

# Networked Distributed Optimization for Demand Response in Residential Load Control

Alex Cohen, Payam Forghani, Wang Xing

**Abstract—**Demand response (DR) will be one of the most important components in future smart grids. DR in a smart grid can reduce peak load and adapt flexible demand to generation fluctuations. Although DR reduces the energy bills for consumers of loads, the utility will also be reduced because of the load reduction. Centralized and distributed control algorithms have been proposed to minimize the utility loss for the consumers, while achieving the load management target. We propose a multi-level distributed load optimization system that serves this purpose. The optimization problem is formulated and solved using primal-dual decomposition. When the number of consumers in the system is very large, an event-triggered method is used to guarantee system performance by reducing the communication frequency.

**Index Terms**—smart grid, demand response, event-triggered control, optimization.

## I. INTRODUCTION

A smart grid is an intelligent network that makes use of advanced information, control and communication technologies and has the ability to integrate traditional backbone power grid with renewable generation resources with high uncertainties. Comparing to traditional power grid it has increased reliability, energy security and transparency, reduced cost and carbon footprint, along with improved stability and flexibility. In electricity grids, demand response (DR) is a mechanism for achieving energy efficiency through managing customers' electricity consumption pattern in response to changes in generation/electricity supply conditions. For example, having end users to reduce their demand at critical time periods when there is a demand response event. In smart grids, the role of demand response is more important because of the ever increasing penetration of distributed generation and the availability of two-way communication between energy provider and the consumers. On the other hand, the high uncertainty that comes with renewable power resources such as wind and PV has posed great challenges in demand response system design. New optimization and control technologies are required to accommodate these difficulties. This explains the recent surge of interests in demand response studies in both industry and academia [1, 2]. As pointed out in many studies, there are severe limitations with state of the art building demand response control strategy. For example, distributed load such as plug loads are becoming a large fraction of building energy growth, if they are included, demand response capacity can

be tremendous increased.

However they are usually not considered in current demand response practices. Consequently, the potential of demand response is strongly constrained since the possibility to dynamically accommodate weather change on building energy usage during demand response event is limited. This not only decreases loads' shedding capacity utilization but also degrades the productivity of some of the building occupants. Motivated by this, we study the dynamic building demand response with the consideration of distributed loads. A multi-agent demand response structure is proposed, where the decision making is distributed to local agents, which summarize and manage the distributed loads around it. There is a master decision maker which generates proper signals to send to the lower level demand response agents. The local agent then react to these signals or demand response events following market mechanisms like in [19], [7] through bidding to decide load shedding scheme. Unlike previous works, we consider the problem of maximum utilization of communication capacity in this system. For example consider two cases: 1). when there are one top-level decision maker but many agents; 2) when the system has many plug-in appliances and relatively few agents. This requires each agent to represent a large number of distributed loads. In both situations, the communication load at the central node (top-level decision maker in the first case and agents in the second) is very high. We propose a distributed optimization using event-trigger formula [11] to reduce the communication load. The idea is to reduce the number of message transmissions by eliminating un-necessary ones.

## II. RELATED WORK

Demand response is receiving fast increasing attention these days and is becoming an integral part of the power system and market operational practice [2]. In [9] the authors presented and analyzed case studies of different electric utility programs. Implementation of advance metering infrastructure and other smart grid technologies further increase the utilization of demand response resources in planning and operation [17]. In [12] the authors discussed the control strategies regarding Heating Ventilation and Air Conditioning (HVAC) system to optimize electricity expense and still maintain users' comfort level. In [8] the authors investigated the use of demand response control strategies in commercial buildings. Event-triggered control [13, 18] and optimization approach was originally developed in the study of networked control systems [10, 14]. It is closely related to the research on control with limited information or unreliable feedback channel [4, 5, 15, 16].

**Manuscript published on 30 September 2013.**

\*Correspondence Author(s)

**Alex Cohen**, Department of Electrical and Computer Engineering, University of Auckland, Auckland, New Zealand.

**Payam Forghani**, College of Information Engineering, Shenzhen University, Shenzhen, China.

**Wang Xing**, College of Computer Science and Technology, Huangshi, China.

© The Authors. Published by Blue Eyes Intelligence Engineering and Sciences Publication (BEIESP). This is an [open access](#) article under the CC-BY-NC-ND license <http://creativecommons.org/licenses/by-nc-nd/4.0/>

### III. PROBLEM FORMULATION

#### A. System Structure

In this paper, we consider a demand response system with hierarchical structure. A central decision maker receives demand response events from utility and based on that controls those centralized appliances, e.g. central lighting, HVAC, etc. There are also multiple agents that aggregates plug appliances and provide control interfaces to the central decision maker. These agents have distributed sensors and can directly control the plug-in appliances. Therefore, the agents form an additional layer between top-level decision makers and the lower level plug-in appliances. The system is illustrated in Figure 1.

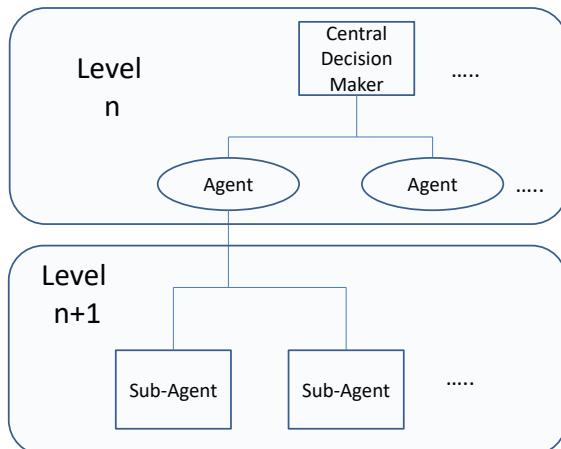


Fig.1 System Structure

We consider the interaction between top-level decision maker and the agents, an optimization problem is formulated and the objective is to minimize the cost of load shedding. The cost is a customized function describing the agent's comfortable level under a given load shedding plan.

#### B. Cost Function Structure

The cost function gives a measure of satisfaction of consumers. It is not easy to quantify in an accurate manner because different consumers have different preferences on different appliances depending on their activities or time of day. Therefore a mathematical formula of consumer preference should be time-varying, nonlinear and case-by-case. But such a function would be difficult to handle in optimization. Here we use a simplified cost function that approximately reflects consumer preference. We assume each controllable appliance has a cost function that is time-invariant. We also assume that all consumers have the same cost function. One rule of thumb is, during a demand response event, if the consumer decides to reduce power consumption by turning off an appliance, his corresponding loss of satisfaction should be reflected as a positive cost of the appliance when the power reduction is positive, and increase with the power reduction. Therefore, the cost is a non-decreasing function of power reduction. Following [10] we choose a continuous cost function:

$$f(s) = as + bs^2 \quad (1)$$

In real-world applications, although the power shedding  $s$  is continuous, it is not the case that consumer's comfort level can always be quantified as continuous value. For instance in

a refrigerator, although the average temperature inside is changing continuously with the power consumption, any temperature within the range of the optimal temperature for the food has no difference [11], however, any temperature outside the range is not acceptable. In this work for convenience of mathematic treatment we assume that the cost function is always continuous. For examples such as the refrigerator case, the discrete cost function can be approximated by convex combination of continuous functions.

#### C. Optimization Problem Formulation

Assume a demand response scenario with  $n$  distributed agents and the total load reduction target  $S$  provided by higher level demand response controller. The cost function of agent  $i$  is denoted by  $f_i(s_i)$  where  $s_i$  represents the load reduction of participant  $i$ . The objective of the demand response control system is to minimize the summed cost function of all the agents (participants of the demand response program) while still meeting the specified demand response target. This optimization problem can be expressed in equation (2)

$$\begin{aligned} & \min_{s_i} \sum_{i=1}^n f_i(s_i) \\ & \text{s.t. } \sum_{i=1}^n s_i = S, L_i \leq s_i \leq U_i, i = 1, 2, \dots, n \end{aligned} \quad (2)$$

where  $L_i$  and  $U_i$  are the upper and lower bounds of load reduction of agent  $i$ , respectively. Note that the objective function is in the form of summation of individual cost functions.

### IV. DISTRIBUTED OPTIMAL DEMAND RESPONSE

In this section, distributed algorithm for solving the optimization demand response problem in equation (2) is introduced in Section 4.1. Event-triggered version of the distributed algorithm is described in Section 4.2 that can generate optimal solution with much reduced communications between central decision maker and agents.

#### A. Distributed Optimization

There are many ways to decompose the optimization problem in equation (2) and solve it in a distributed manner. Consider a two-level control strategy: the central demand response decision maker does not know the cost functions of each agent. The agents need to report the parameters of its cost function to the central decision maker and wait for a decision. Since the agent also is an aggregator which knows the total cost function of its sub-agents (through their reported parameters), it can further decide the load reduction plan for its sub-agents. In this hierarchical distributed control structure the information flowing from bottom to top is about the parameters of cost functions, while the information flowing from top to bottom is the load reduction plans. In order to determine the optimal load reduction for all agents, each agent should generate its cost function according to cost functions of the corresponding sub-agents. Meanwhile, for a load reduction target assigned to an agent, it should minimize the total cost for its sub-agents in order to allocate the load reduction to each subagent.



Published By:

Blue Eyes Intelligence Engineering  
and Sciences Publication (BEIESP)

© Copyright: All rights reserved.

The most usually used method of solving the optimization problem (2) is to treat it as a special case of the Network Utility Maximization (NUM) problem [2], which has the following formula in general:

$$\begin{aligned} \max_{x_i} U(x) &= \sum_{i=1}^n f_i(x_i), \\ \text{s.t. } Ax &\leq U, x \geq 0, i = 1, 2, \dots, n \end{aligned} \quad (3)$$

where  $x = [x_1, x_2, \dots, x_n]$  is a vector of decision variables, it can be solved using the dual decomposition algorithm [17]. The algorithm examines the dual of the problem (3), expressed as

$$\min_{z \geq 0} \{ \sum_{i=1}^n f_i(x_i) - p^T (Ax - U) \}, \text{s.t. } p \geq 0 \quad (4)$$

where  $p = [p_1, p_2, \dots, p_m]$  is a vector of Lagrange multipliers corresponding to the constraint  $Ax \leq U$ . The recursive algorithm that generates a sequence of load reductions for all agents  $\{x[k]\}$  and link prices  $\{p[k]\}$  ( $k$  is the time indices) that asymptotically converges to a solution of (4) can be adopted from NUM literatures such as [17]. For a given initial condition  $x[0]$  and  $p[0]$ , the update procedure is

$$x_i[k+1] = \arg \max_{x_i} \{f_i(x_i[k]) - x_i[k] \sum_j p_j[k]\} \quad (5)$$

$$p_i[k+1] = \max\{0, p_i[k] + \mu(\sum_i x_i[k] - c_j)\} \quad (6)$$

Note that this is a totally distributed algorithm: equation (5) is executed at the agent and equation (6) is executed at the central decision maker.

### B. Event-triggering and Communication Reduction

Event-triggering is an algorithm developed originally in control society for wireless networked control systems (NCS). The idea is to transmit message in the network only when it is "necessary", as defined by the violation of some state-dependent inequalities. With event-triggering, the number of transmissions are reduced but the closed loop control system can still be stable [6, 9, 15]. This idea has also been applied for NCS with a large number of distributed agents [11, 18, 20]. In [17] it is proposed to use event-triggering idea to solve NUM on sensor networks based on primal-dual decomposition. In this section we adopt the algorithm there for our problem (2) to achieve the communication traffic reduction objective.

In our case the topology is simpler than the NUM problem. Unlike regular update laws in (5) and (6), the event-triggered distributed update algorithm is implemented as follows:

Step 1: the discrete update law law (5) and (6) are replaced by their continuous time version (in the form of differential equations), respectively;

Step 2: Based on the continuous time updates, a pair of events are designed  $g_i(x_i) \geq 0, h_i(p_i) \geq 0$  are designed;

Step3: The real-time value  $x_i(t)$  keep changing according to the continuous update law, when the inequalities about  $x_i$  is violated, the current value of  $x_i(t)$  is sent to the central decision maker. Similarly, only when the inequalities about  $p_i$  is violated, the current value of  $p_i(t)$  is sent to the agents.

## V. EXPERIMENTAL RESULTS

In the simulation, the cost functions of the devices (loads) are modeled to be quadratic as in (1). The test case includes 3

agents, the cost functions are provided in Tab. I, and total load reduction requirement is 300W. The optimization results of the two-level system are computed from (2) using two algorithms: one is the regular distributed iteration algorithms in Section 4.1, the other is the event-triggered algorithm in Section 4.2. The experiment has shown a 45% reduction in the number of communications between agents in different layers. This implies that with a given bandwidth limit of the communication link in the distributed demand response system, by applying the event-triggered optimization method, it is possible to double the number of subagents without congesting the link.

Table I: parameter setup for the experiment

Agents	Sub-agents	Parameter a	Parameter b	U (W)
Agent 1	1	0.01	0.04	100
	2	0.1	0.3	160
Agent 2	3	0.04	0	120
	4	0.43	0.1	200
Agent 3	5	0.06	0.21	140

## VI. CONCLUSION

In this work we introduced a multi-level demand response control framework, in each level there are multiple smart agents that serve as central decision maker for lower level agents. An optimal demand response problem is posed as optimization problem with a given quadratic cost model. In the case when many sub-agents belong to one agent, to reduce the traffic in communication networks, an event-triggered distributed optimization algorithm is employed to solve the problem in a networked environment. Experiments show significant traffic reduction while the optimization result is the same as classical algorithm.

## REFERENCES

1. Taqqali, W.M.; Abdulaziz, N., "Smart Grid and demand response technology," IEEE International EnergyCon, pp.710-715, 2010
2. Rahimi, F.; Ipakchi, A., "Overview of Demand Response under the Smart Grid and Market paradigms," Innovative Smart Grid Technologies (ISGT), pp.19-21, 2010
3. Xiaofeng Wang, Yu. Sun, Naira Hovakimyan: Asynchronous task execution in networked control systems using decentralized event-triggering. Systems & Control Letters 61(9), pp. 936-944, 2012
4. G. N. Nair & R. J. Evans, "Stabilization with data-rate-limited feedback: tightest attainable bounds", Systems & Control Letters, vol. 41, no. 1, pp. 49-56, 2000.
5. Sun Yu, Prashant G. Mehta, "Fundamental performance limitations with Kullback-Leibler control cost," IEEE Conference on Decision & Control, pp.7063-7068, 2010.
6. Sun Yu, Prashant G. Mehta, "Bode-Like Fundamental Performance Limitations in Control of Nonlinear Systems". IEEE Transaction on Automatic Control, vol. 55, pp.1390-1405, 2010.
7. Wang, J.; Kennedy, S.; Kirtley, J., "A new wholesale bidding mechanism for enhanced demand response in smart grids," Innovative Smart Grid Technologies (ISGT), pp.19-21, 2010
8. Cobelo, Inigo; Boyra, Maialen; Castellanos, Antonio, "Commercial building load modelling for demand response applications," CIRED 2009, pp.1-4, 2009
9. Wang, J.; Biviji, M.; Wang, W.M., "Case studies of smart grid demand response programs in North America," Innovative Smart Grid Technologies (ISGT), 2011 IEEE PES, pp.1-5, 2011
10. Sekhar Tatikonda, S. Mitter, "Control Over Noisy Channels", IEEE Transactions on Automatic Control, 49, 1196-1201, 2004,



11. Pu Wan; Lemmon, M.D., "Event-triggered distributed optimization in sensor networks," IPSN 2009. pp.49-60, April 2009
12. Bing Dong, "Non-linear optimal controller design for building HVAC systems," Control Applications (CCA), pp.210-215, 2010
13. Tabuada, P., "Event-Triggered Real-Time Scheduling of Stabilizing Control Tasks", IEEE Transactions on Automatic Control, 52(9), pp.1680-1685, 2007
14. Nicola Elia: Remote stabilization over fading channels. Systems & Control Letters, vol.54, no.3, pp. 237-249, 2005.
15. Martins, Nuno C., Dahleh, Murithier A., "Feedback Control in the Presence of Noisy Channels: 'Bode-Like' Fundamental Limitations of Performance", IEEE Transactions on Automatic Control, vol.53, issue.7, pp. 1604 – 1615, 2008
16. Yu. Sun and P. G. Mehta, "Fundamental performance limitations via entropy estimates with hidden Markov models," IEEE Conference on Decision & Control, pp. 3982–3988, 2007.
17. Mak, S.T., "A synergistic approach to implement demand response, asset management and service reliability using smart metering, AMI and MDM systems," IEEE PES '09, pp.1-4.
18. Wang, Xiaofeng; Yu. Sun; Hovakimyan, N., "Relaxing the consistency condition in distributed event-triggered networked control systems," IEEE Conference on Decision and Control (CDC) pp.4727-4732, 2010
19. Arasteh, H. R.; Moghaddam, M. Parsa; Sheikh-El-Eslami, M. K., "Bidding strategy in demand response exchange market," Proceedings of 17th Conference on Electrical Power Distribution Networks (EPDC), 2012