

# Effect of Longitudinal Forces Due to Loads on Prestressed Mono-Block Sleeper Spacing



Mohamed Y Mohsen, Hany I Ahmed, Hany S Riad, Amr A Abdel Rahman

**Abstract:** The present paper proposes scientific and practical methodology to update the concept of the constant sleeper spacing along the railway track to be reset according to the affecting normal forces caused by passenger and freight trains. The proposed methodology has developed a suitable sleeper spacing plan according to three cases which are train acceleration, uniform speed and braking on -5 ‰, 0 ‰ and 5 ‰ grades. The study aims to determine the actual acceleration length, braking length, longitudinal forces, displacement index and finally the suitable sleeper spacing for each part on the track, then calculating the saving in sleepers for the following cases: passenger train runs on single or double track, freight train runs on single or double track and mixed traffic (passenger and freight) runs on single or double track.

**Keywords:** feasibility study on sleeper spacing longitudinal forces on railway track, pre-stressed mono-block concrete sleeper, railway normal forces, sleeper spacing, track creep

## I. INTRODUCTION

The railway track is subjected to forces that are vertical, transversal and longitudinal which are generated by the rolling stock running on the track apart from the forces that are exerted due to earthquake as shown in “Fig. 1”.

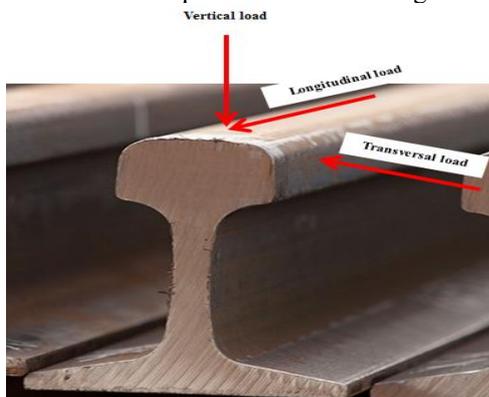


Fig.1. Forces on rail

The present paper deals with the effect of longitudinal forces on both the rail surface and the corresponding contact pressure between sleeper and ballast.

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Some of longitudinal forces are considered as a part of vertical Loads acting on the track, either directly or indirectly [1], while the other part is due to thermal effects. They are playing a very important role in the design, construction, operation and the maintenance of the track, their strain determine the sleepers’ material, spacing, fastenings, dimensioning of the elastic pads and ballast depth [2]. The Longitudinal forces are exerted on the rail rolling surface and are transferred to the subgrade through the track components. During their transfer, the surface area increases, while the developing stresses decrease [3, 4]. The longitudinal forces are transferred by the wheels to the rails through the rail rolling surface; they are transferred to the track bed layers and distributed to a larger number of sleepers compared with the vertical loads.

Longitudinal forces due to vertical loads

- Driving wheels forms a compression when locomotive is pulling the train and make tension when pushing the train
- Idle wheels forms tension when locomotive is pulling the train and make compression when pushing the train and approximately equal to  $R_r W$
- Braked wheels always form tension
- Wheels on an inclined plan forms compression equal to  $R_g W$

Longitudinal forces due to thermal effects

- Splice tapping force forms compression at rail thermal expansion and tension at contraction
- Track creep resistance forms compression at rail thermal expansion and tension at contraction
- A force generated due to the thermal expansion after closing the gap (always compression)
- A force generated due to the thermal contraction after reaching the gap its maximum value  $\Delta_{max}$  (always tension)

## II. LONGITUDINAL FORCES DUE TO VERTICAL LOADS

There are four cases to be studied:

- Longitudinal force result from driving wheel
- Longitudinal force result from idle wheel rotation
- Longitudinal force result from applying either brake shoes or electrical system
- Longitudinal force result from wheel rotation on inclined plan

**A. Longitudinal force result from driving wheel**

Longitudinal forces result from the movement of a driving wheel is due to a couple generated by the locomotive rotating the wheel, it can be analyzed by two opposed direction and equal forces  $F_p$  “(1)”. As well as, a friction force  $F_\mu$  “(2)”, is generated at the wheel – rail contact.

Where:

$$F_p = 270 \eta H_p / S \quad (1)$$

$$F_\mu = \mu_m W_{L2} \quad (2)$$

$\eta$ : Engine power efficiency (taken 0.81 as an average value)

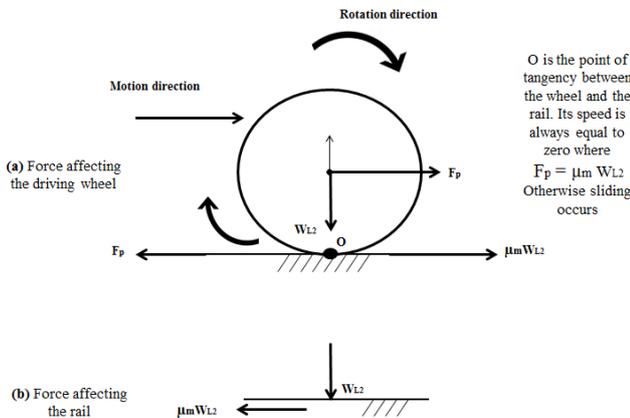
$H_p$ : Locomotive horse power

$S$ : speed Km/hr

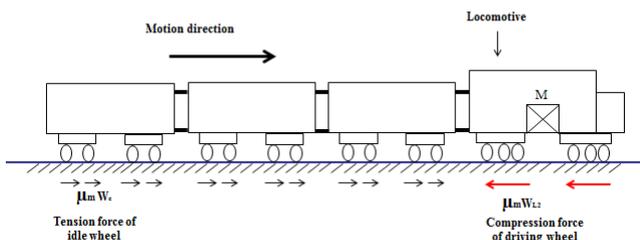
$\mu_m$ : friction coefficient

$W_{L2}$ : weight on the locomotive driving wheel

Frictional force ( $F_\mu$ ) is in the motion direction which causes the wheel transition movement. To make the wheel stable on the track, the speed of the contact point O should be always equal to zero as shown in “Fig. 2”, (a) for driving wheel and (b) for rail surface which means  $F_p = F_\mu$

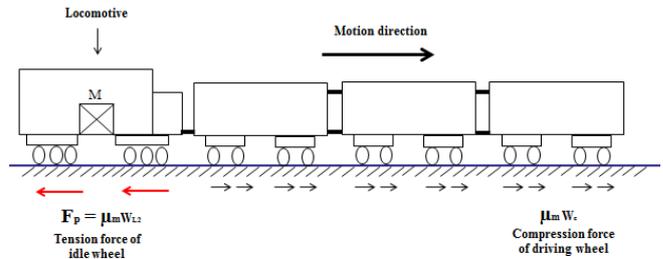


**Fig.2. Forces affecting: (a) Driving wheel, (b) Rail surface**



**Fig.3. Forces generated when locomotive pulls a train**

This frictional force generated from the track, so the wheel reaction will be an opposed direction (to the motion direction). An equivalent force causes a track compression if the locomotive pulling the train as shown in “Fig. 3”, while causes track tension if the locomotive pushing the train as shown in figure “Fig. 4”.



**Fig.4. Forces generated when locomotive pushes a train**

**B. Longitudinal force result from idle wheel rotation**

Idle wheel are not connected to the engine and rotates due to the force  $F$ , and as a result of the wheel forward transfer (not rotation), a backward frictional force is generated  $F\mu_m$  “(1)”, as well as a system of frictional forces generated  $F\mu_c$  “(2)”, as shown in “Fig. 5”, (a) for idle wheel or car wheel and (b) for rail surface.

$$F\mu_m = \mu_m W_{L1} \quad (1')$$

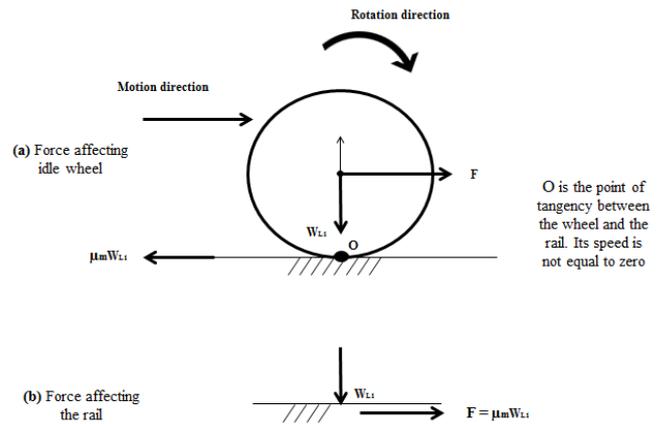
$$F\mu_c = \mu_m W_c \quad (2')$$

Where

$W_{L1}$ : weight on the locomotive idle wheel.

$W_c$ : weight of the train cars.

This frictional force opposes the direction of movement and equals it. It generates a couple causes a rotation beside the transfer generated by the force  $F$ , thus the speed of point O is not equal to zero because it is unbalanced.



**Fig.5. Forces affecting: (a) Idle wheel, (b) Rail surface**

As the frictional force is generated from the track, the wheel reaction to the track will be an opposite direction force and equal in magnitude in the movement direction causing tension force on the track when the locomotive were pulling the train and cause track compression when the locomotive were pushing the train. The effect of driving wheels on the track is opposite to the effect of the idle wheels, as the driving wheel rotates by the couple generated from the engine and ends with friction force in the movement direction helps in transfer, where the idle wheels starts by tension or compression force by the rolling stock movement together and ends with a couple which rotates those wheels. Important note: Friction coefficient  $\mu_m$  is the value of the instantaneous friction between the wheel and rail and it's less than its maximum value  $\mu$  given in “(3)”.

$$\mu = (9000 / (42 + S) + 116) / 1000 \quad (3)$$

**C. Longitudinal force result from applying either brake shoes or electrical system**

- Brake shoes: Braking process generates a couple opposite to wheel rotation direction and can be analyzed into two opposite forces  $F_b$  “(4)”, and equal in magnitude.

$$F_b = f \cdot P \quad (4)$$

Where

$f$ : friction coefficient between the wheel and shoe and can be calculated from Shredder’s formula “(5)”.

$$f = 6.21(0.001 \times S^2) - 1.79(S/1000) + 0.241 \quad (5)$$

$P$ : pressure force generated by brake shoe either applying air compression or suction “(6)”.

$$P = \eta_b W_{eb} \quad (6)$$

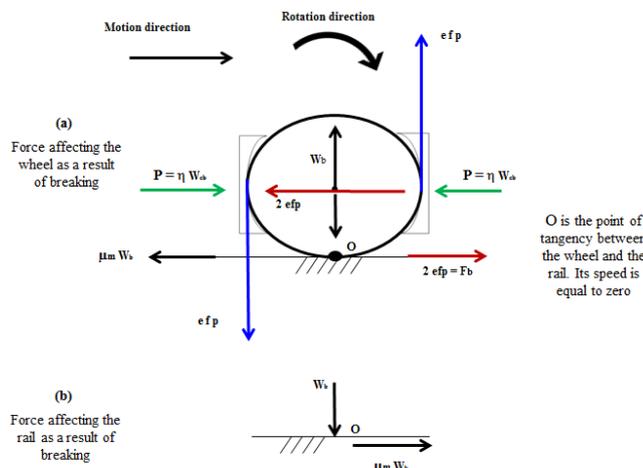
Where:-

$W_{eb}$ : empty weight of the braked wheel

$(\eta_b)$ : percent to  $W_{eb}$ , Taking  $(\eta_b) = 0.45$  (for passenger’s cars and locomotive) and  $= 0.30$  (for freight cars)

$(e)$  = Brake efficiency ranged from 0.92 to 0.98 with an average value 0.95

“Fig. 6”, (a) for braking wheel and (b) for rail surface, shows a generated force equal to  $\mu_m W_b$  at the contact point between the wheel and rail (o) and opposite to the motion direction which resist wheel movement, beside the anti-rotation couple resulting in a fully stop to the wheel. This force affects the rail in the train motion direction causing a tensile force.



**Fig.6. Forces affecting: (a) braking wheel, (b) Rail surface**

- Electrical brake: When using the electrical brakes it is subjected with speed  $S$  through “(7)”.

$$F_b S = Constant = (W_b/g) b S \quad (7)$$

Where:-

$b$ : deceleration due to braking

$W_b$ : weight on the braked wheel

$g = \tan \alpha$  = track gradient

Note: deceleration rate of change is known as Jerk  $J$  and in relation with  $(b)$  at any moment through “(7)”.

$$J \times b = constant \text{ (m/s}^3\text{)} \quad (7')$$

**D. Longitudinal force result from wheel rotation on inclined plan**

The wheel moves downward due to its own weight  $W$  which can be analyzed into two components, the first one parallel to the inclined plan while the other is applied perpendicular to it.

$$\text{The parallel component} = W \sin \alpha \quad (8)$$

Where:

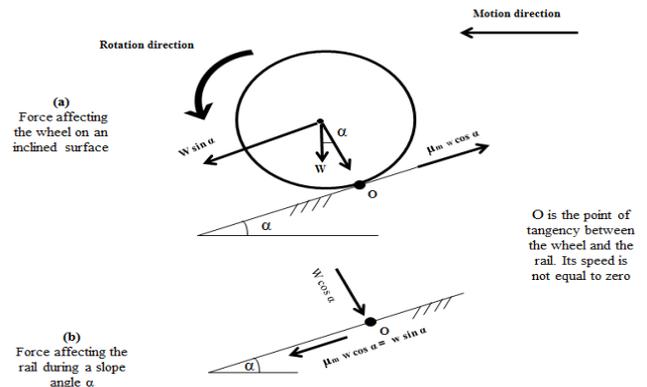
$\alpha$ : track inclination angel to the horizontal

$$\text{The perpendicular component} = W \cos \alpha \quad (9)$$

An upward frictional force is generated as a result of the friction between wheel and rail.

$$\text{Upward frictional force} = \mu_m W \cos \alpha \quad (10)$$

This frictional force in “(10)”, generates a couple with the parallel component in “(8)”, that rotates the wheel anti-clock wise as shown in “Fig. 7”, (a) which represents forces acting wheel rotates on inclined track while (b) represents forces acting on rail surface, hence the rotational motion transfers the wheel downwards.



**Fig.7. Forces affecting: (a) wheel on an inclined track, (b) Inclined rail surface**

The generated couple has two opposite forces  $W \sin \alpha$  and  $\mu_m W \cos \alpha$  having the same value applying on the axle and the contact point between wheel and rail respectively, thus

$$W \sin \alpha = W g = W R_g = \mu_m W \cos \alpha \quad (11)$$

Where:-

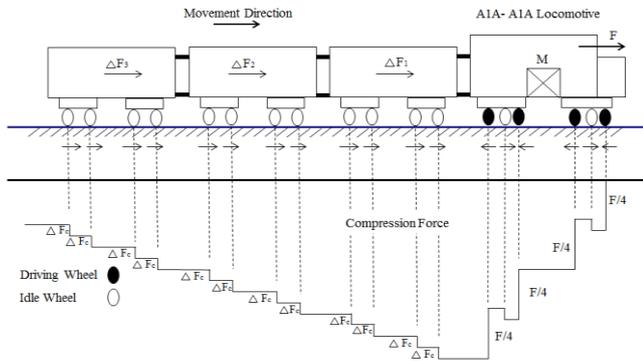
$R_g$ : grade resistance (Kg/ton)

Finally, the effect of wheel on the rail surface can be summarized within the following four cases:

- Case 1: driving wheel  $F_p$  causes a compression if the locomotive is pulling the train but causes a tension if pushing the train.
- Case 2: idle wheel causes a tension if the locomotive is pulling the train but causes a compression if pushing the train.
- Case 3: braking wheel  $F_b$  causes tension
- Case 4: inclined wheel causes compression stresses



# Effect of Longitudinal Forces Due to Loads on Prestressed Mono-Block Sleeper Spacing

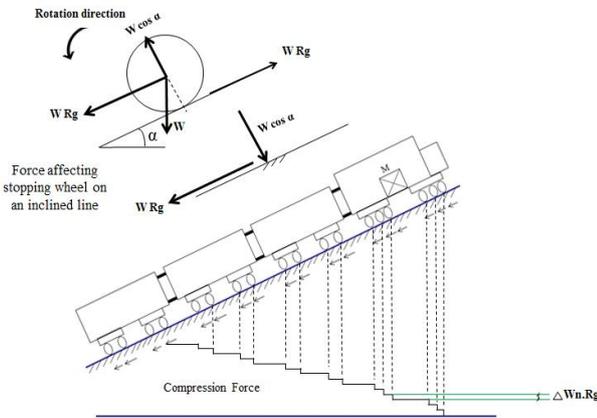


**Fig.8.** N.F.D for railway track due to movement of a train consists of A1A-A1A locomotive and numbers of cars on a horizontal line

### III. APPLICATIONS

#### A. Train runs on horizontal straight track

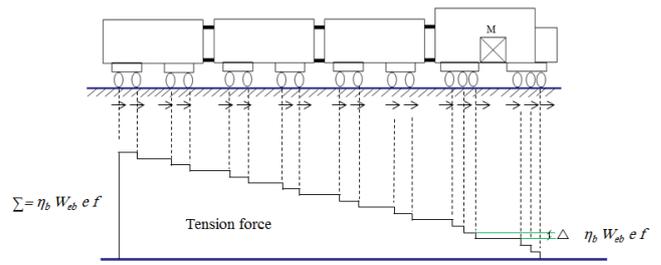
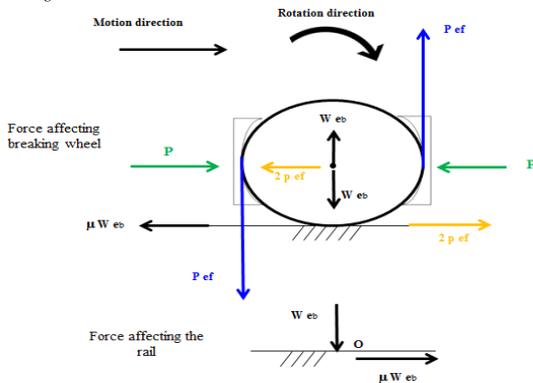
“Fig. 8”, illustrates the normal force diagram (N.F.D) on rail surface for A1A-A1A locomotive due to its movement on a horizontal straight track.



**Fig.9.** N.F.D for railway track due to stopping of a train consists of A1A-A1A locomotive and numbers of cars on an inclined line with grade  $R_g$

#### B. Train stops on an inclined track

“Fig. 9”, illustrates the normal force diagram on rail surface for A1A-A1A locomotive due to stopping on a grade of  $R_g$ .



**Fig.10.** N.F.D for railway track due to brake of a train consists of A1A-A1A locomotive and numbers of cars

Note:

- $R_g$  (gradient forces) =  $\sin \alpha = \tan \alpha$  ( $\alpha$  is very small).
- Force affecting on the track resulting from traction force is downward (compression) if the train is moving up “Fig. 8”, and upward when the train moving down (compression).
- Force affecting on the track resulting from the grade is always downward (compression) regardless the train movement direction up or down or stop “Fig. 9”.
- In case of braking the force affecting on the track is upward when the train moving up and downward when the train moving down (always tension).

#### C. Braked wheel

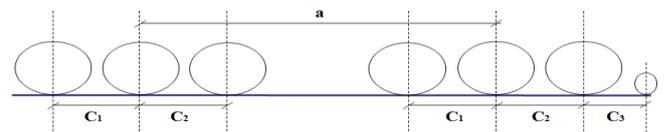
“Fig. 10”, illustrates the normal force diagram on rail surface for A1A-A1A locomotive due to brake.

Note:

- If the locomotive pulls the cars on an upward or downward gradient, the resultant N.F.D is compression and equal to the sum of case A + case B.
- If the locomotive pushes the cars on an upward or downward gradient, the resultant N.F.D is the difference of case A - case B (tension)
- If the train is braked, tensile forces affect the track regardless the direction of movement as shown in “Fig. 10”.
- When train is braked, a force will be generated in the direction of its movement regardless if moving upwards or downwards causing tensile normal stress, thus the resultant force for a train braked on an inclined track, the N.F.D will be obtained as the difference between case B and case C.

### IV. CASE STUDIES

Applying the locomotives having the following dimensions of wheelbase locomotive type A1A-A1A as shown in “Fig. 11” and Table I.



**Fig.11.** Dimensions of wheelbase locomotive type A1A-A1A

**Table- I: Dimensions of wheelbase locomotive type A1A-A1A (mm)**

items	Wheel type		
	disc	11- spoke	14-spoke
a	8788	10668	11601
C1	2133	2286	1800
C2	2133	2286	1800
C3	1676	1676	1676

Where:

a: distance between midpoints of bogies or between center axles for 3-axle bogies

c1: outer to center axle distance on 3-axle bogies.

c2: center to inner axle distance on 3-axle bogies.

c3: outer powered axle to pony distance.

There are two train types to be

studied, passenger train and freight one. Each train will be studied while running with acceleration, running with maximum speed, braking and stopping on 5% upgrade, 0% level, and -5% downgrade.

A passenger train composes of a locomotive type A1A-A1A having 125 ton weights, 2500 Hp power and 18.6 meter length pulls 9 cars with 50 ton weight and 20 meter length. To analyze the longitudinal forces acting on the surface, it is important to determine the axel loads within the whole train and the corresponding distance between two successive axels.

Number axels = 6 (for locomotive) + 4×9 (for cars) = 42 axels

Train weight = 125 + 9×50 = 575 ton

Train length = 18.6 + 9×20 = 198.6 m

A freight train Study of freight train composes of a locomotive type A1A-A1A having 125 ton weights, 2500 Hp power and 18.6 meter length pulls 25 cars with 65 ton weight (15 ton dead weight), 20 cars are braked, 17 meter length and a 20 meter breakvan weighs 45 ton runs on 5% upgrade, 0%, -5% downgrade with its maximum speed, accelerates, brakes and stops for the two cases: loaded train and empty one.

Number axels = 6(for locomotive) + 4×26 (for cars) = 110

Train weight = 125 + 25×65 + 45 = 1795 ton

Train length = 18.6 + 17×25 + 20 = 463.6 m

“Fig. 12”, shows how sleeper spacing will be optimized according to the following conditions:

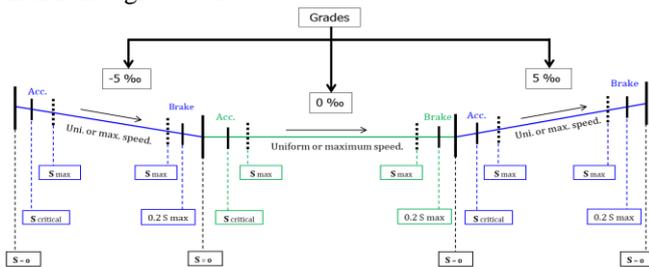
-Train type (passenger and freight)

- Speed (stationary state, critical speed and uniform speed)

- Running state (acceleration, uniform speed and braking)

- Grades (-5 %, 0 % and 5%) which are the three values representing the relation between the sleeper spacing versus the running state

In General, the results for any grade can be obtained either by interpolation or by extrapolation using the three above mentioned grade values.



**Fig.12. Proposed methodology for passenger and freight trains when running on grades -5 %, 0 % and 5% while acceleration, running with maximum speed and braking**

**A. The following equations explains how to get the best sleeper spacing for the both above mentioned trains during acceleration state starting from speed 0 km/hr passing by critical speed until reaching maximum speed.**

- Acceleration length for stage 1: zero to critical speed

$$220 H_p / S_{critical} = (9000 / (42 + S_{critical}) + 116) / 1000 W_{L2} \quad (12)$$

$$220 H_p / S_{max} = W_t (R_{(R+a)} + R_g) \quad (13)$$

$$R_{(R+a)} = 2.2 + 3[(S_{max} + 15) / 100]^2 \quad (14)$$

$$R_{(R+a) average} = (R_{(R+a)S=0} + R_{(R+a)S critical}) / 2 \quad (15)$$

$$\sum R = R_{(R+a) average} + R_g \quad (16)$$

$$F_{\mu average} = (F_{\mu S=0} + F_{\mu S critical}) / 2 \quad (17)$$

$$F_{\mu' average} = F_{\mu average} / W_t \quad (18)$$

$$F_{a' average} = F_{\mu' average} - \sum R \quad (19)$$

$$L_{a \mu} = 4.2 (S_{critical}^2 - (0)^2) / F_{a' average} \quad (20)$$

Where:

$H_p$ : Locomotive engine power in Horsepower

$S_{critical}$ : Critical speed in Km/hr

$S_{max}$ : maximum train speed in Km/hr

$R_{(R+a)S critical}$ : Rolling and air resistance at critical speed kg/ton

$R_{(R+a)S=0}$ : Rolling and air resistance in at 0 km/hr in kg/ton

$R_{(R+a) average}$ : average rolling and air resistance in kg/ton

$\sum R$ : Total resistance

$R_g$ : grade resistance in kg/ton

$F_{\mu average}$ : average friction force in kg

$F_{\mu' average}$ : specific average friction force in kg/ton

$W_t$ : Train weight in ton

$F_{a' average}$ : specific average acceleration force in kg/ton

$L_{a \mu}$ : acceleration length till reaching critical speed in meters

- Acceleration length for stage 2: critical speed to maximum speed.

$$F_{p average} = (F_{pS critical} + F_{pS max}) / 2 \quad (21)$$

$$F_{p' average} = F_{p average} / W_t \quad (22)$$

$$F_{a' average} = F_{p' average} - \sum R \quad (23)$$

$$L_{a f} = 4.2 ((S_{max})^2 - (S_{critical})^2) / F_{a' average} \quad (24)$$

Where:

$F_{p average}$ : average force generated by the locomotive in kg

$F_{pS critical}$ : force generated by locomotive engine at critical speed in kg

$F_{pS max}$ : force generated by locomotive engine at maximum speed in kg

$F_{p' average}$ : specific average force generated by the locomotive in kg

- Longitudinal force for two positions (running on horizontal + stopping on inclined grade) to determine the worst case in stage 1.

Position 1: Running on horizontal

$$F_{L (loco) 1} = F_{\mu average} - (W_L \times R_r) \quad (25)$$

$$F_{L (train) 1} = F_{\mu average} - (W_T \times R_r) \quad (26)$$

Position 2: Stopping on inclined

$$F_{L (loco) 2} = W_L \times R_g \quad (27)$$

$$F_{L (train) 2} = W_T \times R_g \quad (28)$$

Where:

$F_{L (loco) 1}$  &  $2$ : The maximum longitudinal force at the rail surface occurs at the latest locomotive axle for position 1 & position 2 respectively.



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$F_{L (train)1}$  & 2: The maximum longitudinal force at the rail surface occurs at the latest car axle.

$W_L$ : Locomotive weight in ton

$R_r$ : Rolling resistance in kg/ton

Applying super position at the following position

Latest locomotive axle= eq. (25) + eq. (27) (29)

Latest car axle= eq. (26) + eq. (28) (30)

- Longitudinal force for two positions (running on horizontal + stopping on inclined grade) to determine the worst case in stage 2.

Position 1: Running on horizontal

$$F_{L (loco) 1} = F_{paverage} - (W_L \times R_r) \quad (31)$$

$$F_{L (train) 1} = F_{paverage} - (W_T \times R_r) \quad (32)$$

Position 2: Stopping on inclined

$$F_{L (loco) 2} = W_L \times R_g \quad (27)$$

$$F_{L (train) 2} = W_T \times R_g \quad (28)$$

Applying super position at the following position

Latest locomotive axle= eq. (31) + eq. (27) (33)

Latest car axle= eq. (32) + eq. (28) (34)

- Sleeper Spacing  

$$y = (L_s/60) \times F_L / (L_{critical} \times f_s) \quad (35)$$

Where:

$y$  = displacement index due to longitudinal force  $F_L$  relative to

**Fig.13. Procedure of sleeper spacing calculations during train acceleration on -5 %, 0 % and 5% grades**

$L_s$  = sleeper spacing in cm

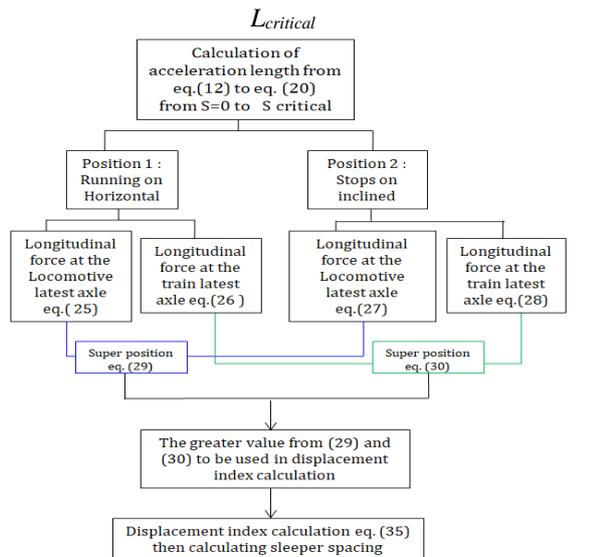
$F_L$  = maximum longitudinal force in kg

$L_{critical}$  = the length corresponding to the maximum force in cm

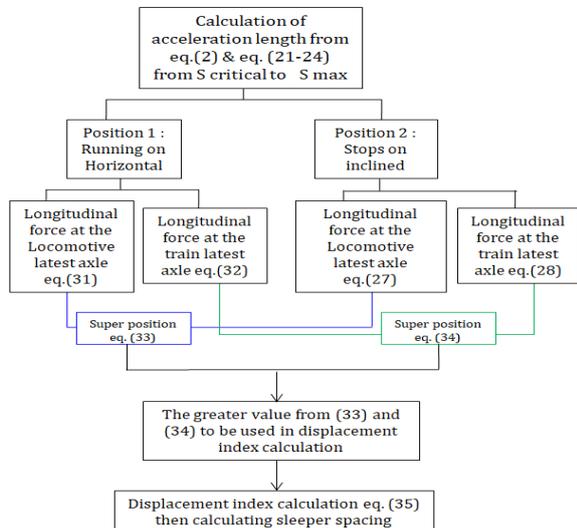
$f_s$  = sleeper longitudinal creep, taken 5 kg/cm for spacing 60 cm for monoblock prestressed concrete sleeper

“Fig. 13”, (a) stage 1 and (b) stage 2 discuss the sequence of the above equations to get the suitable sleeper spacing for passenger and freight trains during acceleration through two stages.

By applying the previous equations on the above mentioned passenger and freight trains and according to the sequence in “Fig. 13”, the final numerical values are as shown in Table II.



**(a) Stage 1 : Sleeper spacing from 0 km/hr to critical speed**



**(b) Stage 2 : Sleeper spacing from critical speed to maximum speed**



**Table- II: The optimum sleeper spacing for passenger and freight trains during acceleration on -5%, 0% and 5% grades**

Train type			Passenger			Freight		
Grade %			5	0	-5	5	0	-5
First speed interval (Km/hr)			0 - 26.72					
Acceleration length (La) (meter)			87.34	76.24	67.64	508.00	274.86	188.480
Longitudinal force (Kg)	Position 1 (running on Horizontal)	Loco.last axle	23776.92	-	23776.92	23776.92	-	23776.92
		Train last Axle	22786.92	-	3962.82	20102.92	-	3962.82
	Position 2 (Stop on inclined)	Loco.last axle	625.00	-	2250.00	625.00	-	1670.00
		Train last Axle	2875.00	-	2875.00	8975.00	-	8975.00
	Super position	Loco.last axle	24401.92	23776.92	26026.92	24401.92	23776.92	25446.92
		Train last Axle	25661.92	-	6837.82	29077.92	-	12937.80
Displacement index (y)			1.96	2.13	0.30	1.96	2.13	0.12
Sleeper Spacing (Ls) cm			45.00	50.00	70.00	45.00	50.00	70.00
Second speed interval (Km/hr)			26.72 - 90.66	26.72 - 122.06	26.72 - 157.12	26.72 - 38.08	26.72 - 70.08	26.72 - 118.88
Acceleration length (La) (meter)			2245.90	3600.00	5288.09	406.00	4039.32	8189.79
Longitudinal force (Kg)	Position 1 (running on Horizontal)	Loco.last axle	13048.56	-	11765.50	27526.14	-	12328.51
		Train last Axle	12058.56	-	1960.91	23852.14	-	2054.75
	Position 2 (Stop on inclined)	Loco.last axle	625.00	-	2250.00	625.00	-	1670.00
		Train last Axle	2875.00	-	2875.00	8975.00	-	8975.00
	Super position	Loco.last axle	13673.58	12268.24	14015.50	28151.14	13939.00	14000.00
		Train last Axle	14933.56	-	4835.91	32827.14	-	11029.75
Displacement index (y)			1.50	1.53	0.17	2.27	1.74	0.07
Sleeper Spacing (Ls) cm			60.00	70.00	75.00	45.00	65.00	75.00

**B. The following equations explain how to get the best sleeper spacing for the both above mentioned trains during running with maximum speed**

- Longitudinal force for two positions (running on horizontal + stopping on inclined grade) to determine the worst case.

Position 1: Running on horizontal

$$F_{L( loco) 1} = F_{pmax} - (W_L \times R_r) \tag{36}$$

$$F_{L( train) 1} = F_{pmax} - (W_T \times R_r) \tag{37}$$

Position 2: Stopping on inclined

$$F_{L( loco) 2} = W_L \times R_g \tag{27}$$

$$F_{L( train) 2} = W_T \times R_g \tag{28}$$

Where:

$F_{pmax}$ : maximum force generated by the locomotive in kg

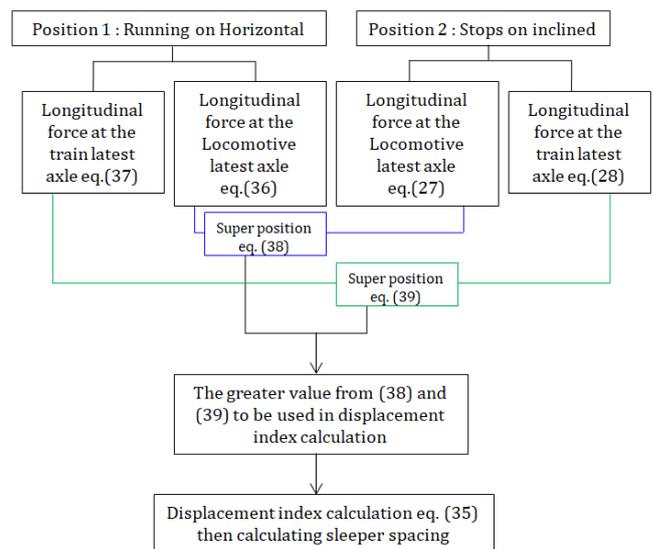
Applying super position at the following position

$$\text{Latest locomotive axle} = \text{eq. (36)} + \text{eq. (27)} \tag{38}$$

$$\text{Latest car axle} = \text{eq. (37)} + \text{eq. (28)} \tag{39}$$

- Sleeper Spacing  $y = (L_s/60) \times F_L / (L_{critical} \times f_s)$   $\tag{35}$

“Fig. 14”, discuss the sequence of the above equations to get the suitable sleeper spacing for passenger and freight trains during running with maximum speed.



**Fig.14. Procedure of sleeper spacing calculations during running with maximum speed on 5%, 0% and -5% grades**

By applying the previous equations on the above mentioned passenger and freight trains and according to the sequence in “Fig. 14”, the final numerical values are as shown in Table III.

## Effect of Longitudinal Forces Due to Loads on Prestressed Mono-Block Sleeper Spacing

**C. The following equations explains how to get the best sleeper spacing for the both above mentioned trains during braking passing by (0.2 maximum speed) until the train stops.**

- Braking length for stage 1: maximum speed to (0.2 maximum speed)

$$F_b' = [2000.e.f.\eta_b (W_{eb}/W_T)] + R_r + R_g \quad (40)$$

$$F_b'_{average} = (F_b'_{S_{max}} + F_b'_{0.2 S_{max}})/2 \quad (41)$$

$$L_b = 4.2(S_{max}^2 - (0.2S_{max})^2)/F_b'_{average} \quad (42)$$

- Braking length for stage 2: (0.2 maximum speed) to zero

$$F_b' = [2000.e.f.\eta_b (W_{eb}/W_T)] + R_r + R_g \quad (40)$$

$$F_b'_{average} = (F_b'_{0.2 S_{max}} + F_b'_{S=0})/2 \quad (43)$$

$$L_b = 4.2((0.2S_{max})^2 - (0)^2)/F_b'_{average} \quad (44)$$

Where:

$F_b'$ : specific brake force of the train in Kg/ton.

$F_b'_{average}$ : average Specific brake force of the train in Kg/ton.

$L_b$ : braking length of the train in meters.

- Longitudinal force for two positions (running on horizontal + stopping on inclined grade) to determine the worst case in stage 1 ( $S_{max}$  to  $0.2 S_{max}$ )

Position 1: Running on horizontal

$$F_{L(train)1} = F_b'_{average} \times W_T \quad (45)$$

Position 2: Stopping on inclined

$$F_{L(train)2} = W_T \times R_g \quad (28)$$

Applying super position at the following position

$$\text{Latest locomotive axle} = \text{eq. (45)} + \text{eq. (28)} \quad (46)$$

- Longitudinal force for two positions (running on horizontal + stopping on inclined grade) to determine the worst case in stage 2 ( $0.2 S_{max}$  to zero)

Position 1: Running on horizontal

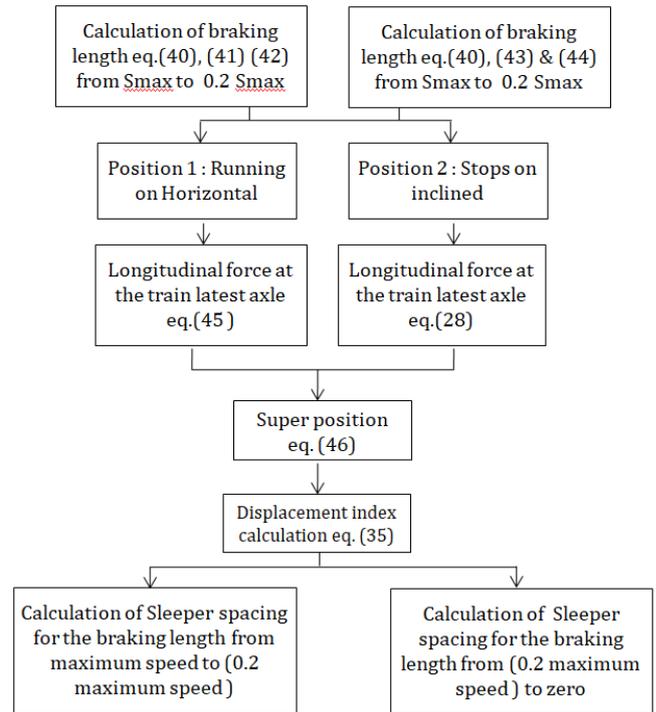
$$F_{L(train)1} = F_b'_{average} \times W_T \quad (45)$$

Position 2: Stopping on inclined

$$F_{L(train)2} = W_T \times R_g \quad (28)$$

Applying super position at the following position

$$\text{Latest locomotive axle} = \text{eq. (45)} + \text{eq. (28)} \quad (46)$$



**Fig.15. Procedure of sleeper spacing calculations during train braking on -5 %, 0 % and 5% grades**

**Table– III: The optimum sleeper spacing for passenger and freight trains during acceleration on -5 %, 0 % and 5% grades**

Train type		Passenger			Freight			
Grade %		5	0	-5	5	0	-5	
Uniform Speed (Km/hr)		90.66	122.06	157.12	38.08	70.08	118.88	
Longitudinal force (Kg)	Position 1 (running on Horizontal)	Loco.last axle	5791.62	4231.00	3225.5	14168.27	7573.17	4351.4
		Train last Axle	4801.62	3241.00	537.5	10494.27	3899.17	725.25
	Position 2 (Stop on inclined)	Loco.last axle	625.00	-	2250.00	625.00	-	8125.00
		Train last Axle	2875.00	-	2875.00	8975.00	-	8975.00
	Super position	Loco.last axle	6416.62	-	5475.50	14793.27	-	12476.40
		Train last Axle	7676.62	-	3412.00	19469.27	-	9700.25
Displacement index (y)		0.80	0.56	0.08	1.85	1.01	0.07	
Sleeper Spacing (Ls) cm		70.00	75.00	80.00	70.00	75.00	80.00	

- Sleeper Spacing
- $$y = (L_s/60) \times F_L / (L_{critical} \times f_s) \quad (35)$$

“Fig. 15”, discuss the sequence of the above equations to get the suitable sleeper spacing for passenger and freight trains during braking through two stages.

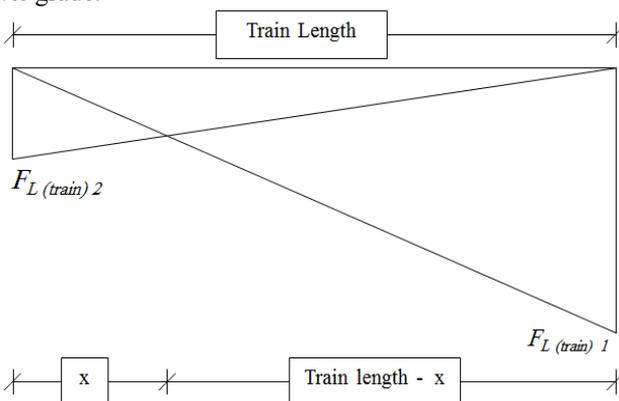
By applying the previous equations on the above mentioned passenger and freight trains and according to the sequence in “Fig. 15”, the final numerical values are as shown in Table IV.



**Table- IV: The optimum sleeper spacing for passenger and freight trains during braking on -5 %, 0 % and 5% grade**

Train type		Passenger			Freight			
Grade %		5	0	-5	5	0	-5	
First speed interval (Km/hr)		90.66 - (0.2)90.66	122.06 - (0.2)122.06	157.12 - (0.2)157.12	38.08 - (0.2)38.08	70.08 - (0.2)70.08	118.88 - (0.2)118.88	
Brake length (Lb) (meter)		217.60	439.77	790.08	140.00	603.28	2380.410	
Longitudinal force (Kg)	Position 1 (running on Horizontal)	Train last axle	87572.50	78539.25	72438.5	74941.25	58911.90	42954.35
	Position 2 (Stop on inclined)	Train last axle	-2875.00	-	-2875.00	-8975.00	-	-8975.00
	Super position	Train last Axle	84697.50	-	*	65966.25	-	*
Displacement index (y)		0.85	0.85	0.88	0.28	0.27	0.26	
Sleeper Spacing (Ls) cm		60.00	65.00	70.00	60.00	65.00	70.00	
Second speed interval (Km/hr)		(0.2)90.66 - 0	(0.2)122.06 - 0	(0.2)157.12 - 0	(0.2)38.08 - 0	(0.2)70.08 - 0	(0.2)118.88 - 0	
Brake length (Lb) (meter)		6.89	13.09	22.81	5.20	20.17	68.50	
Longitudinal force (Kg)	Position 1 (running on Horizontal)	Train last Axle	115112.13	109876.75	104500.50	84023.45	73379.60	62142.90
	Position 2 (Stop on inclined)	Train last Axle	2875.00	-	2875.00	8975.00	-	8975.00
	Super position	Train last Axle	112237.12	-	*	75048.95	-	*
Displacement index (y)		0.84	0.92	0.99	0.24	0.26	0.33	
Sleeper Spacing (Ls) cm		45.00	50.00	55.00	45.00	50.00	65.00	

\* “Fig. 16”, explains the super position and calculating the difference between the longitudinal forces while running on -5 % grade.



**Fig.16. Superposition for both passenger and freight trains during braking on -5 %**

**V. CONCLUSION**

The proposed methodology in this paper discussed how the uniform sleeper spacing (60 cm) which has been used for the prestressed mono-block sleeper is not appropriate in many cases due to the number of forces affecting the track such as the longitudinal force which causes track longitudinal vibration. Thus, to make the track work efficiently against these vibrations, sleeper spacing should be redistributed, and that was the core point of the study. According to the study, the number of prestressed mono-block sleeper is reduced.

“(47)” and “(48)” shows how much saving in sleepers due to the new distribution in 15 kilometers. Table V shows the saving in sleepers for each case.

$$S_n = [(D_s - (L_{b1} + L_{b2} + L_{a\mu} + L_{af})) / L_{s\ uniform}] + [(L_{b1} / L_{s\ b1}) + (L_{b2} / L_{s\ b2}) + (L_{a\mu} / L_{s\ \mu}) + (L_{af} / L_{s\ f}) + (L_{platform} / L_{s\ platform})] \quad (47)$$

$$Saving\ in\ sleepers = [(D_s / 0.6) - S_n] \quad (48)$$

Where:

- \$S\_n\$ : number of sleepers after applying recommended spacing
- \$D\_s\$ : minimum distance between stations (15,000 m)
- \$L\_{b1}\$ : the breaking length taken to reduce train speed from \$S\_{max}\$ to \$0.2 S\_{max}\$
- \$L\_{b2}\$ : the breaking length taken to reduce train speed from \$0.2 S\_{max}\$ to zero
- \$L\_{s\ uniform}\$ : new sleeper spacing under uniform or maximum speed
- \$L\_{s\ b1}\$ : new sleeper spacing through \$L\_{b1}\$
- \$L\_{s\ b2}\$ : new sleeper spacing through \$L\_{b2}\$
- \$L\_{s\ \mu}\$ : new sleeper spacing through \$L\_{a\mu}\$
- \$L\_{s\ f}\$ : new sleeper spacing through \$L\_{af}\$
- \$L\_{platform}\$ : platform length
- \$L\_{s\ platform}\$ : new sleeper spacing through \$L\_{platform}\$

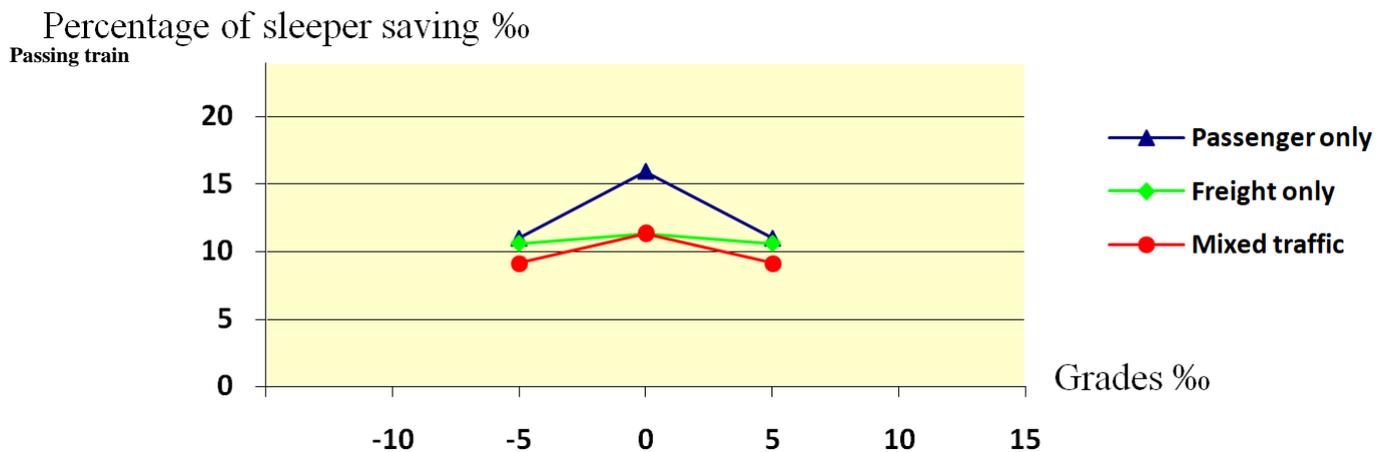
## Effect of Longitudinal Forces Due to Loads on Prestressed Mono-Block Sleeper Spacing

**Table– V: Number of saved sleepers and the corresponding percentage after new spacing for single and double track on (-5 ‰, 0‰ and 5‰ grades) when the passing train is passenger or freight or both of them**

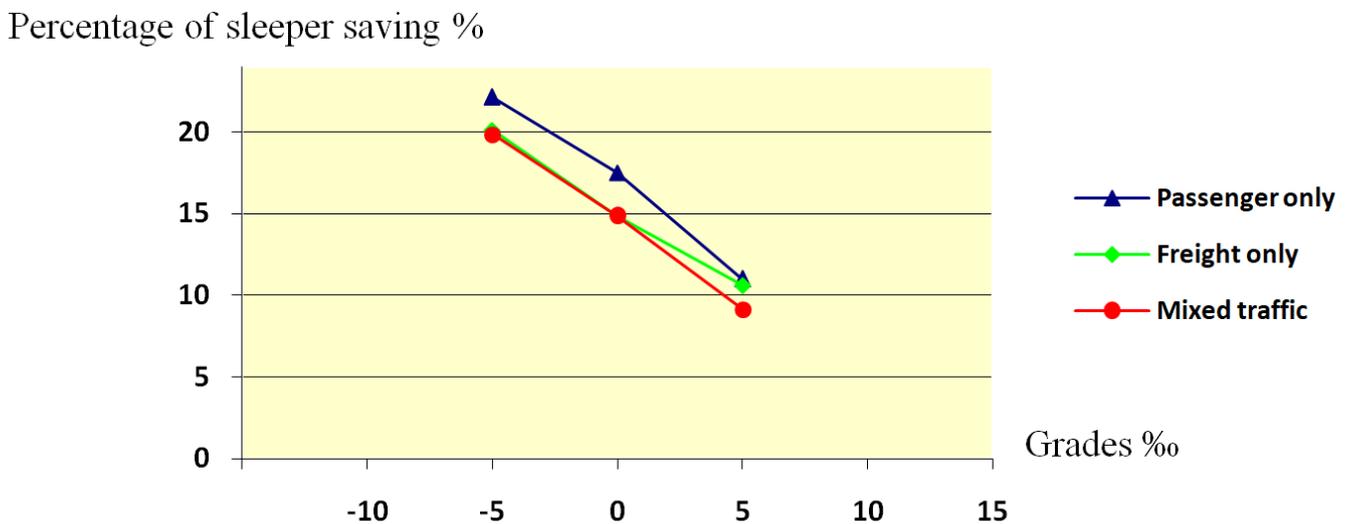
Track type	Single track				Double track					
	Grade				Grade					
	0 ‰		±5‰		-5 ‰		0 ‰		+5 ‰	
	sleeper s	percentage	sleeper s	percentage	sleeper s	percentage	sleeper s	percentage	sleeper s	percentage
Passenger only	3988	15.95%	2750	11.00%	5533	22.13%	8747	17.50% **	2750	11%
Freight only	2830	11.32%	2650	10.60%	5032	20.12%	7436	14.87% **	2650	10.60%
Passenger and freight	2830	11.32%	2292	9.16%	4970	19.88%	7436	14.87% **	2292	9.16%

\*\* Summation of the saved sleepers for both directions.

The data in table V could be represented in a graph as shown in “Fig. 17” and “Fig. 18”



**Fig.17. Relation between grades and percentage of sleeper saving for all traffic types on a single track**



**Fig.18. Relation between grades and percentage of sleeper saving for all traffic types on a double track**

### VI. RECOMMENDATIONS

The suitable sleeper spacing has been calculated from the proposed methodology and mentioned in table II, III and IV. “Fig. 19”, “Fig. 20”, “Fig. 21”, “Fig. 22”, “Fig. 23”, “Fig. 24”, “Fig. 25”, “Fig. 26”, “Fig. 27” and “Fig. 28”, shows the recommended sleeper spacing in the following cases:

- Passenger train runs on double and single track.

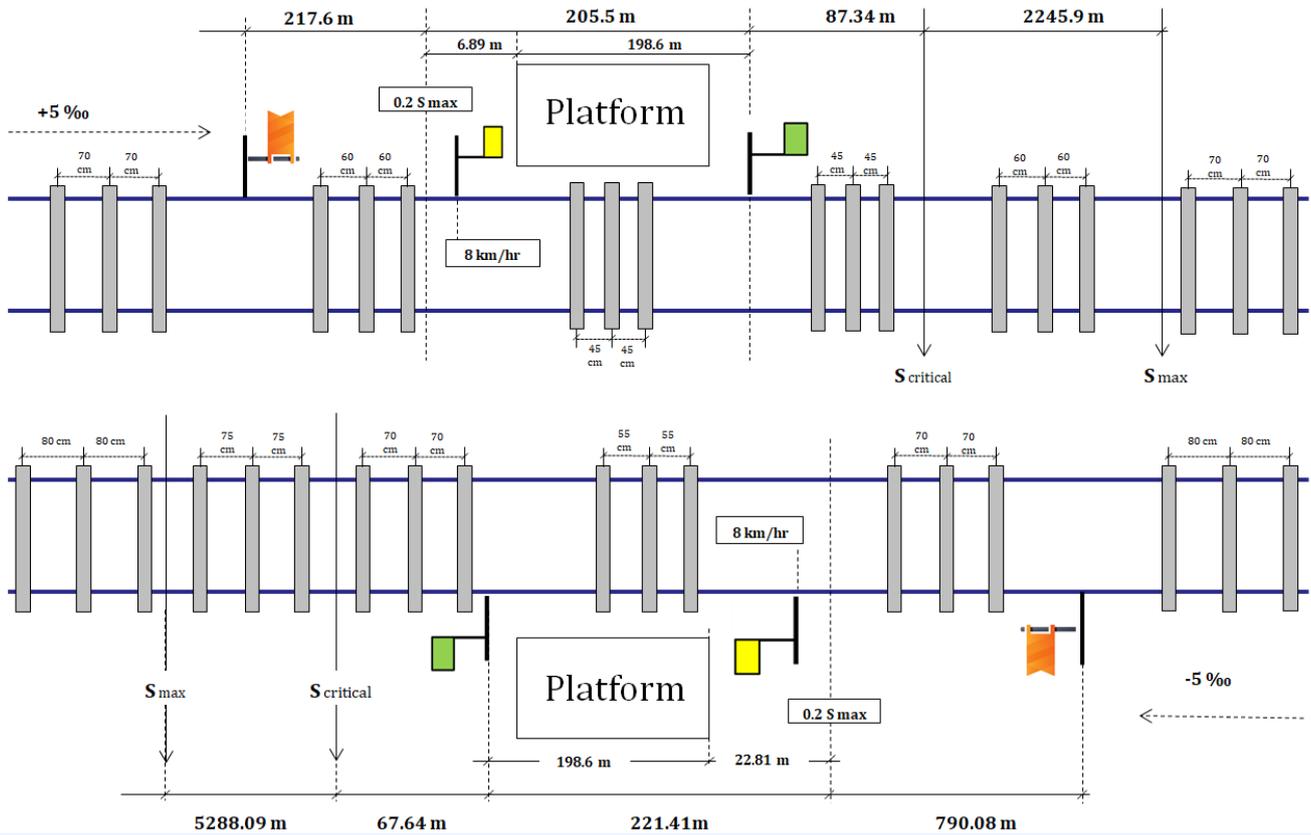


Fig.19. Suitable sleeper spacing for passenger train on double track in both directions (-5 ‰ and 5‰ grades)

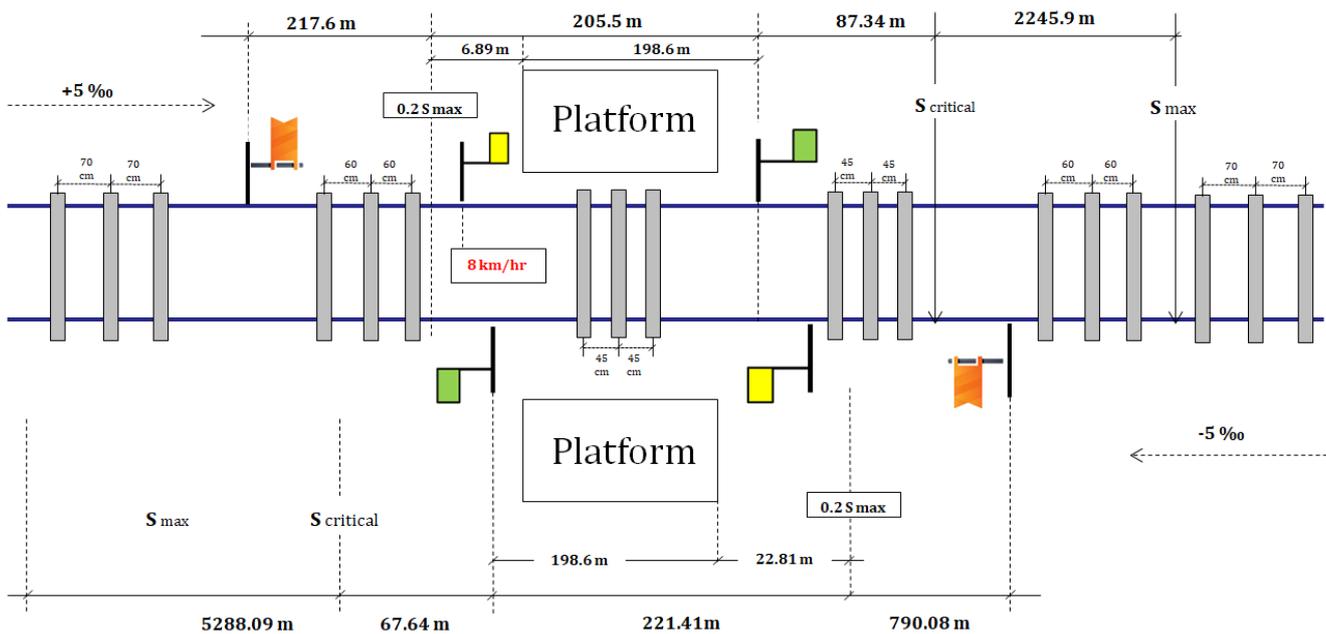
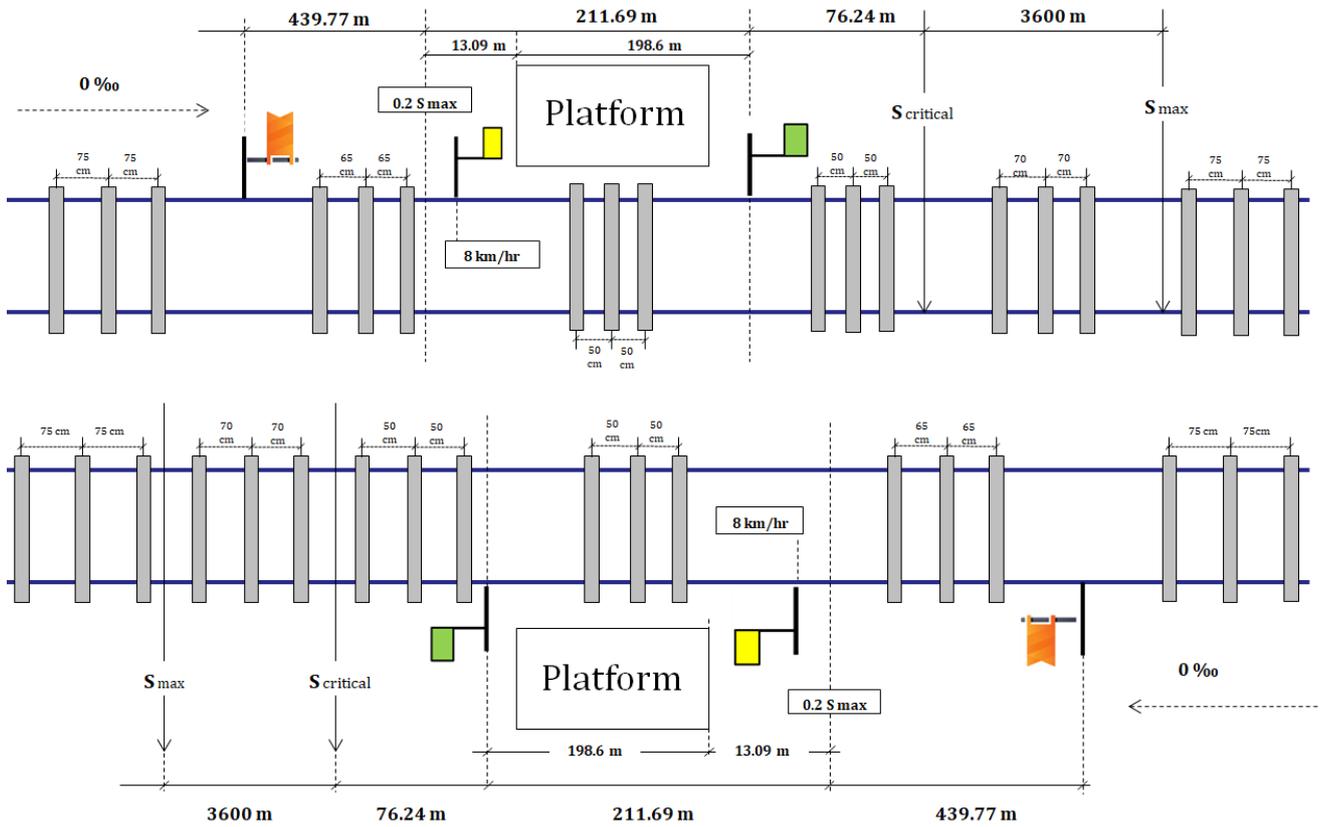
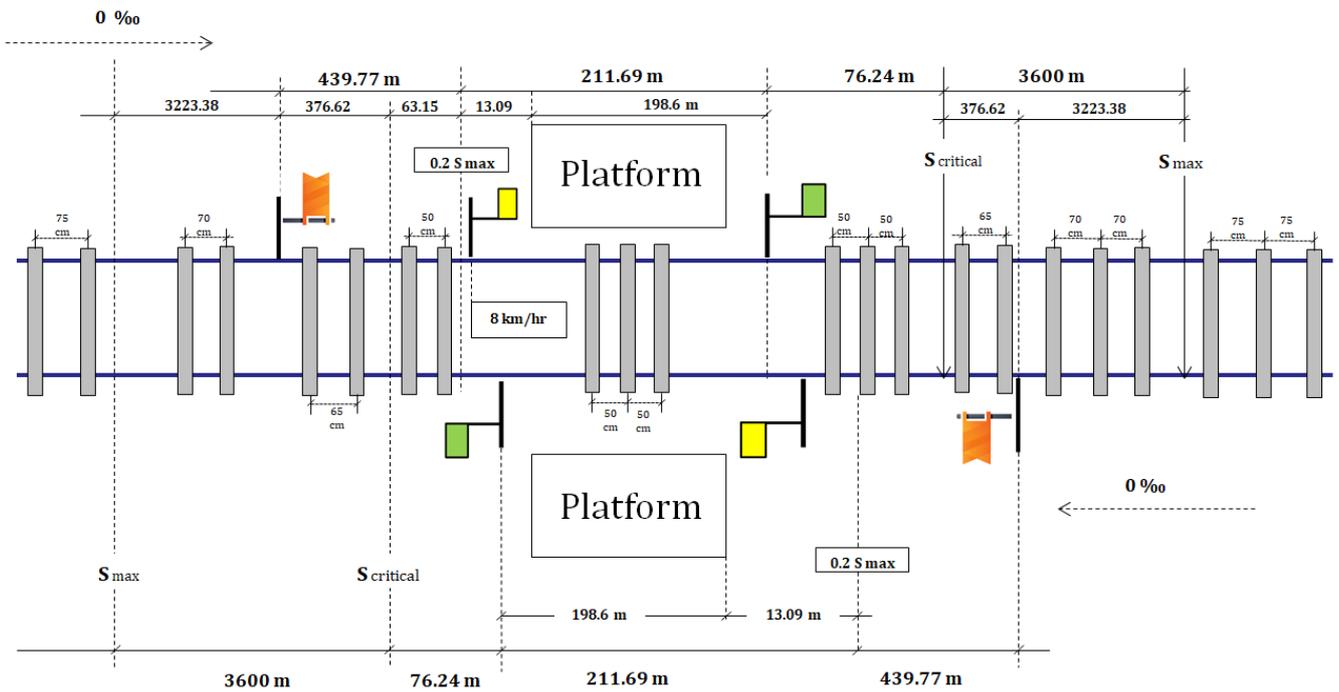


Fig.20. Suitable sleeper spacing for passenger train on single track in both directions (-5 ‰ and 5‰ grades)

# Effect of Longitudinal Forces Due to Loads on Prestressed Mono-Block Sleeper Spacing



**Fig.21. Suitable sleeper spacing for passenger train on double horizontal track in both directions on (0 ‰)**



**Fig.22. Suitable sleeper spacing for passenger train on single horizontal track in both directions on (0 ‰)**

- Freight train runs on double and single track

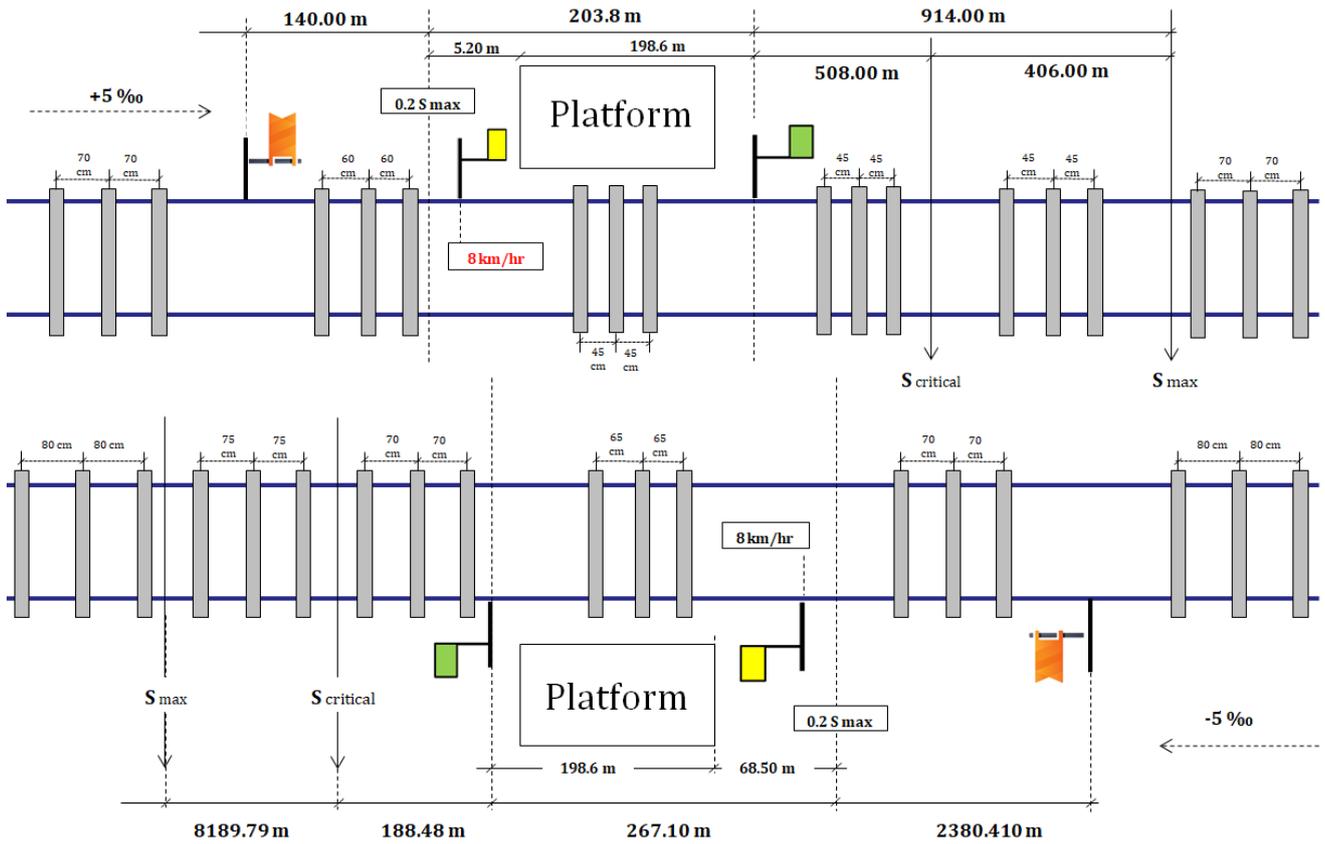


Fig.23. Suitable sleeper spacing for freight train on double track in both directions (-5 ‰ and 5‰ grades)

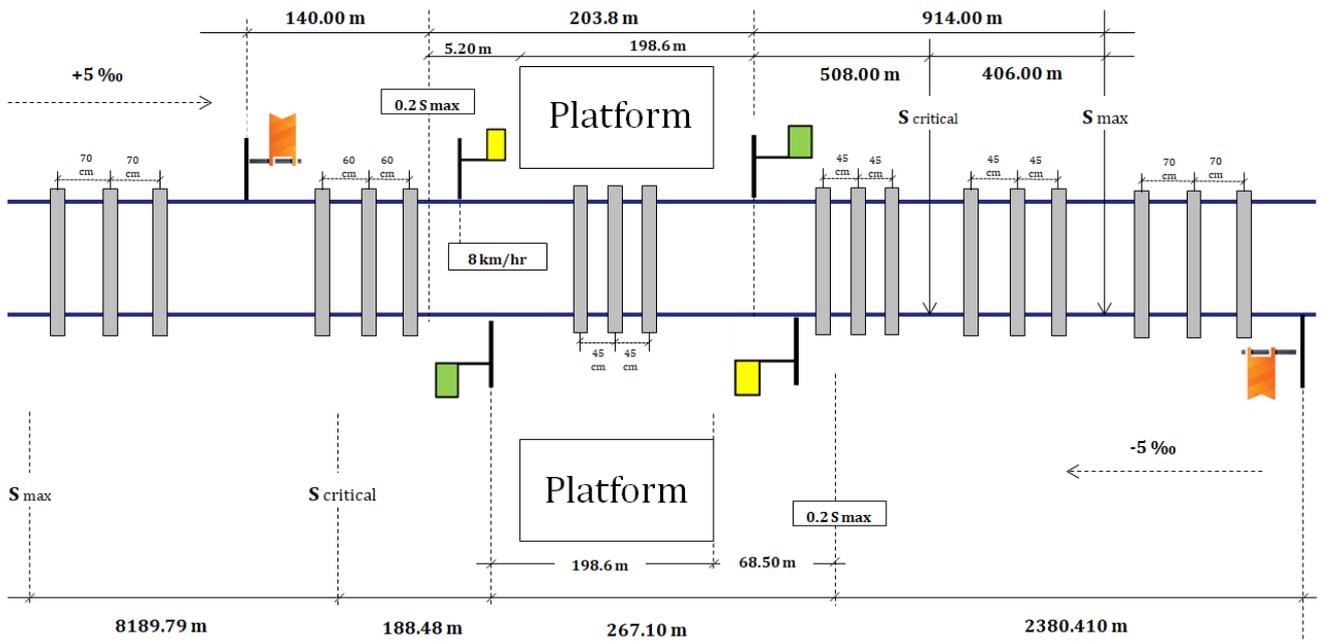
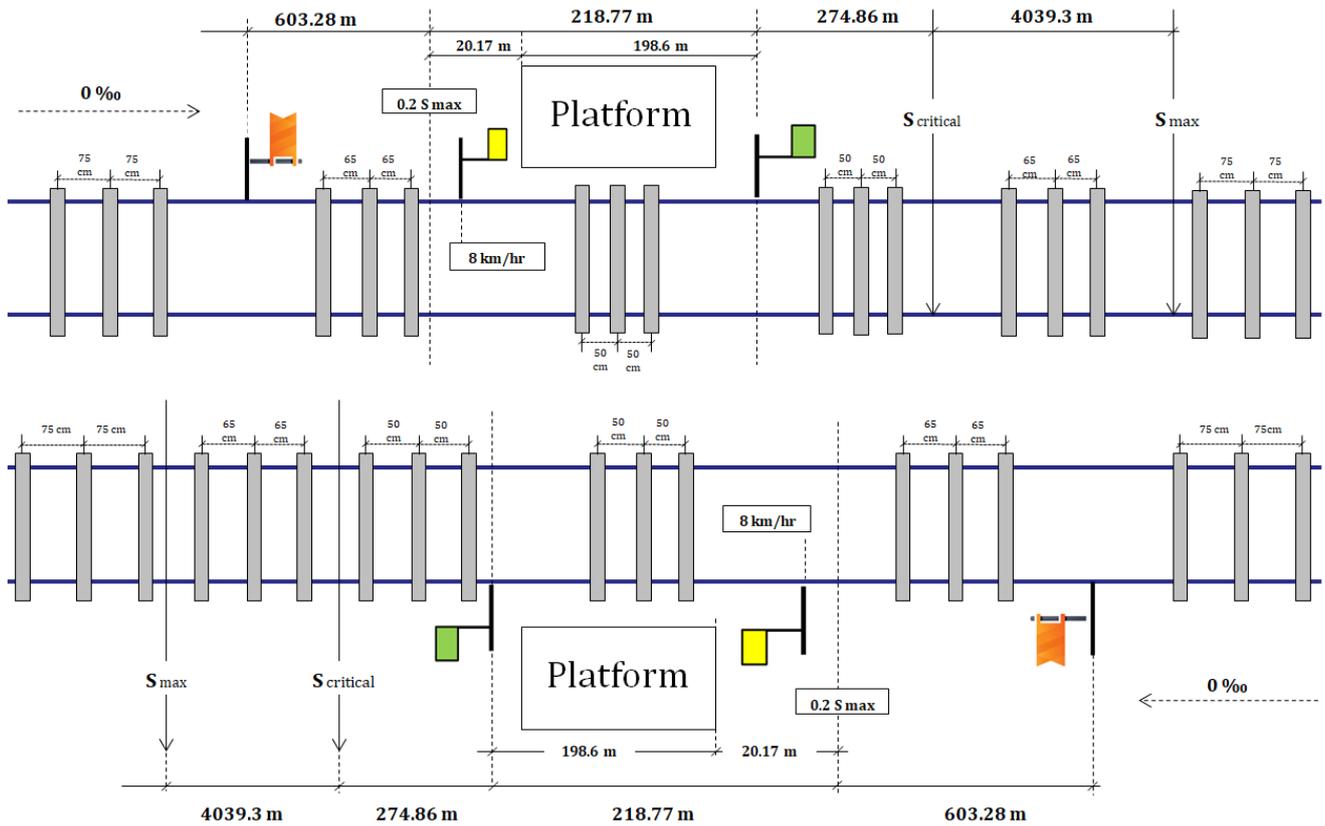
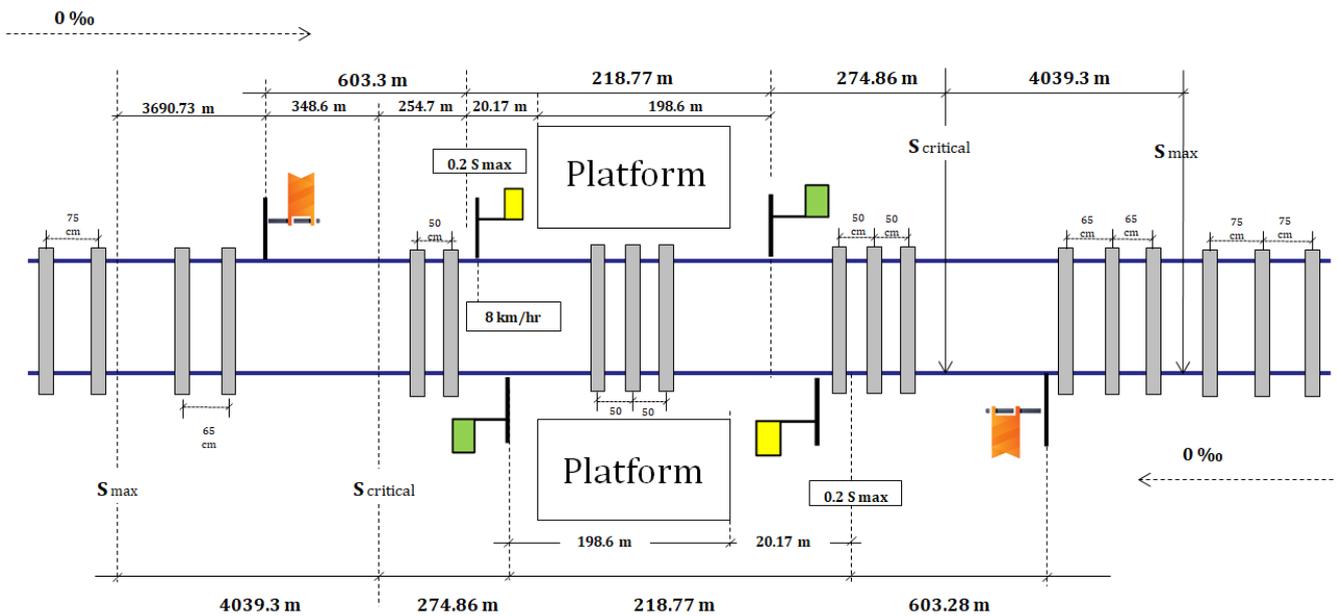


Fig.24. Suitable sleeper spacing for freight train on single track in both directions (-5 ‰ and 5‰ grades)

# Effect of Longitudinal Forces Due to Loads on Prestressed Mono-Block Sleeper Spacing



**Fig.25. Suitable sleeper spacing for freight train on double horizontal track in both directions on (0 ‰)**



**Fig.26. Suitable sleeper spacing for freight train on single horizontal track in both directions on (0 ‰)**

- Passenger and freight train runs on the same track either double or single one

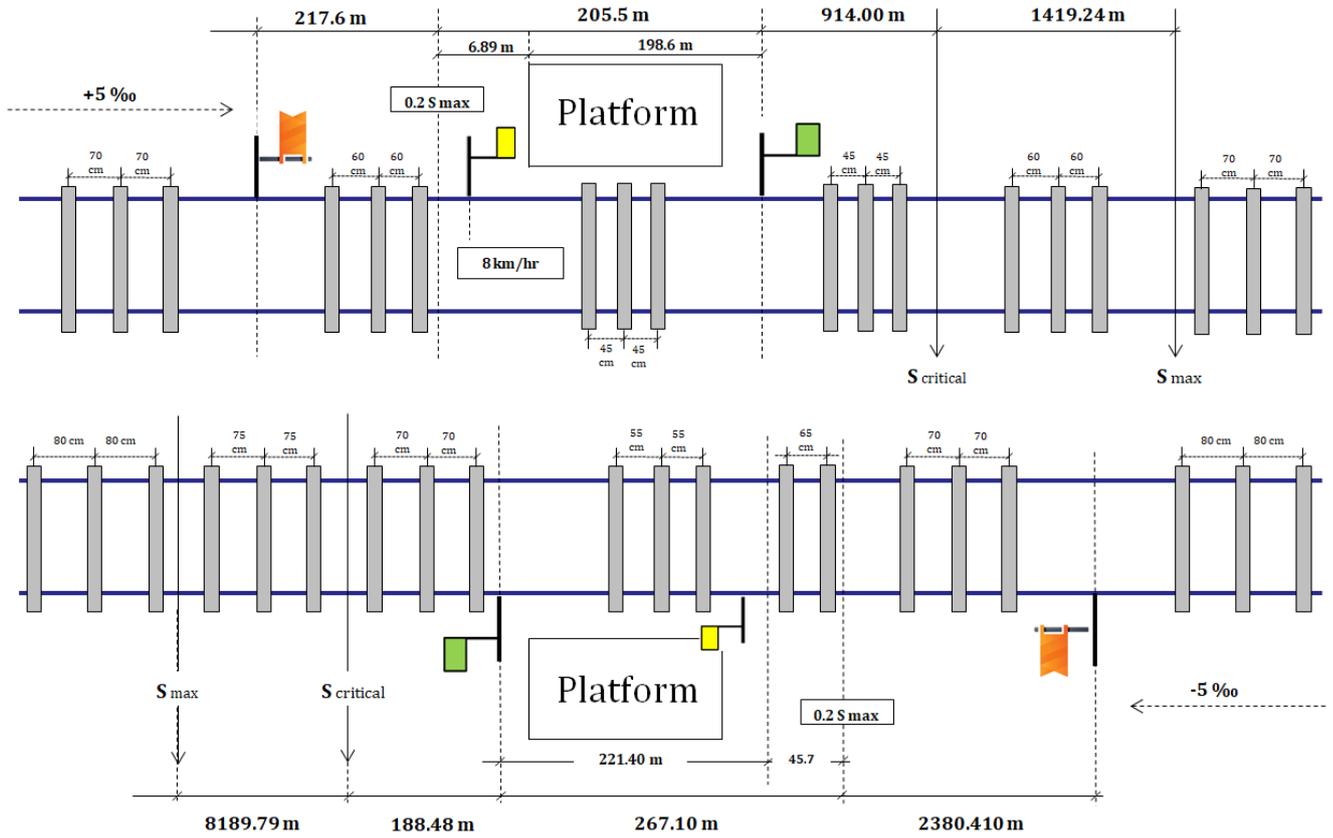


Fig.27. Suitable sleeper spacing for passenger and freight trains on double track in both directions (-5 ‰ and 5‰ grades)

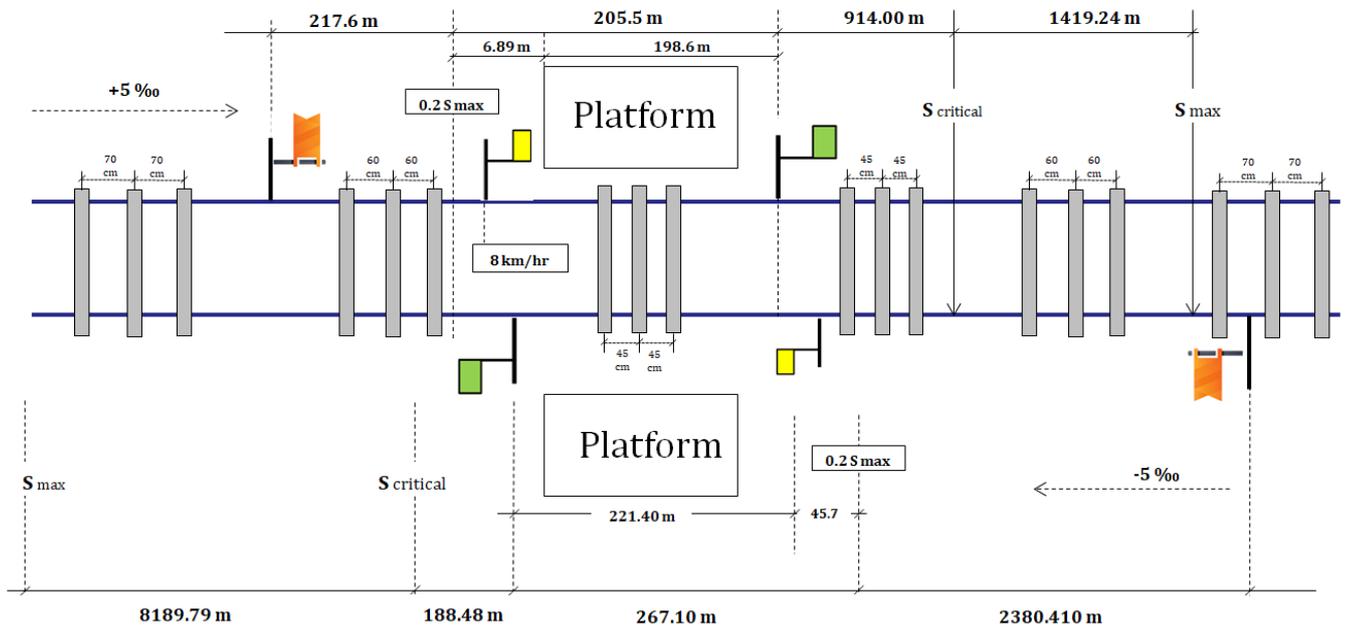


Fig.28. Suitable sleeper spacing for passenger and freight trains on single track in both directions (-5 ‰ and 5‰ grades)

“Fig. 25”, could be applied as a double horizontal track (0 ‰) for both freight and passenger trains altogether, as well as “Fig. 26”, is also applied as a single horizontal track (0 ‰) for mixed traffic.

## REFERENCES

1. American Railway Engineering Association, "Concrete Ties," 1982.
2. Gallego I., "Vertical Track Stiffness as a New Parameter Involved in Designing High-Speed Railway Infrastructure," ASCE, Journ. of Transp. Eng., Vol. 137, No 12, 2011.
3. Esveld, C., "Modern railway track second edition," Zaltbommel, MRT-Productions, 2001.
4. Lichtberger, B., "Track compendium: Formation, permanent way, maintenance, economics," Eurailpress, 2005.
5. Profillidis, V. "Railway Management and Engineering," Section of Transportation, Democritus Thrace University, Greece, Vol. 2, 2014.
6. Buekette J., "Concrete Sleepers," Track Course, RIA, London, 1983.
7. European Standard, "Prestressed Monoblock Concrete Sleepers," European Committee for Standardization, Brussels, 1994.
8. Profillidis, V., "Applications of Finite Element Analysis in the Rational Design of Track Bed Structures," Computers and Structures, Vol. 22, No 3, 1986.
9. Prud'homme A., "La Voie," RGCF, Paris, 1970.
10. UIC, "Factors affecting Track Maintenance Costs and their Relative Importance," Paris, 1992.
11. Panagiotopoulos P., "Hemivariational Inequalities: Applications in Mechanics and Engineering," Springer, Berlin, 1993.
12. <http://www.clag.org.uk/wheelbase.html>

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