Fast Legendre Moments in Securing Digital Image

K. Alice Suresh, S. Thirunavukkarasu, P. Kavitha

Abstract: Moments are set of values used to describe the information contained in the image. In this paper the content of the image is represented with the help of fast legendre moments. Legendre moments has the advantage that these moments are calculated exactly without any loss in information while other moments like geometric moments, zernike moments etc suffer from approximation errors and geometric error when applied to digital images. Legendre moments are also more suitable for image reconstruction. Experimental results show that the proposed system is very efficient in computing moments at much faster time than the Zernike moments since fast legendre moments calculates the moments as two 1D function rather as a 2D function of a digital image and the results of reconstruction in case of tampering is also shown.


I. INTRODUCTION

Digital images are used in a wide range of applications for the past two decades and as a result of that many image editing applications challenges the security and integrity of images. There are many ways a hash can be generated by extracting local and global features [4], fixed point theory[7], watermarking methods [3][8], NMF[9][12], using fourier mellin transformation[12], DFT coefficients, radon transform etc. Hashing is a method to generate hash code because it is time expensive. But with fast legendre moments the time complexity is very much reduced [21]. Two dimensional fast legendre moment function for a 2D image is computed as two, 1D function and hence the number of additions and multiplications is reduced. Further the moment’s basis function can be calculated based on the size of the image and order of moment and can be stored in advance irrespective of image for fast computation.

In image self recovery, geometric moments cannot be used since the basis function \(x_\theta y_\theta^q\) is not orthogonal. Zernike moments though they have an orthogonal basis function image reconstruction is severely handicapped due to the presence of geometric errors and approximation errors and as the moment order increases these approximation errors goes out of control. The more suitable moment for image reconstruction is legendre moments as it does not have approximation errors and image content can be extracted without any information loss [22].

II. LEGENDRE MOMENTS

Moments are set of values used to describe the information contained in the image. There are infinite numbers of moment values for obtaining the information of an image. It is very important to identify specific set of moment values that best describe the information contained in an image.

The set of direct legendre moments for a grey scale image of size \(M \times N\) is given as [22]

\[
L_{pq} = \sum_{i=1}^{M} \sum_{j=1}^{N} I_{p}(x_i)I_{q}(y_j)f(i,j) - \rightarrow 1
\]

Where \(f(i,j)\) is the input image and the moment kernel function is given as

\[
I_{p}(x_i) = \frac{2p+1}{2p+1} (u_{i+1} - u_{i-1}) - (x_i - 1)
\]

\[
I_{q}(y_j) = \frac{2q+1}{2q+1} (v_{j+1} - v_{j-1}) - (y_j - 1)
\]

\[
u_{i+1} = -i + (i-1) \Delta x_i \quad \text{and} \quad \Delta x_i = 2/M
\]

\[
v_{j+1} = -j + (j-1) \Delta y_j \quad \text{and} \quad \Delta y_j = 2/N
\]

The moment kernel function is independent of input image and if the size of the image and the order of moment is known then the moment kernel function can be calculated and can be stored in advanced and reused whenever needed.

Eqn-1 is valid only for \(p \geq 1\) and \(q \geq 1\).

So the first row \(p=0,q=0,1,2,3\ldots\) Max is calculated using

\[
p=0,q=0,1,2,3\ldots\text{Max}
\]
The image function \( f(x,y) \) can be written as an infinite series expansion in terms of the Legendre polynomial over the square \([-1,1] \times [-1,1]\) is given as

\[
f_{\text{max}}(x_i,y_j) = \sum_{p=0}^{\infty} \sum_{q=0}^{\infty} L_{pq}(x_i)p_q(y_j) - \rightarrow 4
\]

If the order of moment is smaller than infinity then the above Eqn 4 can be rewritten as

\[
f_{\text{max}}(x_i,y_j) = \sum_{p=0}^{\text{Max}} \sum_{q=0}^{\text{Max}} L_{pq}(x_i)p_q(y_j) - \rightarrow 5
\]

where Max refers to the maximum order of moments and the total number of moments used in reconstruction of image is given as

\[
N_{\text{TOTAL}}= \frac{(\text{MAX}+1)(\text{MAX}+2)}{2}
\]

### III. DIGITAL IMAGE SECURITY USING FAST LEGENDRE MOMENTS

The image of size M x N is accepted as input and image undergoes a preprocessing stage. In preprocessing the image of size M x N is converted to a fixed size of 256 x 256 using bilinear interpolation. A Gaussian filter is applied to the image to remove any additive noise. Then the image is converted to gray scale image and for this gray scale image fast legendre moments are calculated. The order of moment \((p+q) = n\) is equal to the order of the moment ‘n’. The exact content of the image can be reconstructed using Legendre moment more efficiently than Zernike moments.

### IV. SELF RECOVERY IN CASE OF TAMPERING

For any image if the security code generated varies from that of actual code computed during storing the image then there may be any data change or loss of information in the image. That is this represents that the content of the image is now changed. In that case the original image can be reconstructed using Eqn-5 in which Max is equal to the order of the moment ‘n’. The exact content of the image can be reconstructed using Legendre moment more efficiently than Zernike moments.

### V. PARAMETER SELECTION

The proposed study uses MATLAB (2013a) environment. The image dataset is calculated from internet and CASIA database [17]. The size of the image is considered to be 256 x 256, the reason being, if the size of the image is too small then the features cannot be extracted correctly and if it is too large then it increase the computational complexity. A Gaussian filter is applied to eliminate noise. Fast legendre moments are used to extract the features of the image. Since the number of additions is of \(O(NM^3)\) and multiplications is of \(O(M^2)\) this algorithm is efficient when compared with direct method.

The Euclidean distance between the actual security code \((z_1)\) with that of computed security code \((z_2)\) can be tolerated up to a threshold \(\varepsilon\) to allow content preserving transformation. \(D=||z_1-z_2||\) if the distance D is less than the threshold \(\varepsilon\) then the image is considered to be unaltered. The proposed system considered \(\varepsilon = 7\).

### VI. RESULT AND DISCUSSION

The fast legendre moment for a sample matrix of size 4 x 4 \(A=[3 \ 2 \ 1 \ 4; 2 \ 1 \ 3 \ 2; 4 \ 1 \ 2 \ 1; 5 \ 2 \ 1 \ 2]\) and order 3 is calculated using the formula described in section II, and the moments \(L_{pq}\) are as shown in TABLE-2.
The Kernel generation time for fast Legendre moments for various order for a image size of 64 x 64 is as shown in TABLE-3 and its graph when compared to computation time of Zernike moments and direct Legendre moment is represented as shown in Figure-1.

<table>
<thead>
<tr>
<th>Order</th>
<th>Fast Legendre Moments</th>
<th>Direct Legendre Moments</th>
<th>Zernike Moments</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0.3516</td>
<td>0.6530</td>
<td>0.31897</td>
</tr>
<tr>
<td>20</td>
<td>0.7910</td>
<td>0.6660</td>
<td>0.62938</td>
</tr>
<tr>
<td>30</td>
<td>1.2191</td>
<td>0.6713</td>
<td>0.96111</td>
</tr>
<tr>
<td>50</td>
<td>1.2500</td>
<td>0.6030</td>
<td>0.83219</td>
</tr>
</tbody>
</table>

From Figure-1 it is clear that the Elapsed time increases with increase in moments for all the three types of moments. For fast Legendre moments the elapsed time is less than one second for order up to 100, but for Zernike moments the time increases up to 129 seconds for the same order. In terms of CPU elapsed time fast Legendre moments is most efficient than Zernike moments.

The performance of reconstructed image can be analyzed using some commonly available criteria like Mean square error (MSE) and peak signal to noise ratio (PSNR). For an image of size M x N the MSE and PSNR are given as

\[
MSE = \frac{1}{MN} \sum_{i=1}^{M} \sum_{j=1}^{N} (f_{\text{max}}(x_i,y_j) - f(x_i,y_j))^2
\]

\[
PSNR = 10 \times \log_{10}\left(\frac{2^{16}-1}{\text{MSE}}\right)
\]

Since the moments which describe the global features of the image are only taken into consideration the generated code can miss small local information and also that the reconstruction error increase with the increase in order of moment [22] and thus the robustness of security code is greatly challenged. In future this global feature may be accompanied with content preserving local feature extraction techniques to generate code.

**Conflict of Interest:** Nil

**REFERENCES**

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