Systems with Negative Fuzzy Parameters

Purnima K. Pandit

***Abstract*— Various primitive Engineering and Sciences applications can be modeled using system of linear equations. In such models it can happen that the values of the parameters are not known or they cannot be stated precisely only their estimation due to experimental data or expert knowledge is available. In such situation it is convenient to represent such parameters by fuzzy numbers (refer [14]). The method and the conditions for obtaining the fuzzy solution for the systems with positive fuzzy parameters is given in [10]. There are applications wherein the model involves even negative fuzzy parameters. In this paper, an algorithm for solving such fuzzy systems is proposed and illustrated.**

***Index Terms*—negative fuzzy numbers, α-cut, parallel processing, fully fuzzy systems.**

I. INTRODUCTION

In the mathematical model if there is the imprecise and non probabilistic uncertainties in the parameters then it is convenient to represent them using fuzzy numbers (e.g. Triangular, Trapezoidal, Gaussian etc.).

There are applications which have the simple form as system of linear equations. If such systems involves the above mentioned imprecision then they can be represented by the system of linear equations involving fuzzy parameters. Such systems have become more pervasive in various fields instead of their crisp counter parts, as in [11] wherein circuit analysis is done using fuzzy complex system, [8] demonstrates the use of fuzzy system of linear equations in Economics but such systems involve negative fuzzy numbers.

The solution of the fuzzy systems with the crisp coefficient matrix and fuzzy right-hand side column was first proposed by Freidman et al.[6]. Fully fuzzy system of linear equations is another important class of systems in which all the parameters involved are fuzzy. Solution of such systems using LU decomposition [1], Gram-Schmidt approach to solve such systems [9], and other approaches [3], [4] etc. were proposed. Here many of the stated methods can solve systems involving only positive fuzzy numbers.

Recently, Allahviranloo [2] made a note that technique suggested by Friedman [6] gives only the weak solutions for the fuzzy systems, which may be fuzzy, only in some cases. The result for the existence and uniqueness of the fuzzy solutions for fully fuzzy systems involving positive fuzzy numbers is proved in [10].

In this paper, we consider fully fuzzy linear system of the form $\tilde{A}\tilde{\mathbf{x}} = \tilde{\mathbf{b}}$ wherein the coefficients and RHS both are represented by fuzzy numbers; we extend the results when the coefficients can be even negative fuzzy numbers. The merit of this method is that the fuzzy system is partitioned into two crisp subsystems using α-cut. These partitioned subsystems can be solved individually; also parallel computing algorithms [13] are applicable for large systems.

The paper is organized in the following manner, initially the preliminaries are listed, and then the reduction of fuzzy system into the crisp using α-cut is computed. The next section gives the algorithm for solving the system fuzzy parameters, where the fuzzy coefficients may be negative. The proposed algorithm is used to solve the illustrative example.

II. PRELIMINARIES

A. Definition: Fuzzy number

Let us denote by R_F the class of fuzzy subsets of the real axis (i.e. $u : R \to [0, 1]$) satisfying the following properties:

(i) $\forall u \in R_F$, u is normal, i.e. $\exists x_0 \in R$ with $u(x_0) = 1$;

(ii) $\forall u \in R_F$, u is convex fuzzy set, That is,

$$u(tx + (1-t)y) \geq \min\{u(x), u(y)\},\ \forall t \in [0,1], x, y \in R_F$$

(iii) $u \in R_F$, u is upper semi-continuous on R;

(iv) $\overline{\{x \in R; u(x) > 0\}}$ is compact, where $\overline{A}$ denotes the closure of A.

Then R_F is called the space of fuzzy numbers, as in [2]. Obviously $R \subset R_F$ as R can be regarded as $\{ \chi_x : x$ is any usual real number$\}$.

B. Definition: Non negative fuzzy number

A fuzzy number $\tilde{u}$ is said to be non-negative fuzzy number if $\tilde{u}(x) = 0,\ \forall x < 0$ and $\tilde{u}$ is said to be positive if $\tilde{u}(x) = 0,\ \forall x \leq 0$.

C. Definition: Non positive fuzzy number

A fuzzy number $\tilde{u}$ is said to be non-positive fuzzy number if $\tilde{u}(x) = 0,\ \forall x > 0$ and $\tilde{u}$ is said to be positive if $\tilde{u}(x) = 0,\ \forall x \geq 0$.

D. Definition: Fuzzy matrix

A matrix $\tilde{A} = (\tilde{a}_{ij})$ is called a fuzzy matrix, if each element of $\tilde{A}$ is a fuzzy number.

We may represent $n \times m$ fuzzy matrix $\tilde{A} = (\tilde{a}_{ij})_{n \times m}$, where $\tilde{a}_{ij}$ is a trapezoidal fuzzy number denoted as, $\tilde{a}_{ij} = (b_{ij}, c_{ij}, d_{ij}, e_{ij})$ and defined as

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$$\tilde{a}_{ij} = \begin{cases} \dfrac{(x-b_{ij})}{(c_{ij}-b_{ij})} & b_{ij} < x \le c_{ij} \\ 1 & c_{ij} < x \le d_{ij} \\ \dfrac{(e_{ij}-x)}{(e_{ij}-d_{ij})} & d_{ij} < x \le e_{ij} \\ 0 & otherwise \end{cases}$$

III. OPERATIONS ON FUZZY NUMBERS

A. α-cut / level-cut of fuzzy number:

For 0 < α ≤ 1, denote the α-cut as, $[u]^{\alpha} = \{x \in R : u(x) \ge \alpha\}$ and $[u]^{0} = \overline{\{x \in R : u(x) > 0\}}$.

Then it is well-known that for each, $\alpha \in [0,1]$, the α-cut, $[u]^{\alpha}$ is a bounded closed interval $\left[{}^{\alpha}\underline{u}, {}^{\alpha}\overline{u}\right]$ in *R*.

The addition of the two fuzzy numbers and the scalar multiplication is defined using α-cut, $\forall \alpha \in [0,1]$ as given below, refer [5].

B. *Addition of two fuzzy numbers:*

For $u, v \in R_F$, the sum *u* + *v* is obtained using α-cut as the α-cut of sum of two fuzzy numbers is sum of their α-cuts. That is, $[u+v]^{\alpha} = [u]^{\alpha} + [v]^{\alpha} =$ $\left[{}^{\alpha}\underline{u}, {}^{\alpha}\overline{u}\right] + \left[{}^{\alpha}\underline{v}, {}^{\alpha}\overline{v}\right] = \left[{}^{\alpha}\underline{u} + {}^{\alpha}\underline{v}, {}^{\alpha}\overline{u} + {}^{\alpha}\overline{v}\right]$.

C. *Scalar multiplication with fuzzy number*:

$\lambda \in R^{+}$, the product $\lambda \cdot u$ is given by, $[\lambda \cdot u]^{\alpha} = \lambda [u]^{\alpha}$, $\lambda \left[{}^{\alpha}\underline{u}, {}^{\alpha}\overline{u}\right] = \left[\lambda\, {}^{\alpha}\underline{u}, \lambda\, {}^{\alpha}\overline{u}\right]$.

IV. SYSTEM REDUCTION USING LEVEL-CUTS

Consider the fully fuzzy system in 2-dimension,

$$\tilde{A}\tilde{\mathbf{x}} = \tilde{\mathbf{b}} \tag{1}$$

i.e.

$$\begin{pmatrix} \tilde{a}_{11} & \tilde{a}_{12} \\ \tilde{a}_{21} & \tilde{a}_{22} \end{pmatrix} \otimes \begin{pmatrix} \tilde{x}_1 \\ \tilde{x}_2 \end{pmatrix} = \begin{pmatrix} \tilde{b}_1 \\ \tilde{b}_2 \end{pmatrix}$$

Using the α-cut of the fuzzy elements, we get $\forall \alpha \in [0,1]$,

$${}^{\alpha}\tilde{A}\,{}^{\alpha}\tilde{\mathbf{x}} = {}^{\alpha}\tilde{\mathbf{b}}$$

That is,

$$\left(\begin{bmatrix} {}^{\alpha}\underline{a}_{11}, {}^{\alpha}\overline{a}_{11} \\ {}^{\alpha}\underline{a}_{21}, {}^{\alpha}\overline{a}_{21} \end{bmatrix} \begin{bmatrix} {}^{\alpha}\underline{a}_{12}, {}^{\alpha}\overline{a}_{12} \\ {}^{\alpha}\underline{a}_{22}, {}^{\alpha}\overline{a}_{22} \end{bmatrix}\right) \otimes \left(\begin{bmatrix} {}^{\alpha}\underline{x}_1, {}^{\alpha}\overline{x}_1 \\ {}^{\alpha}\underline{x}_2, {}^{\alpha}\overline{x}_2 \end{bmatrix}\right) = \left(\begin{bmatrix} {}^{\alpha}\underline{b}_1, {}^{\alpha}\overline{b}_1 \\ {}^{\alpha}\underline{b}_2, {}^{\alpha}\overline{b}_2 \end{bmatrix}\right)$$

which becomes,

$$\left[{}^{\alpha}\underline{a}_{11}, {}^{\alpha}\overline{a}_{11}\right] \otimes \left[{}^{\alpha}\underline{x}_1, {}^{\alpha}\overline{x}_1\right] \oplus \left[{}^{\alpha}\underline{a}_{12}, {}^{\alpha}\overline{a}_{12}\right] \otimes \left[{}^{\alpha}\underline{x}_2, {}^{\alpha}\overline{x}_2\right] = \left[{}^{\alpha}\underline{b}_1, {}^{\alpha}\overline{b}_1\right]$$

$$\left[{}^{\alpha}\underline{a}_{21}, {}^{\alpha}\overline{a}_{21}\right] \otimes \left[{}^{\alpha}\underline{x}_1, {}^{\alpha}\overline{x}_1\right] \oplus \left[{}^{\alpha}\underline{a}_{22}, {}^{\alpha}\overline{a}_{22}\right] \otimes \left[{}^{\alpha}\underline{x}_2, {}^{\alpha}\overline{x}_2\right] = \left[{}^{\alpha}\underline{b}_2, {}^{\alpha}\overline{b}_2\right]$$

If each a_{ij} is positive using the operations on the intervals obtained after applying α-cut, the above system can be put into crisp system of linear equations as,

$${}^{\alpha}\underline{a}_{11}\,{}^{\alpha}\underline{x}_1 + {}^{\alpha}\underline{a}_{12}\,{}^{\alpha}\underline{x}_2 = {}^{\alpha}\underline{b}_1$$

$${}^{\alpha}\underline{a}_{21}\,{}^{\alpha}\underline{x}_1 + {}^{\alpha}\underline{a}_{22}\,{}^{\alpha}\underline{x}_2 = {}^{\alpha}\underline{b}_2$$

$${}^{\alpha}\overline{a}_{11}\,{}^{\alpha}\overline{x}_1 + {}^{\alpha}\overline{a}_{12}\,{}^{\alpha}\overline{x}_2 = {}^{\alpha}\overline{b}_1$$

$${}^{\alpha}\overline{a}_{21}\,{}^{\alpha}\overline{x}_1 + {}^{\alpha}\overline{a}_{22}\,{}^{\alpha}\overline{x}_2 = {}^{\alpha}\overline{b}_2$$

Modelling of various real life applications may involve even negative fuzzy numbers. To obtain fuzzy solution for such systems after taking α-cut, for the negative parameters interchange $\underline{a}_{ij}$ with $\overline{a}_{ij}$.

That is the system to be solved is:

$$\begin{pmatrix} {}^{\alpha}\underline{a}_{11} & {}^{\alpha}\underline{a}_{12} & 0 & 0 \\ {}^{\alpha}\underline{a}_{21} & {}^{\alpha}\underline{a}_{22} & 0 & 0 \\ 0 & 0 & {}^{\alpha}\overline{a}_{11} & {}^{\alpha}\overline{a}_{12} \\ 0 & 0 & {}^{\alpha}\overline{a}_{21} & {}^{\alpha}\overline{a}_{22} \end{pmatrix} \begin{pmatrix} {}^{\alpha}\underline{x}_1 \\ {}^{\alpha}\underline{x}_2 \\ {}^{\alpha}\overline{x}_1 \\ {}^{\alpha}\overline{x}_2 \end{pmatrix} = \begin{pmatrix} {}^{\alpha}\underline{b}_1 \\ {}^{\alpha}\underline{b}_2 \\ {}^{\alpha}\overline{b}_1 \\ {}^{\alpha}\overline{b}_2 \end{pmatrix}$$

with the entries for the negative fuzzy numbers in the coefficient matrix being interchanged.

Using,

$$A = \begin{pmatrix} {}^{\alpha}\underline{a}_{11} & {}^{\alpha}\underline{a}_{12} & 0 & 0 \\ {}^{\alpha}\underline{a}_{21} & {}^{\alpha}\underline{a}_{22} & 0 & 0 \\ 0 & 0 & {}^{\alpha}\overline{a}_{11} & {}^{\alpha}\overline{a}_{12} \\ 0 & 0 & {}^{\alpha}\overline{a}_{21} & {}^{\alpha}\overline{a}_{22} \end{pmatrix}, \mathbf{x} = \begin{pmatrix} {}^{\alpha}\underline{x}_1 \\ {}^{\alpha}\underline{x}_2 \\ {}^{\alpha}\overline{x}_1 \\ {}^{\alpha}\overline{x}_2 \end{pmatrix} \text{ and } \mathbf{b} = \begin{pmatrix} {}^{\alpha}\underline{b}_1 \\ {}^{\alpha}\underline{b}_2 \\ {}^{\alpha}\overline{b}_1 \\ {}^{\alpha}\overline{b}_2 \end{pmatrix}$$

We get crisp system of the form

$$A \times \mathbf{x} = \mathbf{b} \tag{2}$$

where, the coefficient matrix, *A* is of dimension $2n \times 2n$, **x** and **b** are column vectors of dimension 2*n*.

Observe that (2) can be partitioned into two systems as:

$$\begin{pmatrix} {}^{\alpha}\underline{a}_{11} & {}^{\alpha}\underline{a}_{12} \\ {}^{\alpha}\underline{a}_{21} & {}^{\alpha}\underline{a}_{22} \end{pmatrix} \begin{pmatrix} {}^{\alpha}\underline{x}_1 \\ {}^{\alpha}\underline{x}_2 \end{pmatrix} = \begin{pmatrix} {}^{\alpha}\underline{b}_1 \\ {}^{\alpha}\underline{b}_2 \end{pmatrix}, \begin{pmatrix} {}^{\alpha}\overline{a}_{11} & {}^{\alpha}\overline{a}_{12} \\ {}^{\alpha}\overline{a}_{21} & {}^{\alpha}\overline{a}_{22} \end{pmatrix} \begin{pmatrix} {}^{\alpha}\overline{x}_1 \\ {}^{\alpha}\overline{x}_2 \end{pmatrix} = \begin{pmatrix} {}^{\alpha}\overline{b}_1 \\ {}^{\alpha}\overline{b}_2 \end{pmatrix}$$

That is, $\underline{A} \times \underline{\mathbf{x}} = \underline{\mathbf{b}}$ and $\overline{A} \times \overline{\mathbf{x}} = \overline{\mathbf{b}}$. The solution of these crisp systems determines ${}^{\alpha}\underline{x}_i$ and ${}^{\alpha}\overline{x}_i$, which allows us to construct the fuzzy solution of the system (2) if the conditions stated in the following result is satisfied.

A Theorem: The fuzzy solution to (2) exists if the following conditions are satisfied:

(i) $\forall \alpha \in [0,1]$

$$\begin{pmatrix} {}^{\alpha}\underline{a}_{11} & {}^{\alpha}\underline{a}_{12} \\ {}^{\alpha}\underline{a}_{21} & {}^{\alpha}\underline{a}_{22} \end{pmatrix}^{-1} \begin{pmatrix} {}^{\alpha}\underline{b}_1 \\ {}^{\alpha}\underline{b}_2 \end{pmatrix} \le \begin{pmatrix} {}^{\alpha}\overline{a}_{11} & {}^{\alpha}\overline{a}_{12} \\ {}^{\alpha}\overline{a}_{21} & {}^{\alpha}\overline{a}_{22} \end{pmatrix}^{-1} \begin{pmatrix} {}^{\alpha}\overline{b}_1 \\ {}^{\alpha}\overline{b}_2 \end{pmatrix}$$

(ii) $\forall \alpha, \beta \in [0,1], \alpha \le \beta$

$$\begin{pmatrix} {}^{\alpha}\underline{a}_{11} & {}^{\alpha}\underline{a}_{12} \\ {}^{\alpha}\underline{a}_{21} & {}^{\alpha}\underline{a}_{22} \end{pmatrix}^{-1} \begin{pmatrix} {}^{\alpha}\underline{b}_1 \\ {}^{\alpha}\underline{b}_2 \end{pmatrix} \le \begin{pmatrix} {}^{\beta}\underline{a}_{11} & {}^{\beta}\underline{a}_{12} \\ {}^{\beta}\underline{a}_{21} & {}^{\beta}\underline{a}_{22} \end{pmatrix}^{-1} \begin{pmatrix} {}^{\beta}\underline{b}_1 \\ {}^{\beta}\underline{b}_2 \end{pmatrix}$$

$$\le \begin{pmatrix} {}^{\beta}\overline{a}_{11} & {}^{\beta}\overline{a}_{12} \\ {}^{\beta}\overline{a}_{21} & {}^{\beta}\overline{a}_{22} \end{pmatrix}^{-1} \begin{pmatrix} {}^{\beta}\overline{b}_1 \\ {}^{\beta}\overline{b}_2 \end{pmatrix} \le \begin{pmatrix} {}^{\alpha}\overline{a}_{11} & {}^{\alpha}\overline{a}_{12} \\ {}^{\alpha}\overline{a}_{21} & {}^{\alpha}\overline{a}_{22} \end{pmatrix}^{-1} \begin{pmatrix} {}^{\alpha}\overline{b}_1 \\ {}^{\alpha}\overline{b}_2 \end{pmatrix}$$

Proof: Proof can be given on similar line as in [10]. Also, if the conditions given in the statement are satisfied then for each *i* = 1,2

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(i) $\forall \alpha \in [0,1],\ {}^{\alpha}\underline{x}_i \le {}^{\alpha}\overline{x}_i$

(ii) $\forall \alpha, \beta \in [0,1], \alpha \le \beta,\ {}^{\alpha}\underline{x}_i \le {}^{\beta}\underline{x}_i \le {}^{\beta}\overline{x}_i \le {}^{\alpha}\overline{x}_i$

Hence, $\forall \alpha \in [0,1],\ \tilde{x}_i = \left[{}^{\alpha}\underline{x}_i, {}^{\alpha}\overline{x}_i\right]$, for i = 1,2 where,

- ${}^{\alpha}\underline{x}_i$ is a bounded left continuous non-decreasing function over [0,1].
- ${}^{\alpha}\overline{x}_i$ is a bounded left continuous non-increasing function over [0,1].

Thus, the solution of the system (2) satisfying the above conditions would indeed generate the components of fuzzy solution vector $\tilde{\mathbf{x}}$ for the system (1).

The components of fuzzy $\tilde{\mathbf{x}}$ are reconstructed as shown by the lemma below.

B. Lemma: The i^{th} component of the fuzzy solution vector, $\tilde{\mathbf{x}}$ of the fully fuzzy system (1) can be reconstructed form the components ${}^{\alpha}\underline{x}_i$ and ${}^{\alpha}\overline{x}_i$ of the crisp system (2) and is given as

$$\tilde{x}_i = \bigcup_{\alpha\in[0,1]} {}_{\alpha}\tilde{x}_i$$

where, ${}_{\alpha}\tilde{x}_i = \alpha \cdot {}^{\alpha}\tilde{x}_i$ and ${}^{\alpha}\tilde{x}_i = \left[{}^{\alpha}\underline{x}_i, {}^{\alpha}\overline{x}_i\right]$.

Proof: For each particular $y \in R$, let $a = \tilde{x}_i(y)$.

Then

$$\left(\bigcup_{\alpha\in[0,1]} {}_{\alpha}\tilde{x}_i\right)(y) = \sup_{\alpha\in[0,1]} {}_{\alpha}\tilde{x}_i(y)$$

$$= \max\left[\sup_{\alpha\in[0,a]} {}_{\alpha}\tilde{x}_i(y), \sup_{\alpha\in(a,1]} {}_{\alpha}\tilde{x}_i(y)\right]$$

For each $\alpha \in (a, 1]$, we have $\tilde{x}_i(y) = a < \alpha$ and, therefore ${}_{\alpha}\tilde{x}_i(y) = 0$. On the other hand for each $\alpha \in [0, a]$, we have $\tilde{x}_i(y) = a \ge \alpha$, therefore ${}_{\alpha}\tilde{x}_i(y) = \alpha$.

Hence,

$$\left(\bigcup_{\alpha\in[0,1]} {}_{\alpha}\tilde{x}_i\right)(y) = \sup_{\alpha\in[0,1]} \alpha = a = {}_{\alpha}\tilde{x}_i(y)$$

V. COMPUTATIONAL ALGORITHM

Step:1 Identify the problem whose mathematical representation is in the form of system of linear equations.

Step:2 If some of the coefficients or the resource vector are unknown of imprecisely known then such a system can be appropriately represented by fuzzy numbers, they give rise to fuzzy system of linear equations given by $\tilde{A}\tilde{\mathbf{x}} = \tilde{\mathbf{b}}$.

Step:3 Taking α-cut of the fuzzy system gives

$$\left[{}^{\alpha}\underline{A}, {}^{\alpha}\overline{A}\right] \otimes \left[{}^{\alpha}\underline{x}, {}^{\alpha}\overline{x}\right] = \left[{}^{\alpha}\underline{b}, {}^{\alpha}\overline{b}\right].$$

From this produce crisp system in terms of parameter α in the form

$$\begin{pmatrix} {}^{\alpha}\underline{A} & 0 \\ 0 & {}^{\alpha}\overline{A} \end{pmatrix} \begin{pmatrix} {}^{\alpha}\underline{x} \\ {}^{\alpha}\overline{x} \end{pmatrix} = \begin{pmatrix} {}^{\alpha}\underline{b} \\ {}^{\alpha}\overline{b} \end{pmatrix}$$

here, for the negative fuzzy numbers in the coefficient matrix interchange $\underline{a}_{ij} < 0$ with $\overline{a}_{ij} < 0$.

Step:4 Solve the crisp system generated in step 3, to produce $\underline{\mathbf{x}}, \overline{\mathbf{x}}$.

Step:5 If the given conditions are satisfied for all $\alpha \in [0,1]$, the components of the fuzzy vector can be constructed, refer [7], using the relation

$$x_j = \bigcup_{\alpha\in[0,1]} \alpha \cdot {}^{\alpha}x_j$$

where, ${}^{\alpha}x_j = \left[{}^{\alpha}\underline{x}_j, {}^{\alpha}\overline{x}_j\right]$.

VI. NUMERICAL ILLUSTRATION

Step:1 Consider the system of linear equations involving parameters represented by trapezoidal fuzzy numbers.

(-9, -7, -5 , -3) $\tilde{x}_1$ + (10, 12, 14, 16) $\tilde{x}_2$ = (1, 3, 4, 6)

(18, 20, 23, 25) $\tilde{x}_1$ + (-9, - 8, -7, -5) $\tilde{x}_2$ = (6, 8, 13, 16)

Step:2 Taking the α -cut of the system we get the corresponding crisp system as follows:

$$\begin{pmatrix} 2\alpha-9 & 2\alpha+10 & 0 & 0 \\ 2\alpha+18 & \alpha-9 & 0 & 0 \\ 0 & 0 & -2\alpha-3 & 16-2\alpha \\ 0 & 0 & 25-2\alpha & -\alpha-5 \end{pmatrix} \begin{pmatrix} {}^{\alpha}\underline{x}_1 \\ {}^{\alpha}\underline{x}_2 \\ {}^{\alpha}\overline{x}_1 \\ {}^{\alpha}\overline{x}_2 \end{pmatrix} = \begin{pmatrix} 2\alpha+1 \\ 2\alpha+6 \\ 6-2\alpha \\ 16-3\alpha \end{pmatrix}$$

Step:3 Since we observe that the coefficients a_{11} and a_{22} are negative we interchange $\underline{a}_{11}$ with $\overline{a}_{11}$ and $\underline{a}_{22}$ with $\overline{a}_{22}$. Hence the crisp system to be solved becomes:

$$\begin{pmatrix} -2\alpha-3 & 2\alpha+10 & 0 & 0 \\ 2\alpha+18 & -\alpha-5 & 0 & 0 \\ 0 & 0 & 2\alpha-9 & 16-2\alpha \\ 0 & 0 & 25-2\alpha & \alpha-9 \end{pmatrix} \begin{pmatrix} {}^{\alpha}\underline{x}_1 \\ {}^{\alpha}\underline{x}_2 \\ {}^{\alpha}\overline{x}_1 \\ {}^{\alpha}\overline{x}_2 \end{pmatrix} = \begin{pmatrix} 2\alpha+1 \\ 2\alpha+6 \\ 6-2\alpha \\ 16-3\alpha \end{pmatrix}$$

Step:4 Solving the crisp system we get the components of fuzzy vector as

$${}^{\alpha}x_1 = \left[\frac{6\alpha+13}{2\alpha+33}, \frac{8\alpha^2-104\alpha+310}{2\alpha^2-55\alpha+319}\right]$$

$${}^{\alpha}x_2 = \left[\frac{8\alpha^2+56\alpha+36}{2\alpha^2+43\alpha+165}, \frac{10\alpha^2-121\alpha+294}{2\alpha^2-55\alpha+319}\right]$$

Step:5 Since the conditions for the existence of the fuzzy solution is satisfied we can construct $\tilde{x}_1$ and $\tilde{x}_2$ using the relation, for j = 1,2 as

$$x_j = \bigcup_{\alpha\in[0,1]} \alpha \cdot {}^{\alpha}x_j$$

This gives the fuzzy solution represented in the trapezoidal form as $\tilde{x}_1$ = [0.3939, 0.5429, 0.8045, 0.9718] and $\tilde{x}_2$ = [0.2182, 0.4762, 0.6880, 0.9216] as shown in Figure 1.

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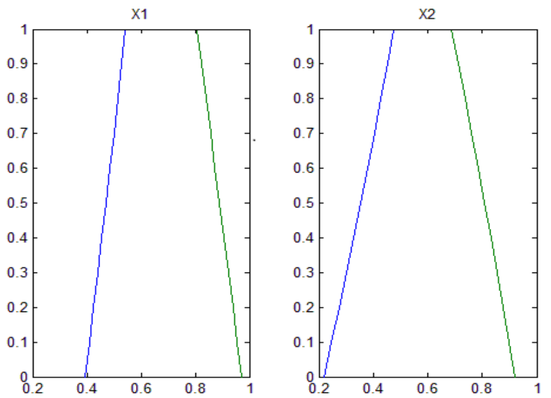

Figure 1. Components of the fuzzy vector x

VII. CONCLUSION

In this paper, we have given method to solve the system of linear equations with negative coefficient parameters. The computational algorithm for finding the fuzzy solution for such system is proposed. A numerical example is solved using the proposed algorithm.

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