# A Paper on System Stability (First Order and Second Order) Using PID Controller

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*Abstract: PID Controller is used for tuning of three constants (P,I&D).It stabilize the system by reducing oscillations and settling time. In the proposed method, new tuning rules based on the exact satisfaction of gain and phase margin specifications using proportional-integral (PI) and proportional-integral-differential (PID) type controllers are used for unstable first-order plus dead-time (UFOPDT) processes. The tuning rules are given in the form of iterative algorithms, as well as in the form of accurate, analytical approximations. Moreover, several specific functions, related to the crossover frequencies of the Nyquist plot and to the feasible design specifications for a given process, are derived. These functions, which are particularly useful for the general design of PI and PID-type controllers for UFOPDT processes are accurately approximated, in order to simplify the tuning procedure. With the proposed approximations, the tuning rules require relatively small computational effort and are particularly useful for online applications.*

*Index Terms—Differential, Proportional, Integral, Delay, First and Second Order System.*

## I. INTRODUCTION

The UFOPDT/USOPDT system is unsteady first/second order system having specific dead time. Two- or three-term controllers for UFOPDT processes have been tuned according to numerous methods, one of the most accepted of them being several variations of the direct synthesis tuning method and the ultimate cycle method is based on the minimization of different integral criteria, etc. However in a Proportional-integral (PI) tuning method, based only on the phase and increasing gain margin specifications is projected. This method uses some approximations of the tan-1 function to simplify the PI controller design, but due to the less accuracy of approximations used, it is not appropriate for large set of values of the time delay and for small gain and phase margin specifications. In an inner feedback loop with a PI controller is used to steady the FOPDT processes and subsequently an outer PID controller is designed using identified tuning methods for stable processes in order to achieve one particular design specification (PM=600, GM=3).The problem of designing PI controllers for UFOPDT process, based on phase and increasing gain margin specifications could also be treated by applying these methods.

## II. EFFECT IN DELAY ON SYSTEM

Delay effects on the strength and control of dynamical systems are problems of recurring interest since the delay presence may persuade complex behaviors (oscillations, instability, bad performances) for the (closed-loop) schemes: ``small'' delays may undermine some systems, but "large" delays may stabilize others. Indeed, for example, a string of delay (stability) `switches' (stability to instability or instability to stability) may appear with the second order even for a single distinct (or point) delay in a linear differential-difference equation, if the delay value, seen as a parameter, is greater than before. Delays are there in a system when a signal or physical variable originating in one part of system becomes presented in another part after a lapse of time. Delay can also turn out due to the time coupled with the transmission of information to secluded locations and in digital control systems due to the time implicated in computing control signal from measured data.

**PID CONTROLLER-**A proportional-integral-derivative controller (PID controller) is a widespread feedback loop component in industrial control systems. The Controller compares a measured value from a process (typically an industrial process) with a mention set point value. The difference (or "error" signal) is then used to calculate a new value for a manipulable input to the process that brings the process calculated value back to its desired defined point. Unlike the simple control algorithms, the PID controller can regulate processed output based on the history and rate of change of the error signal, which gives exact and stable control.

## III. PROPOSED TUNING METHODS

In particular, for a certain normalized time delay and given desired gain margin specifications $GM_{inc}^{des}$ and $GM_{dec}^{des}$ or/and phase margin specification $PM^{des}$, the methods presented below provide the controller that satisfy these specifications.

## IV. TUNING METHOD FOR PI-TYPE CONTROLLERS BASED ON SIMULTANEOUS GAIN AND PHASE MARGIN SPECIFICATIONS

Clearly, it is not always possible to design a PI controller that gives a closed-loop system with stability margins $GM_{inc}$, $GM_{dec}$ and PM equal to some pre-specified desired values $GM_{inc}^{des}$, $GM_{dec}^{des}$ and $PM^{des}$.This is due to the fact that it is not always probable to exactly assign three independent specifications with only two independent controller parameters namely $K_C$ and $\tau_I$. In this respect, our intention here is to present a PI controller tuning method that does not necessarily provide stability margins equal to the desired ones, but, in the general case, it attains actual stability margins larger than or at least equal to their pre-specified desired values. Therefore, from this point on, simultaneous stability margins satisfaction is considered in the above sense, unless otherwise stated.The basic steps of the method are the following.

**PGM Tuning Method-**

Step 1) First, verify that the desired specifications are admissible, namely, if

$0 < PM^{des} < PM^{max}$ and $1 < GM_{prod}^{des} < GM_{prod,max}^{des}$

And if there exists a value of $K_C$ that can satisfy all three specifications, when $\tau_I \to \infty$.

Step 2) Calculate the two controllers obtained by the PM and the GM methods. If the controller with the largest value of $\tau_I$ satisfies all three specifications, then this is the requested controller. In the opposite case continue with Step 3).

Step 3) Assume that $K_{C,PM}$ and $\tau_{I,PM}$ are the controller parameters obtained from the application of the PM-tuning method and $K_{C,PM}$ and $\tau_{I,PM}$ are the controller parameters obtained from the GM-tuning method. Then, if none of these two controllers satisfy all specifications, check which controller gives the largest gain $K_c$. In the case where:
i) $K_{C,PM} > K_{C,GM}$, then in order to satisfy all specifications with the smallest value of $\tau_I$, regularly increase $\tau_I$ (starting from the $\max(\tau_{I,PM}, \tau_{I,GM})$, while maintaining the same $GM_{inc}$ (by choosing $K_C = K_{max}/GM_{inc}^{de}$, until the phase margin specification is also satisfied;
ii) $K_{C,PM} < K_{C,GM}$, then regularly increase $\tau_I$ (starting from the max $(\tau_{I,PM}, \tau_{I,GM})$, while maintaining the same $GM_{dec}$ (by choosing $K_C = K_{max}/GM_{dec}^{de}$, until the phase margin specification is also satisfied.

Step 4) This completes the method.

Although there are numerous ways to select the controller parameters in order to satisfy (of course not exactly) all three specifications, the method presented here is preferred because it requires the smallest computational effort, since for a given $\tau_I, PM(d, \tau_I)$ can be calculated exactly without the use of iterative algorithms. It is noted here that when the tuning methods are applied using the approximations proposed in the brief, it is suggested to select the specifications slightly larger (i.e., by 5%) to compensate for the errors of the approximations. Moreover, in all PI tuning methods mentioned above, if the response obtained is too oscillatory (due to the small value of $\tau_I$), then, by increasing the value of $\tau_I$, the damping of the closed-loop system increases. From the analysis presented in Section 2, it is clear that, when $\tau_I$ is increased, the resulting closed-loop system is more robust, and all the stability robustness specifications are still satisfied.

**PM TUNING METHOD-**

Step 1) Check if the phase margin specification is achievable by a PI controller (i.e. $0 < PM^{des} < PM^{max}(d)$)

Step 2) Given the time delay and the phase margin specification $PM^{des}$, calculate the integral time constant $\tau_I(d, PM^{des})$ of the PI controller using either (23), if $PM^{des} > 0.2\,PM^{max}$ or for better accuracy, the PM algorithm, if if $PM^{des} > 0.2\,PM^{max}$.

Step 3) with $\tau_I$ known, compute the corresponding frequency $\omega_p$ and the controller gain $K_C$ from relations. This completes the method.

The advantage of this method is that the obtained $GM_{dec}$ and $GM_{inc}$ are quite proportioned, especially in the case where the stability region is small (large d or small $PM^{des}$). It should be noted here, that it is not suggested to choose $PM^{des} < 0.2PM_{max}$, since, in this case, the system is not sufficiently robust

**GM TUNING METHOD-**

Step 1) Calculate the desired gain margin product $GM_{prod}^{des}$ and check if this product is practicable by a PI type controller.(i.e. , check if $1 < GM_{prod}^{des} < GM_{prod,max}^{des}$ )

Step 2) Given the time delay d and $GM_{prod}^{des}$, calculate the integral time constant $\tau_I(d, GM_{prod}^{des})$ of the PI controller, using either equation, $GM_{prod}^{des} > 1 + 0.2GM_{prod,max} - 1)$ or for better accuracy, the GM algorithm, $GM_{prod}^{des} < 1 + 0.2GM_{prod,max} - 1)$.

Step 3) If $\tau_I > 1.2\tau_{I,min}$, evaluate the crossover frequencies $\omega_{min}$ and $\omega_{max}$ using equations (24) and (25), and the critical gains $K_{min}$ and $K_{max}$ using (5). If $\tau_I > 1.2\tau_{I,min}$, then the $\omega_{min}$ algorithm and the $\omega_{max}$ algorithm should be used for the estimation of $\omega_{min}$ and $\omega_{max}$, respectively. Note that when the GM algorithm is used to obtain $\tau_I$, the algorithm produces the values of $\omega_{min}$, $\omega_{max}, K_{min}$ and $K_{max}$.

Step 4) The gain of the controller can now be evaluated from one of the following relations:

$K_C = K_{max}/GM_{inc}$ or $K_C = GM_{dec}K_{min}$

This completes the method.

When applying the GM tuning method, it is preferable to select symmetrical gain margin specifications $GM_{inc} = GM_{dec}$. In this case, the phase margin of the closed-loop system is close to the maximum phase margin obtained when $\omega_G = \omega_p$, particularly in the case of small phase margins (large d and small $GM_{prod}$)

**Extension To Pid-Type Controllers And Usopdt Systems-** The simplest way to tune the PID controller is to first tune the parameters $\tau_I$ and $K_C$ of a PI controller, in order to satisfy the desired specifications, on the basis of the tuning methods presented above and to add the derivative action. If $\tau_D < \tau_{Dmax}$, then the PID controller has a larger stability region than that of the initially tuned PI controller and it satisfies the desired gain and phase margin specifications. To additional improve the design of the PID controller $\tau_D$ should a priori be selected on the basis of the designer's knowledge relative to the process. If there are no limitations imposed by the process, then it is recommended to select $\tau_D$ as large as possible in the range defined by (30). This way, the resulting closed-loop system has the fastest possible responses, from both the set-point tracking and the load attenuation point of view, and the minimum possible error in the case of regulatory control. In the case where the time response of the system is too oscillatory, as it was previously suggested, it is preferable to increase the parameter $\tau_I$. In this case, the response of the system becomes smoother and the resulting closed-loop system is more robust. In practical applications, derivative action of the PID controller is often combined with a first-order filter.

The PID-tuning methods presented here can easily be applied in combination with such a filter if the time delay used in the design is preferred larger than the real process dead time by a factor of $\tau_f$, where $\tau_f$ is the time constant of the filter (e.g. $10\tau_D = \tau_f$)

In the case of USOPDT systems, the loop transfer function is given by

$$G_{L,S}(s) = \frac{K_C(\tau_I s + 1)(\tau_D s + 1)}{\tau_I s(\tau_S s + 1)(s-1)} \exp(-ds)$$

From equation (1) it is clear that a PID controller can be tuned based on the PI-tuning methods presented in this concise if the derivative time constant $\tau_D$ of the controller is selected as . $\tau_D = \tau_s$ .

**ADVANCED PID TUNING FOR INTEGRATING AND UNSTABLE PROCESS WITH GAIN AND PHASE MARGIN SPECIFICATIONS-**In the process control, more than 95% of the control loops are of the proportional-integral-derivative (PID) type. The main motive is its comparatively defined structure, which can be easily understood and implemented in practice. Over the years, there are many formulas derived to tune the PID controllers for stable processes such as Ziegler-Nichols, Cohen-coon, internal model control, integral absolute error optimum (ISE, IAE, and IATE), and recently proposed tuning methods. How-ever, it is not easy to control, integrating and unsteady processes with time delay. Advance tuning system methods for the PID controller with set point weighting are proposed for integrating and unstable processes meet both gain and phase margin specifications, which are the primary measure for control system robustness. The control scheme 1st adopts the internal loop design strategy. Then, simple and effective PID-type controllers with set point weighting and design based on gain and phase margin specifications. This control scheme is divided in two parts:
1. Internal loop design strategy
2. PID type controller design.

**Pid Tuning Method Controller Design Strategy-** Represent controller transfer functions by $G_c(s)$ and integrating or unstable process transfer functions by $G_p(s)$, where $G_c(s)$ is given by

$$G_c(s) = K_p + \frac{K_i}{s} + K_d s \qquad (2)$$

We first introduce an inner feedback loop; the block diagram of controller design strategy is shown in figure 4.Here the p-controller ($K_1$) in the inner feedback loop plays an important role in changing the integrating or unstable process in stable onKL.

**INTEGRATING PROCESS-** For controller design purposes, we adopt the following simple integrating model

$$G_p(s) = \frac{K}{s(Ts+1)} e^{-Ls} \qquad (3)$$

With the P controller in the inner feedback loop, the internal closed loop transfer function $G_1(s)$ can be obtained as

$$G_1(s) = \frac{G_p(s)}{1 + K_1 G_p(s)} = \frac{Ke^{-Ls}}{Ts^2 + s + KK_1 e^{-Ls}} \qquad (4)$$

Using a Taylor series expansion, the time-delay term in the denominator of eq. 49 can be approximated by

$$e^{-Ls} \cong 1 - Ls + 0.5L^2 S^2 \qquad (5)$$

Substituting equation (5) into equation (4) $G_1(s)$ can be expressed by

$$G_1(s) \cong G'_p(s) = \frac{Ke^{-Ls}}{(T + 0.5KK_1 L^2)s^2 + (1 - KK_1 L)s + KK_1} \qquad (6)$$

Here, $G'_p(s)$ denote the second order plus time delay model obtained from the Taylor series expansion method. Because the characteristic equation of $G'_p(s)$ should have negative pole to be stable, the following condition must be satisfied from the Routh-Hurwith stability criterion:

$$K_1 < 1/KL \qquad (7)$$

For optimum disturbance rejection, the P controller is given by.

$$K_1 = 0.2/KL \qquad (53)$$

This satisfies the stability criterion (52). If we choose equation (53) as a design value of the P controller gain in the inner feedback loop, then eq. 51 is given by

$$G'_p(s) = \frac{e^{-Ls}}{\frac{T + 0.1L}{K}s^2 + \frac{0.8}{K}s + \frac{0.2}{KL}} \qquad (8)$$
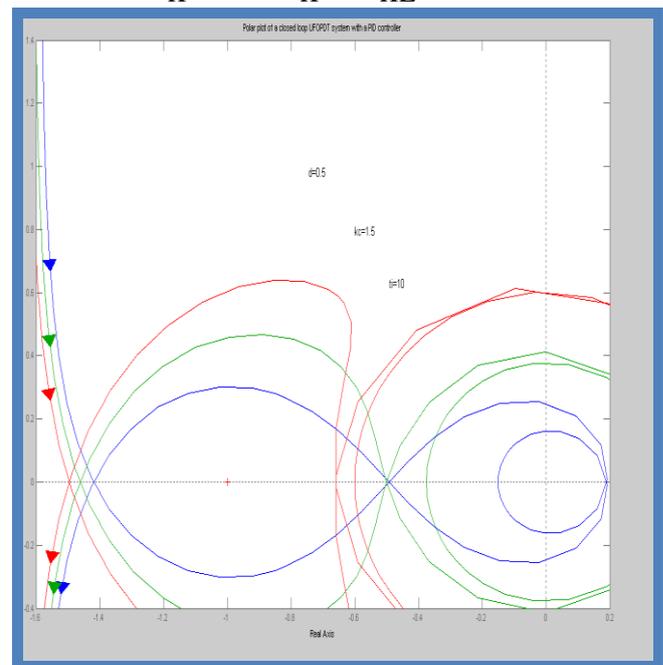


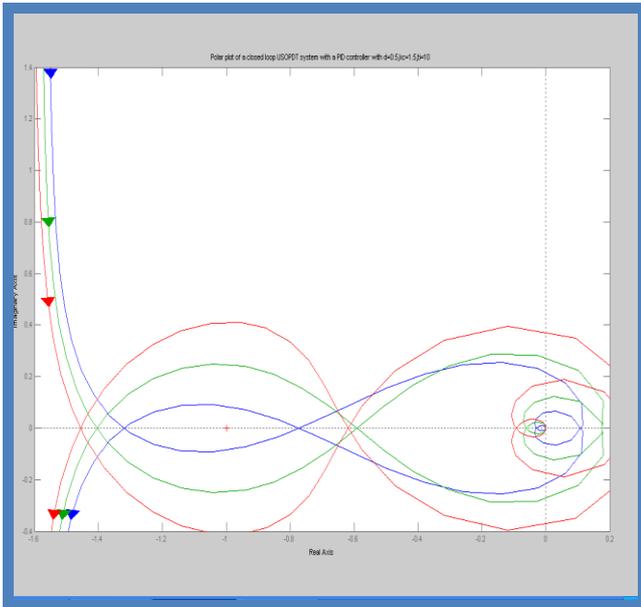*Figure: Polar plot of a closed loop UFOPDT system with PID controller.*

*Figure: Polar plot of a closed loop USOPDT system with PID controller.*

## V. CONCLUSION & FUTURE SCOPE

New tuning rules for PID controllers are developed for UFOPDT/USOPDT processes, based on the precise satisfaction of phase and gain margin specifications. These all methods take into an account all three desired specifications and they are valid and accurate for a wider range of process and controller parameters than other existing methods based on steadiness margin specifications. The proposed tuning methods have the advantage that derivative term is not compulsory by the design methods and its choice is left to the process designer. Although iterative algorithms are essential to accurately solve tuning problem, analytical equations are also provided to circumvent iterations in the case of online tuning. The simulation results obtained from the application of the proposed tuning methods to a variety of UFOPDT/USOPDT process models show accuracy of the proposed methods and their effectiveness in meeting the desired design specifications. In advanced PID tuning for integrating and unstable process with gain and phase margin specifications, simple tuning formulas have been derived. I adopted an additional inner feedback loop design technique and a designing controller based on both gain and phase margin specification and finally obtained simple tuning formulas of PID controllers with set point weighting which can overcome the structural limitation of a typical PID controller for integrating and unstable processes. With a proposed PID tuning method, we can also obtain a loop transfer function with a good shape, like phase margin $60^0$, gain margin over 3, and the real part close to -0.5 in low frequencies, which guarantees both robustness and performance. Simulations results have been given to show the performance that can be achieved.

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KSHITIZ AGARWAL Student of Mtech Final Year from Suresh Gyan Vihar University, Jaipur pursuing in **Digital & Wireless Communication Engineering**. I had a three year experience in the field of Electronics & Communication Engineering. I have presented several papers in National as well as in International Conference like **All India Seminar** held by **Institution of Engineers,** "VCAN" held at IET, Alwar (Raj.), and Malviya National Institute of Technology. Attended Several Lectures given by Senior Professor in technical topics organized by MNIT, AMITY UNIVERSITY etc.