Small Signal Stability Analysis of a Wind Penetrated Electricity Distribution System

P. Venkata Narayana, K. H. Phani Sree

Abstract—The new types of generating systems such as wind generators, PV based static generators, diesel generators, and power from cogeneration plants have been introduced in to the system resulting in new challenges to stability, operation and control of the power system and its components. The reason being intermittent nature of the such types of generation. Due to their unregulated operation, the generators may impose a serious threat to the small signal stability. This paper analyses the small signal stability of the test distribution system at various penetration levels of wind generation in to the test system. For this purpose, eigen values and participation factor approaches have been chosen for analysis.

Keywords— Distributed generation, small signal stability, eigen value analysis, participation factor, Power System Analysis Toolbox (PSAT)

I. INTRODUCTION

Power systems are steadily growing with ever larger capacity. Formerly separated systems are interconnected to each other. Modern power systems are evolved into systems of very large size, stretching out hundreds and thousands of kilometers. With growing generation capacity, different areas in a power system are added with even larger inertia. With electric utility restructuring, public environmental policy, and expanding power demand, small distributed generators are in great need in order to satisfy on-site customer energy needs. Major improvements in the economic, operational, and environmental performance of small, modular units have been achieved through decades of intensive research. The use of renewable energy technologies exhibits a significant growth in nowadays power systems mainly due to critical factors such as limited available primary energy resources used in conventional power plants, the fast increase in fuel prices, and environmental concerns. Wind power constitutes the renewable generation technology which has experienced the fastest growing among all types of renewable generation technologies currently investigated. Extrapolating the current trend into the future, it is easy to foresee installed wind capacities exceeding 50 % of the overall capacity in some countries in the not too distant future.

Large integration of wind power into power networks will affect considerably the dynamic behavior of the power system since wind based generation systems and conventional synchronous generators exhibit fundamentally different transient responses. This stems first and foremost from their inherently different dynamic characteristics. Additionally, wind generation systems result in the reduction of the overall system inertia pegged to the network in relation to the installed capacity. Furthermore, modern power networks are operated close to their security limits due to economical and technical considerations.

It is increasingly recognized that the investigation of the small-signal stability of power systems yields important results that are complementary to those yielded by the usual transient stability investigations. The advantage of studying the small-signal stability by using eigenvalue analysis when compared to transient stability investigations is that it gives a complete overview of the small-signal stability of the current system operating state, whereas in transient stability investigations only one event at a time can be simulated. The drawback of eigenvalue analysis is, that a linearized set of equations is used and that the higher order terms are neglected, which may lead to erroneous results, particularly when a system is described by strong non-linear equations. Keeping in mind these limitations, eigenvalue analysis is nevertheless considered a powerful tool.

The small signal stability (SSS) problem of a power system occurs usually due to insufficient damping of electromechanical oscillations. However, further research is required for better understanding of the main factors influencing the impact of large scale wind power integration on small signal stability. This paper provides an attempt to assess how large scale wind power integration influences the small signal stability, since these stability constraints are essential for power system security, as evidenced in recent blackouts throughout the world.

The small signal stability of a 15 bus test distribution comprising conventional thermal power plants and a SCIG based wind power plant were evaluated through modal analysis and time domain simulations respectively.

II. SYSTEM MODELLING

The modelling approach adopted in this paper is as explained below.
A. Synchronous Generator

The sixth order model of synchronous generator is considered for stability analysis in this paper. The Park-Concordia model is used for synchronous machine equations, whose scheme is depicted in Fig 1.

\[ v_d = V \sin(\delta - \theta) \]
\[ v_q = V \cos(\delta - \theta) \]

Power injections are expressed in the form:

\[ P = v_d i_d + v_q i_q \]
\[ Q = v_q i_d - v_d i_q \]

This model is obtained assuming the presence of a field circuit and an additional circuit along the d-axis and two additional circuits along the q-axis. The system has six state variables \((\delta, \omega, e_d, e_q, e_d', e_q')\) and the following equations:

\[ \delta = \Omega_b(\omega - 1) \]
\[ \dot{\omega} = (P_m - P_e - D(\omega - 1))/M \]
\[ e_d' = (e_d' - \left( x_d - x_d' - \frac{T_{do d'}}{T_{do}} x_d \right) i_d + (1 - \frac{T_{do d'}}{T_{do}}) v_d') \]
\[ e_q' = (e_q' - \left( x_q - x_q' - \frac{T_{do q}}{T_{do}} x_q \right) i_q + (1 - \frac{T_{do q}}{T_{do}}) v_q') \]

\[ e_d'' = (-f_s(e_d')) + \left( x_q - x_q' - \frac{T_{qo q}}{T_{qo}} x_q \right) i_q / T_{qo} \]
\[ e_q'' = (-e_d'' + e_q' - \left( x_d - x_d' - \frac{T_{do d'}}{T_{do}} x_d \right) i_d + (1 - \frac{T_{do d'}}{T_{do}}) v_d') / T_{do} \]
\[ e_d''' = \left( -e_d'' + e_q'' + \left( x_q - x_q' - \frac{T_{qo q}}{T_{qo}} x_q \right) x_d - x_d' \right) / T_{qo} \]

The electrical power is given as:

\[ P_e = (v_q + r_s i_q)i_q + (v_d + r_s i_d)i_d \]

B. Induction Generator

The induction generators are popularly employed in wind power generation applications, small and micro hydro and some thermal plant. The mechanical torque is assumed to be constant. The squirrel cage induction generator (SCIG) model has been considered. The steady state equivalent circuit of an SCIG is shown in Fig. 2.

\[ \frac{x_q = x_s + x_{rq}}{x'r + x_{rm}} \]
\[ T_{so} = \frac{x_{rq} x_{rm}}{a_{gr} a_{gm}} \]

On the other hand, to simulate the mechanical parts as gear box and shafts, the mechanical equations which are taken into account are the turbine inertia \(H_{tr}\) and rotor inertia \(H_{m}\), and shaft stiffness \(K_s\) as shown below:

\[ w_{wr} = \frac{(T_{wr} - K_s \gamma)}{2H_{wr}} \]
\[ w_{wm} = \frac{(K_s \gamma - T_p)}{2H_{m}} \]

where \(\gamma\) is the slip factor and \(\omega\) represents rotor speed indices.

C. Description of the Distribution System

A 15 bus distribution system is adopted for performing the small signal stability analysis with increased wind penetration. The test system is shown in Fig. 2 which is based on the distribution feeder in the Kumamoto area of Japan. To perform small signal stability analysis, a wind generation unit is connected on bus 15.
In this system, two pairs of complex low frequency oscillations were observed which are summarized in the following table.

### The Oscillatory Modes Existing in the Distribution System

<table>
<thead>
<tr>
<th>Modes</th>
<th>Real Part</th>
<th>Imaginary Part</th>
<th>Damping Ratio</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>1,2</td>
<td>-0.48374</td>
<td>14.8556</td>
<td>0.0325</td>
<td>2.3656</td>
</tr>
<tr>
<td>3,4</td>
<td>-0.72993</td>
<td>12.8273</td>
<td>0.0568</td>
<td>2.0448</td>
</tr>
</tbody>
</table>

#### B. Participation Factor

The contributions of states on oscillations were observed by evaluating the participation factors of each state on a particular mode. The participation factor of the kth state in the ith eigen mode may be given by

\[ p_{ki} = \phi_{ki} \psi_{ik} \]

where

- \( \phi_{ki} \) = the element on the kth row and ith column of the modal matrix \( \Phi \)
- \( \psi_{ik} \) = the element on the ith row and kth column of the modal matrix \( \Psi \)

\[ \phi_{ki} = kth \ entry \ of \ the \ right \ eigenvector \ \Phi \]
\[ \psi_{ik} = kth \ entry \ of \ the \ left \ eigenvector \ \Psi \]

#### C. Eigen Value Analysis

Eigen value or Modal analysis describes the small signal behaviour of the system i.e. the behavior linearised around one operating unit. The Eigenvalue analysis investigates the dynamic behavior of a power system under different characteristic frequencies (‘modes’). In a power system, it is required that all modes are stable.

The following tables show the participation factor of various states on different oscillation modes.

### Impact of Penetration of Wind energy on Participation factors of State Variables for Modes 1,2

<table>
<thead>
<tr>
<th>State</th>
<th>5% Wind</th>
<th>10% Wind</th>
<th>15% Wind</th>
<th>20% Wind</th>
<th>25% Wind</th>
<th>30% Wind</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.01359</td>
<td>0.01541</td>
<td>0.0148</td>
<td>0.01416</td>
<td>0.01348</td>
<td>0.01272</td>
</tr>
<tr>
<td>2</td>
<td>0.01359</td>
<td>0.01541</td>
<td>0.0148</td>
<td>0.01416</td>
<td>0.01348</td>
<td>0.01272</td>
</tr>
<tr>
<td>3</td>
<td>0.01813</td>
<td>0.02947</td>
<td>0.03907</td>
<td>0.04728</td>
<td>0.05442</td>
<td>0.0607</td>
</tr>
<tr>
<td>4</td>
<td>0.00371</td>
<td>0.00303</td>
<td>0.00236</td>
<td>0.00176</td>
<td>0.00135</td>
<td>0.00129</td>
</tr>
<tr>
<td>5</td>
<td>0.00223</td>
<td>0.00096</td>
<td>0.00014</td>
<td>0.00002</td>
<td>0.00004</td>
<td>0.00008</td>
</tr>
<tr>
<td>6</td>
<td>0.00297</td>
<td>0.00244</td>
<td>0.0019</td>
<td>0.00142</td>
<td>0.00108</td>
<td>0.00103</td>
</tr>
<tr>
<td>7</td>
<td>0.00029</td>
<td>0.00048</td>
<td>0.00063</td>
<td>0.00076</td>
<td>0.00088</td>
<td>0.00097</td>
</tr>
<tr>
<td>8</td>
<td>0.000802</td>
<td>0.01327</td>
<td>0.01761</td>
<td>0.02132</td>
<td>0.02458</td>
<td>0.02751</td>
</tr>
<tr>
<td>9</td>
<td>0.01784</td>
<td>0.02962</td>
<td>0.03935</td>
<td>0.04761</td>
<td>0.05477</td>
<td>0.06106</td>
</tr>
<tr>
<td>10</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>11</td>
<td>0.03108</td>
<td>0.02973</td>
<td>0.02863</td>
<td>0.02793</td>
<td>0.02754</td>
<td>0.02742</td>
</tr>
<tr>
<td>12</td>
<td>0.42613</td>
<td>0.41187</td>
<td>0.39828</td>
<td>0.38662</td>
<td>0.37627</td>
<td>0.36672</td>
</tr>
<tr>
<td>13</td>
<td>0.25375</td>
<td>0.24372</td>
<td>0.23527</td>
<td>0.229</td>
<td>0.22449</td>
<td>0.22146</td>
</tr>
<tr>
<td>14</td>
<td>0.20352</td>
<td>0.1957</td>
<td>0.18833</td>
<td>0.18195</td>
<td>0.1763</td>
<td>0.17116</td>
</tr>
<tr>
<td>15</td>
<td>0.00049</td>
<td>0.00283</td>
<td>0.00472</td>
<td>0.00566</td>
<td>0.00559</td>
<td>0.00475</td>
</tr>
</tbody>
</table>

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**Fig. 2.** Single line diagram of the test distribution system

**Fig. 3.** Eigen values of the distribution system

The Eigen values of the system gives the information about the small signal stability. The system under study has fifteen eigen values with the wind generator connected to the system, which is shown in Fig.3. Since all the Eigen values lie on the left side of the imaginary axis, the system is said to be asymptotically stable. Eigen value analysis may be used to determine the acceptable renewable energy penetration before the system loses its small signal stability.
OBSERVATIONS
For Modes 1, 2, the PFs of state-5 increased from 0.00223 to 0.03049. Furthermore, the PFs of state-8 increased significantly from 0.00802 to 0.02751 and the PFs of state-9 were increased significantly from 0.01784 to 0.06106. Also, the PFs of state-15 increased significantly from 0.00049 to 0.00475.

Impact of Penetration of Wind energy on Participation factors of State Variables for Modes 3,4
<table>
<thead>
<tr>
<th>Stat</th>
<th>5% Wind</th>
<th>10%W</th>
<th>15%W</th>
<th>20%W</th>
<th>25%W</th>
<th>30%W</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.0005</td>
<td>0.0001</td>
<td>0.0001</td>
<td>0.0001</td>
<td>0.0001</td>
<td>0.0001</td>
</tr>
<tr>
<td>2</td>
<td>0.0005</td>
<td>0.0001</td>
<td>0.0001</td>
<td>0.0001</td>
<td>0.0001</td>
<td>0.0001</td>
</tr>
<tr>
<td>3</td>
<td>0.3139</td>
<td>0.3055</td>
<td>0.2981</td>
<td>0.2917</td>
<td>0.2865</td>
<td>0.2821</td>
</tr>
<tr>
<td>4</td>
<td>0.00461</td>
<td>0.00501</td>
<td>0.00528</td>
<td>0.00541</td>
<td>0.00543</td>
<td>0.00552</td>
</tr>
<tr>
<td>5</td>
<td>0.15434</td>
<td>0.14986</td>
<td>0.14584</td>
<td>0.14246</td>
<td>0.13967</td>
<td>0.13741</td>
</tr>
<tr>
<td>6</td>
<td>0.00257</td>
<td>0.0028</td>
<td>0.0029</td>
<td>0.00306</td>
<td>0.00331</td>
<td>0.00308</td>
</tr>
<tr>
<td>7</td>
<td>0.00408</td>
<td>0.00397</td>
<td>0.00389</td>
<td>0.00383</td>
<td>0.00378</td>
<td>0.00374</td>
</tr>
<tr>
<td>8</td>
<td>0.16417</td>
<td>0.15965</td>
<td>0.15547</td>
<td>0.15166</td>
<td>0.14819</td>
<td>0.14504</td>
</tr>
<tr>
<td>9</td>
<td>0.31512</td>
<td>0.3066</td>
<td>0.29914</td>
<td>0.29275</td>
<td>0.28736</td>
<td>0.28286</td>
</tr>
<tr>
<td>10</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>11</td>
<td>0.00223</td>
<td>0.00392</td>
<td>0.00527</td>
<td>0.00635</td>
<td>0.00722</td>
<td>0.00709</td>
</tr>
<tr>
<td>12</td>
<td>0.01112</td>
<td>0.01973</td>
<td>0.02699</td>
<td>0.03328</td>
<td>0.03889</td>
<td>0.04399</td>
</tr>
<tr>
<td>13</td>
<td>0.01186</td>
<td>0.02091</td>
<td>0.02832</td>
<td>0.03446</td>
<td>0.03961</td>
<td>0.04396</td>
</tr>
<tr>
<td>14</td>
<td>0.00158</td>
<td>0.00747</td>
<td>0.01253</td>
<td>0.01674</td>
<td>0.0202</td>
<td>0.02292</td>
</tr>
<tr>
<td>15</td>
<td>0.0141</td>
<td>0.01438</td>
<td>0.01588</td>
<td>0.01774</td>
<td>0.01952</td>
<td>0.02101</td>
</tr>
</tbody>
</table>

Observations
For Modes 3, 4, the PFs of state-12 increased from 0.01112 to 0.04399. Furthermore, the PFs of state-13 increased significantly from 0.01186 to 0.04396 and the PFs of state-14 were increased significantly from 0.00158 to 0.02292. Thus, it has been observed that the Participation of some states have significantly increased with increased wind penetration. The participation of the remaining states changes slightly in either direction.

IV. CONCLUSIONS
The small signal stability of a test distribution system is investigated with increased penetration of wind energy. The impact of wind energy penetration on the distribution system is estimated by calculating the participation factors of different states at different oscillation modes. The penetration is varied to study the impact on small signal stability. The sensitivity parameter and time domain simulation are used for stability analysis. Low frequency oscillation modes with approximate frequency of 3 Hz were observed. The results show that rotor flux variables of wind generators participate significantly in the system oscillations. The oscillatory modes dominated by wind generator states are less sensitive with power fluctuations and relatively well damped as compared to the modes dominated by synchronous generator states. The increased penetration of wind power has a positive impact on the oscillation damping of the synchronous generator.

APPENDICES
A. Test Distribution System Data
The distribution system data is given in the following tables. The per unit values are based on 30 MVA and 11.432 kV.

REFERENCES


AUTHOR PROFILE

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