatrix} + Bu(t) \tag{7}$$

with:

$$B = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{8}$$

The synchronization of the coupled master-slave Chen chaotic dynamical system needs the stabilization of the error system (6), which can be achieved when the following nonlinear active feedback control laws:

$$u_i(t) = -\sum_{j=1}^{3} k_{ij}(.)e_{Sj}(t) \quad \forall i = 1,2 \tag{9}$$

are applied; $K(.) = \left\{k_{ij}(.)\right\}$, $\forall i = 1,2$ and $\forall j = 1,2,3$, the instantaneous control gain matrix.

Then, it comes:

$$\begin{cases} \dot{e}_{S1}(t) = \alpha\left(e_{S2}(t) - e_{S1}(t)\right) \\ \dot{e}_{S2}(t) = \left(\gamma - \alpha - x_{s3}(t) - k_{11}(.)\right)e_{S1}(t) + \left(\gamma - k_{12}(.)\right)e_{S2}(t) \\ \qquad -\left(x_{m1}(t) + k_{13}(.)\right)e_{S3}(t) \\ \dot{e}_{S3}(t) = \left(x_{s2}(t) - k_{21}(.)\right)e_{S1}(t) + \left(x_{m1}(t) - k_{22}(.)\right)e_{S2}(t) \\ \qquad -\left(\beta + k_{23}(.)\right)e_{S3}(t) \end{cases} \tag{10}$$

Therefore, the controlled dynamical error system can be described by:

$$\dot{e}_S(t) = A_c(.)e_S(t) \tag{11}$$

with:

$$A_c(.) = \begin{bmatrix} -\alpha & \alpha & 0 \\ \gamma - \alpha - x_{s3}(t) - k_{11}(.) & \gamma - k_{12}(.) & -x_{m1}(t) - k_{13}(.) \\ x_{s2}(t) - k_{21}(.) & x_{m1}(t) - k_{22}(.) & -\beta - k_{23}(.) \end{bmatrix} \tag{12}$$

So, the nonlinear matrix elements $a_{c_{ij}}(.)$, $A_c(.) = \left\{a_{c_{ij}}(.)\right\}$, $\forall i,j = 1,2,3$, depend on control gains, slave and master state variables.

Now, in order to study the stability of (10), our task is restricted to choose the control gains, in one hand, to simplify the complexity of the dynamical error system, and to make efficient the following proposed stability method, in the other hand.

B. Proposed Sufficient Stability Conditions

When the considered system (10) is stabilized by the state feedback control law u, the error will converge to zero as $t \to +\infty$; then, the systems (3) and (4) will be globally synchronized.

Taking into account the importance of arrow form choice for instantaneous characteristic matrices, to obtain useful sufficient stability conditions for nonlinear systems, as shown in [3,8-9], let design a suitable state feedback controller of system (4), so that the closed-loop system (11) being described by the following nonlinear differential equations form:

$$\begin{cases} \dot{e}_{S1}(t) = a_{c_{11}}(.)e_{S1}(t) + a_{c_{12}}(.)e_{S2}(t) + a_{c_{13}}(.)e_{S3}(t) \\ \dot{e}_{S2}(t) = a_{c_{21}}(.)e_{S1}(t) + a_{c_{22}}(.)e_{S2}(t) \\ \dot{e}_{S3}(t) = a_{c_{31}}(.)e_{S1}(t) + a_{c_{33}}(.)e_{S3}(t) \end{cases} \tag{13}$$

which leads to an instantaneous characteristic matrix under the arrow form, such that non zero elements are located in its main diagonal, its first row and its first column.

The application of the aggregation techniques [3,8-9], for the stability study, associated to the arrow form matrix, for the system description, leads to the following theorem.

Theorem. *The error system, described by (6) is stabilized by the nonlinear state feedback control law defined by (9), if the matrix $A_c(.)$, given by (12), is under the arrow form (13) and such that:*

i. the nonlinear elements are isolated in either one row or one column of the matrix $A_c(.)$,

ii. the diagonal elements, $a_{c_{ii}}(.)$, of the matrix $A_c(.)$ are such that :

$$a_{c_{ii}}(.) < 0 \quad \forall i = 2,3 \tag{14}$$

iii. there exist $\varepsilon > 0$, for which:

$$a_{c_{11}}(.) - \sum_{i=2}^{3}\left(\left|a_{c_{1i}}(.)a_{c_{i1}}(.)\right|\right)a_{c_{ii}}^{-1}(.) \le -\varepsilon \tag{15}$$

Proof. The overvaluing system $M\left(A_c(.)\right)$, associated to the vectorial norm $p(z) = \left[|z_1| \quad |z_2| \quad |z_3|\right]^T$, $z = \left[z_1 \quad z_2 \quad z_3\right]^T$, is defined, in this case, by the following system of differential equations:

$$\dot{z}(t) = M\left(A_c(.)\right)z(t) \tag{16}$$

such that the elements $m_{ij}(.)$ of $M\left(A_c(.)\right)$ are

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deduced from the ones of the matrix $A_c(.)$ by substituting the off-diagonal elements by their absolute values, such that:

$$\begin{cases} m_{ii}(.) = a_{c_{ii}}(.) \quad \forall i = 1,2,3 \\ m_{ij}(.) = \left|a_{c_{ij}}(.)\right| \quad \forall i,j = 1,2,3,\ i \neq j \end{cases} \tag{17}$$

The system (6) is then stabilized by (9), if the matrix $M\left(A_c(.)\right)$ is the opposite of an $M-$ matrix [3,8-9], or if, by application of the aggregation techniques, the sufficient stability conditions, for $\varepsilon > 0,$ are formulated in the subsequent manner:

$$\begin{cases} a_{c_{ii}}(.) \leq -\varepsilon \quad \forall i = 2,3 \\ (-1)^3 \det\left(M\left(A_c(.)\right)\right) \geq \varepsilon \end{cases} \tag{18}$$

The following development of the first member of the last inequality (18):

$$(-1)^3 \det\left(M\left(A_c(.)\right)\right) = (-1)\left(a_{c_{11}}(.) - \sum_{i=2}^{3}\left(\left|a_{c_{1i}}(.)a_{c_{i1}}(.)\right|\right)a_{c_{ii}}^{-1}(.)\right)(-1)^2\prod_{j=2}^{3}a_{c_{jj}}(.) \tag{19}$$

achieves easily the proof of the theorem.

C. Application of the proposed stability conditions to synchronize the coupled master-slave Chen chaotic system

The concept of chaos synchronization emerged much later, not until the gradual realization of the usefulness of chaos by scientists and engineers. Synchrony is the simplest effect of coupled identical systems: two identical systems display the same dynamical pattern in their common phase space [3]. For that reason, the developed state feedback control technique is applied, in this subsection, to achieve chaos synchronization of two identical Chen systems.

In fact, the characterization of the closed-loop system (11) by an arrow form matrix is easily checked by choosing the correction parameters $k_{13}(.)$ and $k_{22}(.)$ such that:

$$\begin{cases} k_{13}(.) = -x_{m1}(t) \\ k_{22}(.) = x_{m1}(t) \end{cases} \tag{20}$$

To satisfy the assumption (i) as well as the constraints (14) of the above-mentioned theorem, the two gain parameters k_{12} and k_{23} are chosen linear, such that:

$$\begin{cases} \gamma - k_{12} < 0 \\ -\beta - k_{23} < 0 \end{cases} \tag{21}$$

Then, it remains only to fulfil the condition (15), expressed in this case as follows:

$$-\alpha - \left(\left|\alpha\left(\gamma - \alpha - x_{s3}(t) - k_{11}(.)\right)\right|\left(\gamma - k_{12}\right)^{-1}\right) < 0 \tag{22}$$

to guarantee the asymptotic stability of the dynamical error system (10).

So, $\forall\ k_{21}(.)$ and for $\beta > 0,$ one possible choice of the other gain parameters is given by:

$$\begin{cases} k_{11}(.) = -x_{s3}(t) \\ k_{12} = 2\gamma \\ k_{23} = 0 \end{cases} \tag{23}$$

Hence, the instantaneous control gain matrix $K(.)$ can be chosen as:

$$K(.) = \begin{bmatrix} -x_{s3}(t) & 2\gamma & -x_{m1}(t) \\ 0 & x_{m1}(t) & 0 \end{bmatrix} \tag{24}$$

This choice ensures that the synchronization between systems (3) and (4) is achieved. This is also confirmed by the exponential convergence of the synchronization quality defined by the error propagation on the error states:

$$\left\|e_S(t)\right\| = \sqrt{e_{S1}^2(t) + e_{S2}^2(t) + e_{S3}^2(t)} \tag{25}$$

Fig. 1. shows the error dynamics in the uncontrolled state, while Fig. 2. illustrates the error dynamics when controller is switched on.

Obviously, the two chaotic Chen systems evolve in the same direction as well as the same amplitude; that's to say, they are globally asymptotically synchronized by means of the proposed nonlinear state feedback controller.

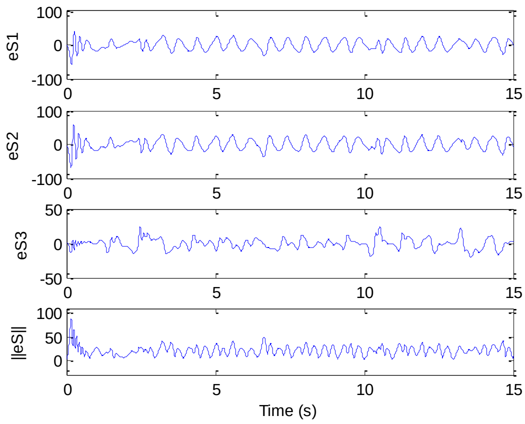

Fig. 1. Error dynamics of the coupled master-slave Chen system when controller is deactivated

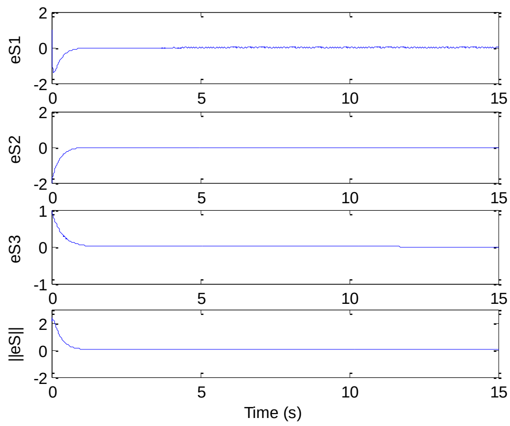

Fig. 2. Error dynamics of the coupled master-slave Chen system when controller is switched on

III. OBSERVER-BASED SYNCHRONIZATIONFOR COUPLED CHAOTIC SYSTEMS

A. Nonlinear observer design

Continuous time chaotic systems are generally described by a set of nonlinear difference equations. It is convenient to separate the dynamics into linear and nonlinear parts and to find an appropriate observer intended to synchronize two coupled, drive and response, chaotic systems.

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Let us consider the class of continuous time chaotic drive systems expressed by the following state space representation:

$$\begin{cases} \dot{x}_d(t) = Ax_d(t) + f\left(x_d(t)\right) \\ y_d(t) = Cx_d(t) \end{cases} \quad (26)$$

where x_d is the state vector, $x_d \in \mathbb{R}^n$, A a constant matrix, $A = \{a_{ij}\}$, $\forall i, j = 1,\ldots,n$, $f(.)$ a nonlinear n vector function, y_d the p output vector, $y_d \in \mathbb{R}^p$ and C a $p \times n$ constant matrix.

Then, as a response system, let consider the proposed nonlinear observer candidate, designed as follows:

$$\begin{cases} \dot{x}_r(t) = Ax_r(t) + f\left(x_r(t)\right) + L(.)\left(y_d(t) - y_r(t)\right) \\ y_r(t) = Cx_r(t) \end{cases} \quad (27)$$

where $x_r \in \mathbb{R}^n$, $y_r \in \mathbb{R}^p$, are respectively the state vector and the output vector of the nonlinear observer-based response system.

The $n \times p$ nonlinear matrix of observer gains $L(.)$ has to be chosen so that the dynamical observer-based synchronization error $e(t)$, $e(t) = \left[e_1(t) \quad e_2(t) \quad e_3(t)\right]^T$, between system (26) and system (27), defined by (28), being such that:

$$\lim_{t \to +\infty} e_i(t) = \lim_{t \to +\infty} \left(x_{ri}(t) - x_{di}(t)\right) = 0 \ \forall i = 1,\ldots,n \quad (28)$$

So, in order to study the synchronization property of the two considered drive and response chaotic systems, the following error system of differential equations is taken into account:

$$\dot{e}(t) = \dot{x}_r(t) - \dot{x}_d(t) \quad (29)$$

By substituting the terms $\dot{x}_d$ and $\dot{x}_r$ by their values according to the differential equations given by (26) and (27), we can easily obtain the dynamical error system description below:

$$\dot{e}(t) = \left(A - L(.)C\right)e(t) + f\left(x_r(t)\right) - f\left(x_d(t)\right) \quad (30)$$

Consequently, when this dynamical error system (30) is asymptotically stable, the synchronization error will converge to zero as time $t \to +\infty$, which implies that the observer-based response system (27) is globally synchronized with the drive system described by (26).

In the particular case, where there exist an $n \times n$ instantaneous matrix $F(.)$, such that:

$$f\left(x_r(t)\right) - f\left(x_d(t)\right) = F(.)e(t) \quad (31)$$

we can rearrange the description (30) in the following way:

$$\dot{e}(t) = \left(A - L(.)C + F(.)\right)e(t) \quad (32)$$

Hence, in this stage, we need only to determine, conveniently, the nonlinear observer matrix so that the instantaneous characteristic matrix of the closed-loop system (32), by respect to (31), being under the arrow form [3,8-9], to assure the synchronization property between drive and response Chen chaotic systems (26) and (27), respectively.

B. Application to the case of two coupled chaotic Chen systems

The proposed observer-based synchronization approach is considered, here, for the chaotic Chen system, described by (1) and (2).

Let consider a drive Chen system given by:

$$\begin{bmatrix} \dot{x}_{d1}(t) \\ \dot{x}_{d2}(t) \\ \dot{x}_{d3}(t) \end{bmatrix} = \begin{bmatrix} -\alpha & \alpha & 0 \\ \gamma - \alpha & \gamma & 0 \\ 0 & 0 & -\beta \end{bmatrix} \begin{bmatrix} x_{d1}(t) \\ x_{d2}(t) \\ x_{d3}(t) \end{bmatrix} + \begin{bmatrix} 0 \\ -x_{d1}(t)x_{d3}(t) \\ x_{d1}(t)x_{d2}(t) \end{bmatrix} \quad (33)$$

and:

$$\begin{bmatrix} y_{d1}(t) \\ y_{d2}(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_{d1}(t) \\ x_{d2}(t) \\ x_{d3}(t) \end{bmatrix} \quad (34)$$

for which, a similar response Chen system is described as follows:

$$\begin{bmatrix} \dot{x}_{r1}(t) \\ \dot{x}_{r2}(t) \\ \dot{x}_{r3}(t) \end{bmatrix} = \begin{bmatrix} -\alpha & \alpha & 0 \\ \gamma - \alpha & \gamma & 0 \\ 0 & 0 & -\beta \end{bmatrix} \begin{bmatrix} x_{r1}(t) \\ x_{r2}(t) \\ x_{r3}(t) \end{bmatrix} + \begin{bmatrix} 0 \\ -x_{r1}(t)x_{r3}(t) \\ x_{r1}(t)x_{r2}(t) \end{bmatrix} + \begin{bmatrix} l_{11}(.) & l_{12}(.) \\ l_{21}(.) & l_{22}(.) \\ l_{31}(.) & l_{32}(.) \end{bmatrix} \begin{bmatrix} y_{d1}(t) - y_{r1}(t) \\ y_{d2}(t) - y_{r2}(t) \end{bmatrix} \quad (35)$$

and:

$$\begin{bmatrix} y_{r1}(t) \\ y_{r2}(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_{r1}(t) \\ x_{r2}(t) \\ x_{r3}(t) \end{bmatrix} \quad (36)$$

In this case, it is obvious that the dynamic error system can be written in the following compact state space description:

$$\dot{e}(t) = A_e(.)e(t) \quad (37)$$

with:

$$A_e(.) = A - L(.)C + F(.) \quad (38)$$

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$$A_e(.) = \begin{bmatrix} -\alpha & \alpha - l_{11}(.) & -l_{12}(.) \\ \gamma - \alpha - x_{r3}(t) & \gamma - l_{21}(.) & -x_{d1}(t) - l_{22}(.) \\ x_{r2}(t) & x_{d1}(t) - l_{31}(.) & -\beta - l_{32}(.) \end{bmatrix} \quad (39)$$

and:

$$F(.) = \begin{bmatrix} 0 & 0 & 0 \\ -x_{r3}(t) & 0 & -x_{d1}(t) \\ x_{r2}(t) & x_{d1}(t) & 0 \end{bmatrix} \quad (40)$$

First of all, and by respect to the stabilisability conditions announced in the above-mentioned theorem, relatively to the instantaneous matrix $A_e(.)$ instead of $A_c(.)$, the dynamic error system (37) must be characterized by an instantaneous arrow form matrix $A_e(.)$, that is to say, the main requirements, concerning the choice of the instantaneous observer gains $l_{22}(.)$ and $l_{31}(.)$, are given by:

$$\begin{cases} l_{22}(.) = -x_{d1}(t) \\ l_{31}(.) = x_{d1}(t) \end{cases} \quad (41)$$

Furthermore, by adopting the previous choices of $l_{22}(.)$ and $l_{31}(.)$, the remaining parameters l_{11}, l_{12}, l_{21} and l_{32} are chosen linear in order to isolate all the nonlinearities in the first column of $A_e(.)$.

Indeed, the two last diagonal elements of the characteristic matrix $A_e(.)$ have to fulfil the inequalities (14), namely:

$$\begin{cases} \gamma - l_{21} < 0 \\ -\beta - l_{32} < 0 \end{cases} \quad (42)$$

For this purpose, and by taking into account the fact that $\beta > 0$, one possible solution is expressed by:

$$\begin{cases} l_{21} = 2\gamma \\ l_{32} = 0 \end{cases} \quad (43)$$

Finally, it is relevant to denote that to satisfy the observer-based synchronization property of the two coupled drive and response chaotic Chen systems, it is necessary to tune the remaining design observer gains parameters in the system (37-40), explicitly characterized by (39), such that the following stability condition is true:

$$\left(-\alpha - \left(\begin{array}{l} \left|(\alpha - l_{11})(\gamma - \alpha - x_{r3}(t))\right|(-\gamma)^{-1} \\ +\left|x_{r2}(t) l_{12}\right|(-\beta)^{-1} \end{array} \right) \right) < 0 \quad (44)$$

From various possible solutions, let consider $L(.)$ defined by:

$$L(.) = \begin{bmatrix} \alpha & 0 \\ 2\gamma & -x_{d1}(t) \\ x_{d1}(t) & 0 \end{bmatrix} \quad (45)$$

Through the illustration of both drive and response states evolutions, without observer gains, Fig. 3., it is noticeable that these states aren't yet synchronized and grow with time chaotically, with different amplitudes.

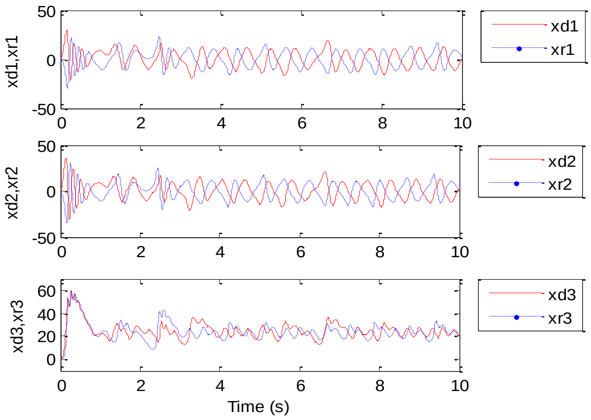

Fig. 3. Error dynamics between the drive and the response Chen systems when observer gains are switched off

Therefore, with the adequately designed observer-based response system, we clearly show that under mild conditions, the response Chen system (35) and (36) traces the dynamics of the drive Chen system (33) and (34) as they achieve synchronous states shown in Fig. 4, in which, both systems oscillate in a synchronized manner within a shorter time, with an exponentially decaying of the synchronization quality defined by the error propagation on the dynamical error states, expressed by $\|e(t)\| = \sqrt{e_1^2(t) + e_2^2(t) + e_3^2(t)}$, and presented in Fig. 5., below. Thus, the required synchronization has been provided thanks to our judiciously designed nonlinear observer.

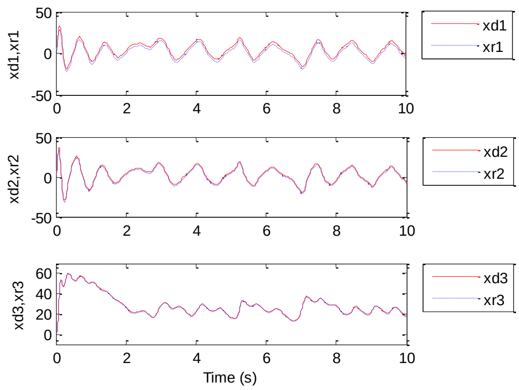

Fig. 4. Observer-based synchronization between the two coupled drive and response chaotic Chen systems

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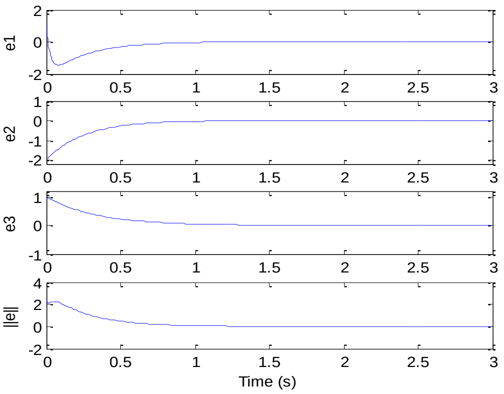

Fig. 5. Exponential convergence of the error dynamics thanks to the adequate observer-based response chaotic Chen system

Hence, the observer-based proposed method arises two main advantages. First, due to a suitable choice of nonlinear observer gains, the synchronization is achieved within a short time. Second, our scheme covers chaotic systems that comprise nonlinearities that are non Lipschitzian with respect to the state, which is rather requested by many synchronizing schemes in literature.

IV. CONCLUSION

In this paper, the synchronization is achieved for two identical coupled chaotic systems. Under some structural assumptions of the drive system and based on aggregation techniques associated to the arrow form matrix, an observer-based response system is designed to assure that the property of synchronization is successfully reached. The simulation results demonstrate the feasibility of the proposed approaches to synchronize two Chen chaotic systems, within shorter time.

REFERENCES

1. L. M. Pecora and T. L. Carroll, “Synchronization in chaotic systems”, Phys Rev Lett, vol. 64, 1990, pp. 821-824.
2. M. Ogorzalek, “Taming chaos-I: Synchronization”, IEEE Trans Circ Syst-I, vol. 40, 1993, pp. 693-699.
3. S. Hammami, K. Ben Saad and M. Benrejeb, “On the synchronization of identical and non-identical 4-D chaotic systems using arrow form matrix”, Chaos, Solitons & Fractals, vol. 42, 2009, pp. 101-112.
4. M. Hasler, “Synchronization principles and applications”, in IEEE international symposium on circuits and systems, 1994, vol. 3, pp. 314-327, New York.
5. H. Xiping and Q. Zhang, “Image encryption based on chaotic modulation of wavelet coefficients”, in Congress on IEEE Image and Signal Processing, 2008, vol. 1, pp. 622-626, Sanya, Hainan.
6. L. Kocarev and U. Parlitz, “Generalized synchronization, predictability, and equivalence of unidirectionally coupled dynamical systems”, Phys Rev Lett, vol. 76, 1996, pp. 1816-1819.
7. S. S. Yang and K. Duan, “Generalized synchronization in chaotic systems”, Chaos, Solitons & Fractals, vol. 10, 1998, pp. 1703-1707.
8. S. Hammami, M. Benrejeb, M. Feki and P. Borne, “Feedback control design for Rössler and Chen chaotic systems anti-synchronization”, Phys Lett A, vol. 374, 2010, pp. 2835-2840.
9. S. Hammami and M. Benrejeb, “Coexistence of synchronization and anti-synchronization for chaotic systems via feedback control”, Chaotic systems, Croatia: Editions INTECH, 2011, pp. 203-224.
10. G. H. Li, “Synchronization and anti-synchronization of Colpitts oscillators using active control”, Chaos, Solitons & Fractals, vol. 26, 2005, pp. 87-93.
11. Ö. Morgül and E. Solak, “Observer-based synchronization of chaotic signals”, Phys Rev E, vol. 54, 1996, pp. 4803-4811.
12. Ö. Morgül and E. Solak, “On the synchronization of chaotic systems by using state observers”, Int J Bifurcation Chaos, vol. 7, 1997, pp. 1307-1322.
13. H. Nijmeijer and I. Mareels, “An observer looks at synchronization”, IEEE Trans Circ Syst-I, vol. 44, 1997, pp. 882-890.

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