
Nilesh P. Patil, V.A. Kulkarni

Abstract: Due to the measurement errors objectively existing, measurement result deviates from “true value” of the measurand. As measurement devices also shows measurement errors, a measurement result will never be exact. This means there will always be a measurement uncertainty that must be taken into account while evaluating conformance of products to tolerances. If the measurement uncertainty is neglected, this can result in false rejection or false acceptance of products, with possibly far-reaching consequences. Measurement uncertainty determination for coordinate measuring machines is difficult because of the many uncertainty contributors such as CMM hardware errors, temperature, measurement strategy etc. that are involved and these are affecting on the performance of the CMM which is nothing but the uncertainty involved in the process. This paper indicates what the uncertainty in the measurement is, standard types of uncertainty and how to calculate the budget for uncertainty with standard component i.e. single cylinder engine block assembly. With reference to the Guide to the expression for uncertainty in measurement (ISO GUM), the process is followed with Type-A and Type-B standard uncertainty by the statistical analysis of series of observations taken and by means other than the statistical analysis of series of observations by analyzing the results, we had found that the max. Expanded uncertainty for bore diameter, concentricity & cylindricity is observed as ± 2.9331 μm, ± 2.9721 μm & ± 2.9250 μm respectively.

Index Terms: Coordinate Measuring Machine (CMM), ISO GUM, Measurement uncertainty, required accuracy, Uncertainty Analysis

I. INTRODUCTION

All measurement processes and instruments have some extent of uncertainty. When reporting the measurement result, it is necessary to report the uncertainty associated with the measurement. No perfect measurement exits. Instead of that, the result of measurement is only an approximation of the value of the quantity being reported. Therefore, the measurement result is not complete without the addition of a quantitative statement of its uncertainty. The evaluation of measurement uncertainty is complex as there are a variety of sources of errors and variation. There is no any single uncertainty statement which can be generally applied to all CMMs. The uncertainty for every individual CMM is specific to a particular task and it depends on the various issues like sampling strategies, probing errors, residual errors, fitting algorithms used and the ambient conditions.

A. Measurement Uncertainty

Measurement uncertainty is a parameter associated with measurement result and presents an assessment that is characterized by a range of values within which the measured value is. Complete measurement reports must contain statement about the measurement uncertainty. This fact is relevant due to two important metrological concepts, i.e. decision on “the acceptance or rejection” and maintaining traceability in accordance with ISO 9000 standards. [1][2]

In order to be able to evaluate measurement uncertainty for CMM feature measurements, all uncertainty contributors should be identified and quantified. Some of them are Work piece itself includes temperature, form deviations, roughness, flexibility. Also Hardware, sampling strategy & evaluation strategy these are the contributors to the measurement uncertainty for CMM.

II. PROBLEM STATEMENT AND OBJECTIVE

Generally the inspection results of CMMs are not stated with the confidence level of measurement process. Some research has been done and has demonstrated the use of CMM

In automotive domain for surface wear monitoring. He only considered Type-A uncertainty not used Type-B uncertainty but needs to be, because the measured results would have been used for Decision making for tolerance design by design engineers, pre and post maintenance plan activities, manufacturing resource planning etc. and it is of great significance. Uncertainty if stated clearly and practically one can take logical and effective decisions.

In this project work, coordinate measuring machine (CMM) is used with single cylinder engine block to observe the bore surface.

A. Work Objective

➢ Identification of various types of uncertainty contributors during measurement process done with coordinate measuring machine.
➢ To study and follow the ISO-GUM procedure of uncertainty estimation.
➢ To estimate uncertainty budget for single cylinder engine block followed with ISO-GUM procedure.
III. EVALUATION OF MEASUREMENT UNCERTAINTY

A. Steps to evaluating uncertainty
The main steps to evaluating the overall uncertainty of a measurement are as follows.

1. Express mathematically the relationship between the measurand Y and the input quantity Xi on which Y depends.
2. Obtain estimated values of input quantity Xi.
3. Evaluate the standard uncertainty \( u(x_i) \) of each input estimate (Type-A evaluation of standard uncertainty).
4. Calculate the covariances associated with any input estimates that are correlated.
5. Obtain the result of the measurement.
6. Determine the combined standard uncertainty \( u(y) \) from the standard uncertainties and covariances associated with the input estimates.
7. Multiply the combined standard uncertainty \( u(y) \) by a coverage factor which gives expanded uncertainty (C).
8. Result the Uncertainty clearly.

Figure No.1: Procedure for evaluating Uncertainty

Uncertainty is inevitable in measurement. Measurement uncertainty is the most important concept in GPS according to the GUM. Whether the measurement results are applicable or not is determined upon the value of uncertainty to a great extent. Rational evaluation of uncertainty can report the quality and reliability of the measured value.

B. Type-A evaluation of standard uncertainty
The type-A method of evaluation uses statistical procedures to obtain the best estimate and the standard measurement uncertainty. Under the independent measurement conditions of equal precision, the series values \( x_i \) are obtained by \( n \) statistically independent observations carried out to the measurand X, the best estimate value \( x \) is the arithmetic mean of the observed values \( x_i (i = 1, 2, \ldots, n) \):

\[
\bar{x} = \frac{1}{n} \sum_{i=1}^{n} x_i.
\] (1)

Where, \( x \) is an estimate for true value of the measurand (X) and known as measurement result.

The standard measurement uncertainty \( u(x) \) is:

\[
u(x) = \frac{s(x)}{\sqrt{n}} = \sqrt{\frac{1}{n(n-1)} \sum_{i=1}^{n} (x_i - \bar{x})^2} \] (2)

Where \( s(x) \) is experimental standard deviation of \( x \),

\[
s(x) = \sqrt{\frac{1}{n(n-1)} \sum_{i=1}^{n} x_i (x_i - \bar{x})^2} \] (3)

C. Type-B evaluation of standard uncertainty
When the estimate value \( x \) of the measurand X is not obtained by repeated observation, the standard measurement uncertainty \( u(x) \) can be gained by collecting information that influences measurement result. This is the type-B evaluation of standard uncertainty, in which the best estimate and the standard uncertainty is obtained by method different from statistical procedures. The information sources of type B evaluation of uncertainty involve in measurement data before; experience and general knowledge about measuring instrument; technical specifications offered by manufacturing plants; certificates of inspection, certificates of calibration, reports of testing and other correlative files. Based on above information sources, the interval (-e,e) limited the measured quantity should be determined reasonably, make an assumption of the probability distribution of the measured quantity, estimate the coverage factor \( k \) according to required coverage probability, and then standard uncertainty can be calculated as:

\[
u(x) = e / k \] (4)

Where, \( e \) is the half-width of the interval.

According to ISO/TS 14253-2, for type B evaluation of standard uncertainty, experience shows that in most cases it is sufficient to use only three types of distributions for transforming limits of variation into standard deviation. These three types of distribution are Gauss distribution, rectangular or uniform distribution and U-shaped distribution.

Normal distribution is one way to evaluate uncertainty contributors so that they can be quantified and budgeted for. It allows a manufacturer to take into account prior knowledge, manufacturer's specifications, etc. Normal distribution helps understand the magnitude of different uncertainty factors and understand what is important.

Triangular distribution is most often used in evaluations of noise and vibration. The manufacturer must be more comfortable estimating the width of \( n \) variation using "hard" limits rather than a certain number of standard deviations.

Rectangular distribution is fairly conservative. The manufacturer has an idea of the variation limits, but little idea as to the distribution of uncertainty contributors between these limits. It is often used when information is derived from calibration certificates and manufacturer’s specifications.
U-shaped distribution is not as rare as it seems. Cyclic events, such as temperature, often yield uncertainty contributors that fall into this sine wave pattern.

**Table No. 1: PDF with Different Correction Factors**

<table>
<thead>
<tr>
<th>Sr.No</th>
<th>Distribution function</th>
<th>Divisor</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Rectangular</td>
<td>(\sqrt{3})</td>
</tr>
<tr>
<td>2</td>
<td>Triangular</td>
<td>(\sqrt{6})</td>
</tr>
<tr>
<td>3</td>
<td>U-shaped</td>
<td>(\sqrt{2})</td>
</tr>
<tr>
<td>4</td>
<td>Normal</td>
<td>2-for 95% confidence level &amp; 3-for 99% confidence interval</td>
</tr>
</tbody>
</table>

**D. Combined standard uncertainty**

For direct measurement, if the influence from each contributor on the uncertainty of the measurement result is evaluated as a standard uncertainty component \(u_i\) \((i = 1, 2, 3, \ldots, N)\), where \(N\) is the number of uncertainty contributors, and all the components are uncorrelated, then the combined standard uncertainty attributed to the measurement result is obtained as square root of the sum of the squares of the uncertainty components.

\[
u_c(y) = \sqrt{u_1^2 + u_2^2 + u_3^2 + \ldots + u_N^2}\]  

**E. Degrees of Freedom (DOF)**

For Type A evaluation of uncertainty, if the measurand \(x_i\) is observed independently for \(n\) times, the standard uncertainty \(u(x_i)\) can be obtained, and then its DOF can be calculated as:

\[v(x_i) = n - 1\]

For Type B evaluation of uncertainty, the DOF of standard uncertainty \(u(x_i)\) can be calculated as:

\[v(x_i) \approx \left(\frac{1}{2}\right)\left[\frac{\Delta u(x_i)}{u(x_i)}\right]^2\]  

Where \(\Delta u(x_i)\) is the relative standard uncertainty of \(u(x_i)\), its value can be obtained by experience.

According to Welch–Satterthwaite formula, effective DOF of \(\nu_c(y)\) can be calculated as:

\[V_{eff}(y) = \frac{\nu_c^4(y)}{\sum_{i=1}^{n} \frac{u^4(x_i) \left(\frac{\partial f}{\partial x_i}\right)^4}{v(x_i)}}\]  

Where \(N\) is the number of uncertainty components; \(u(x_i)\) is the standard uncertainty in both Type A and Type B evaluation for uncertainty components; and \(v(x_i)\) is the DOF of the standard uncertainty \(u(x_i)\).

**F. Evaluation of expanded uncertainty**

The expanded uncertainty \(U\) is given by,

\[U = k \cdot u_c(y)\]  

Where, \(k\) – coverage factor, numerical factor used as a multiplier of the combined standard uncertainty in order to obtain an expanded uncertainty and \(u_c(y)\) is the combined uncertainty

**G. Mathematical model for measurement**

The mathematical model in the uncertainty evaluation is,

\[Y = X_i + \Delta X \pm U\]  

Where,

\[Y = \text{True value of measurand}\]

\[X_i = \text{dimension of the measurand reported by actual inspection}\]

\[\Delta X = \text{errors in measurements and}\]

\[U = \text{expanded uncertainty}\]

**IV. EXPERIMENTAL WORK**

**Table No. 2: CMM Specifications**

<table>
<thead>
<tr>
<th>Sr. No.</th>
<th>Particular</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Size</td>
<td>1200 x 800 x 600 mm</td>
</tr>
<tr>
<td>2</td>
<td>Volumetric Accuracy</td>
<td>(2.9 + L/300) (\mu m)</td>
</tr>
<tr>
<td>3</td>
<td>Resolution</td>
<td>2 (\mu m)</td>
</tr>
<tr>
<td>4</td>
<td>Repeatability*MPE_E</td>
<td>2 (\mu m)</td>
</tr>
<tr>
<td>5</td>
<td>Software</td>
<td>VirtualDmis 5.5</td>
</tr>
</tbody>
</table>

**A. Other Specifications**

1. Scanning speed of probe: 10mm/sec
2. Stylus used: Silicon Nitride Stylus M3 XXT,
3. Uncertainty in Calibration Gauge Block of 50 mm : 1.1 \(\mu m\)
4. Room Initial temperature (T₁): 20 °C
5. Room Temperature at end(T₂): 21°C
6. Reference Temperature(Tref): 20 °C
7. Humidity: 60 ±5 %
8. Thermal coefficient of Silicon Nitride Sphere(\(\alpha_p\)): 2.9x10-6 mm/m °C

**Figure No.2: Experimental Setup**

**Stylus**

**V-Clamping**

**Square block**

**Cylindrical block**
V. INSPECTION DETAILS, RESULTS AND CALCULATIONS

A. Standard Uncertainty for Inspection reading measured for Bore Diameter, Concentricity & Cylindricity i.e. Type-A Uncertainty

Table No. III: Inspection report for Bore diameter, Concentricity & Cylindricity

<table>
<thead>
<tr>
<th>Readings Sr. No.</th>
<th>Dimensions Measured</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>I.D OF H-1</td>
<td>CONCENTRICITY OF I.D (H-1)</td>
</tr>
<tr>
<td></td>
<td>(mm)</td>
<td>(μm)</td>
</tr>
<tr>
<td>1</td>
<td>52.4128</td>
<td>21</td>
</tr>
<tr>
<td>2</td>
<td>52.4133</td>
<td>22</td>
</tr>
<tr>
<td>3</td>
<td>52.4133</td>
<td>23</td>
</tr>
<tr>
<td>4</td>
<td>52.4139</td>
<td>24</td>
</tr>
<tr>
<td>5</td>
<td>52.4129</td>
<td>23</td>
</tr>
<tr>
<td>6</td>
<td>52.4121</td>
<td>22</td>
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<td>7</td>
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<td>22</td>
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<tr>
<td>8</td>
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<td>52.4133</td>
<td>23</td>
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<tr>
<td>10</td>
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<tr>
<td>11</td>
<td>52.4142</td>
<td>22</td>
</tr>
<tr>
<td>12</td>
<td>52.4144</td>
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<tr>
<td>13</td>
<td>52.4133</td>
<td>23</td>
</tr>
<tr>
<td>14</td>
<td>52.4151</td>
<td>23</td>
</tr>
<tr>
<td>15</td>
<td>52.4123</td>
<td>26</td>
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<td>16</td>
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<tr>
<td>17</td>
<td>52.4110</td>
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<tr>
<td>18</td>
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<tr>
<td>19</td>
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<td>52.4129</td>
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</tr>
<tr>
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<td>52.4128</td>
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<tr>
<td>23</td>
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<tr>
<td>24</td>
<td>52.4131</td>
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<td>27</td>
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<td>28</td>
<td>52.4151</td>
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<tr>
<td>29</td>
<td>52.4139</td>
<td>23</td>
</tr>
<tr>
<td>30</td>
<td>52.4158</td>
<td>22</td>
</tr>
<tr>
<td>Mean(mm)</td>
<td>52.4135</td>
<td>23.033</td>
</tr>
</tbody>
</table>

B. Type-B Uncertainty Calculation

1. Standard uncertainty due to change in lab Temperature $u(x_i)$ and assuming rectangular distribution

$$ u(x_i) = \Delta T / \sqrt{3} $$

$$ = 1 / \sqrt{3} $$

$$ = 0.5774 \text{ μm} $$

Standard uncertainty due to Thermal expansion of probe material $u(x_i)$.

By considering only 10% of $\alpha_p$,

Estimated uncertainty = $0.1 \times 0.0029 = 0.00029 \text{ μm} $

Assuming ‘U’ distribution,

$$ u(x_i) = 0.00029 / \sqrt{2} = 0.00021 \text{ μm} $$

2. Standard uncertainty due to Resolution of Equipment $u(x_i)$,

Estimated uncertainty = 2 μm

Assuming Rectangular distribution,

$$ u(x_i) = 2 / \sqrt{3} = 1.1547 \text{ μm} $$

3. Standard uncertainty due to Form error in Reference Sphere $u(x_i)$,

Estimated uncertainty = 0.071 μm

Assuming Rectangular distribution,

$$ u(x_i) = 0.071 / \sqrt{3} = 0.0409 \text{ μm} $$

4. Standard uncertainty due to calibration of Gauge blocks $u(x_b)$,

Estimated uncertainty = 1.1 μm (from Calibration certificate),

Assuming Rectangular distribution,

$$ u(x_b) = 1.1 / \sqrt{3} = 0.6350 \text{ μm} $$

VI. RESULT AND DISCUSSION

A. Expanded Uncertainty for Bore Diameter measurement

Table No. IV: Expanded uncertainty sheet for Bore Diameter
B. Expanded uncertainty for Concentricity measurement

Table No. V: Expanded uncertainty sheet for Concentricity

<table>
<thead>
<tr>
<th>Source of Uncertainty for Concentricity Width</th>
<th>Estimate (μm)</th>
<th>PPM Typical correction factor</th>
<th>Standard Uncertainty</th>
<th>Sensitivity Coefficient</th>
<th>Degrees of Freedom</th>
<th>Uncertainty (μm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type-A</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Temperature</td>
<td>0.0023</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Thermal expansion of gauge</td>
<td>0.0025</td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mechanical error</td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gauge on Reference Standard</td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Calibration of Gauge &amp; Work Tool</td>
<td></td>
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</tr>
</tbody>
</table>

Maximum Expanded Uncertainty for Concentricity Width:

\[ U = \pm 2.9721 \, \text{μm} \]

Maximum Expanded Uncertainty for Concentricity Width:

\[ U = \pm 2.9250 \, \text{μm} \]

C. Expanded uncertainty for Cylindricity measurement

Table No. VI: Expanded uncertainty sheet for Cylindricity

<table>
<thead>
<tr>
<th>Source of Uncertainty for Cylindricity Width</th>
<th>Estimate (μm)</th>
<th>PPM Typical correction factor</th>
<th>Standard Uncertainty</th>
<th>Sensitivity Coefficient</th>
<th>Degrees of Freedom</th>
<th>Uncertainty (μm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type-A</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Temperature</td>
<td>0.0022</td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Thermal expansion of gauge</td>
<td>0.0025</td>
<td></td>
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<tr>
<td>Mechanical error</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gauge on Reference Standard</td>
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<td>Calibration of Gauge &amp; Work Tool</td>
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<td></td>
</tr>
</tbody>
</table>

Maximum Expanded Uncertainty for Cylindricity Width:

\[ U = \pm 2.9721 \, \text{μm} \]

Maximum Expanded Uncertainty for Cylindricity Width:

\[ U = \pm 2.9250 \, \text{μm} \]

D. Measurement Results

After Following ISO-GUM Procedure to find out Type-A and Type-B uncertainty results, for the expanded uncertainty are stated as below:

Maximum Expanded Uncertainty for Bore Diameter, \( U = \pm 2.9331 \, \text{μm} \).

Maximum Expanded Uncertainty for Concentricity, \( U = \pm 2.9721 \, \text{μm} \).

Maximum Expanded Uncertainty for Cylindricity, \( U = \pm 2.9250 \, \text{μm} \).

VII. CONCLUSION

By analyzing the above said experimental results of uncertainty estimation, following points needs to be considered:

- Inspection Laboratories must calculate the uncertainty
- Budget as a part of measurement evaluation to provide high quality measurement results.
- When designer takes into consideration Tolerance, he must be aware to the measurement uncertainty and must specify this.

- It is seen that while measuring critical dimensions i.e. Very small dimensions, the value of uncertainty has significant impact on measurement result, so it should be stated on inspection result.
- Time to time calibration of measuring instrument is to be done to improve the quality of measurement, correspondingly achieve the optimum economic efficiency of technology.

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REFERENCES


AUTHOR PROFILE
