Pentagonal Graph for Designing Manufacturing Cellular System

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Abstract: In this paper we have analyzed that pentagonal snake \( PS_k \), subdivision of a pentagonal snake \( S(PS_k) \) and alternate pentagonal snake \( A(PS_k) \) are mean square cordial graphs.

Keywords: Mean square cordial labeling, Pentagonal snake, Subdivision of a pentagonal snake, alternate pentagonal snake.

I. INTRODUCTION

One of the most engrossing and booming areas of mathematics is graph theory which is an analysis of graph that deals with the relationship of vertices and edges. Due to the involvement of the researchers for the past 60 years, over 200 graphs labeling techniques [1] have been discussed in thousands of research papers. Graph with labeling serves as a useful model with greater number of applications like the concepts of coding, study of crystals, radar detections, astro studies, designing circuits, communication network addressing, managing data bases, sharing secret messages and it simulates many constrained programming in finite number of domains. Along with these applications, this technique is applied in disk redundancies, manufacturing design in drilling machines, circuit board designs, network configuration etc Here Harary[2] is followed for basic notations. Cordial labeling was introduced by Cahit[3] and Ponraj et al[4] were initiated the mean cordial labeling of a graph. Mean square cordial labeling introduced by A.Nellai murugan et al and they have discussed it for some special graphs[5]. Moreover they have discussed the mean square cordial labeling for some tree and cycle related graphs [6,7]. Dhanalakshi et al have discussed mean square cordial labeling related to some cyclic and acyclic graphs and its rough approximations [8,9]. In this paper we analysed that pentagonal snake \( PS_k \), subdivision of a pentagonal snake \( S(PS_k) \) and alternate pentagonal snake \( A(PS_k) \) are mean square cordial graphs.

II. PRELIMINARIES

Definition 1: Let \( G = (V, E) \) be a graph with \( p \) vertices and \( q \) edges. “A Mean Square Cordial labeling of a Graph \( G(V, E) \) with \( p \) vertices and \( q \) edges is a bijection from \( V \) to \{0, 1\} such that each edge \( uv \) is assigned the label \( \left\lfloor \sqrt{(f(u)^2 + f(v)^2)/2} \right\rfloor \) where \( x \) (ceil( x )) is the least integer greater than or equal to \( x \) with the condition that the number of vertices labeled with 0 and the number of vertices labeled with 1 differ by at most 1 and the number of edges labeled with 0 and the number of edges labeled 0 and the number of edges labeled with 1 differ by at most 1”.

Definition 2: The pentagonal snake \( PS_k \) is obtained from a path \( u_1, u_2, …, u_k \) by joining \( u_i \) and \( u_{i+1} \) for \( 1 \leq i \leq k-1 \) to two new vertices \( v_1, w_i, x_i \) and then joining \( v_1, x_i \) and \( x_i, w_i \). That is the path \( P_k \) by replacing each edge of the path by a cycle \( C_5 \).

Definition 3: Let \( G \) be a graph. The subdivision graph \( S(G) \) is obtained from \( G \) by subdividing each edge of \( G \) with a vertex

Definition 4: An alternate pentagonal snake \( A(PS_k) \) is obtained from a path \( u_1, u_2, …, u_k \) by joining \( u_i \) and \( u_{i+1} \) to two new vertices \( v_i, w_i \) and by joining \( v_i \) and \( w_i \) to a new vertex \( x_i \) respectively. That is, every alternate edge of a path is replaced by a cycle \( C_5 \).

III. MAIN RESULTS

Theorem 1: Pentagonal snake \( PS_k \) admits mean square cordial labeling \( k \geq 2 \).

Proof: Let \( P_k \) be the path \( u_1, u_2, …, u_k \). Let \( V(PS_k) = V(P_k) \cup \{v_i, w_i, x_i : i \) varies from 1 to \( k-1 \} \) and \( E(PS_k) = \{(u_i, u_{i+1}) : i \) varies from 1 to \( k-1 \} \cup \{(v_i, w_i) : i \) varies from 1 to \( k-1 \} \cup \{(v_i, x_i) : i \) varies from 1 to \( k-1 \} \cup \{(w_i, x_i) : i \) varies from 1 to \( k-1 \} \} \). Here \( |V| = 4k - 3 \) and \( |E| = 5k - 5 \).

Define \( f \) maps \( V(PS_k) \) to \{0, 1\}.

Case (i) \( k \) is odd

\[
\begin{align*}
    f(v_i) & = 0, i \text{ varies from } 1 \text{ to } (k-1)/2 \\
    & = 1, i \text{ varies from } (k+1)/2 \text{ to } k-1 \\
    f(u_i) & = 0, i \text{ varies from } 1 \text{ to } (k+1)/2 \\
    & = 1, i \text{ varies from } (k+3)/2 \text{ to } k \\
    f(w_i) & = 0, i \text{ varies from } 1 \text{ to } (k+1)/2 \\
    & = 1, i \text{ varies from } (k+1)/2 \text{ to } k-1 \\
    f(x_i) & = 0, i \text{ varies from } 1 \text{ to } (k-1)/2 \\
    & = 1, i \text{ varies from } (k+1)/2 \text{ to } k-1
\end{align*}
\]

The corresponding edge labeling is as follows

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The following table expresses the cardinality of vertices and edges of MSCL for the above graph.

<table>
<thead>
<tr>
<th>T</th>
<th>0</th>
<th>1</th>
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<tbody>
<tr>
<td></td>
<td>2k-1</td>
<td>2k-2</td>
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Hence pentagonal snake $PS_k$ admits mean square cordial labeling $\forall k \geq 2$.

**Illustration**:

![Figure 1: Mean square cordial labeling of pentagonal snake $PS_5$](image1)

**Figure 2: Mean square cordial labeling of pentagonal snake $PS_4$**

**Theorem:** 2 Subdivision of a pentagonal snake $S(PS_k)$ admits mean square cordial labeling $k \geq 3$ and $k$ is odd.

**Proof:** Let $P_k$ be the path $u_1, u_2, u_k$ . Let $V(PS_k) = V(P_k) \cup \{v_i, w_i, x_i; \text{ varies from } 1 \text{ to } k-1\}$ and $V(S(PS_k)) = V(PS_k) \cup \{a_i, b_i, c_i, d_i, e_i; \text{ varies from } 1 \text{ to } k\}$ Then $E(S(PS_k)) = \{(a_i,b_i); \text{ i varies from 1 to k-1} \cup \{(b_i,v_i); \text{ i varies from 1 to k-1} \cup \{(c_i,x_i); \text{ i varies from 1 to k-1} \cup \{(d_i,w_i); \text{ i varies from 1 to k-1} \cup \{(w_i,e_i); \text{ i varies from 1 to k-1} \cup \{(e_i,u_i); \text{ i varies from 1 to k-1} \cup \{(a_i,u_i); \text{ i varies from 1 to k-1} \cup \{(u_i,v_i); \text{ i varies from 1 to k-1} \}

Here $|V|=9k-8$ and $|E|=10k-10$

Define $f$ maps $V(S(PS_k)) \to \{0,1\}$

<table>
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<tr>
<td></td>
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The following table expresses the cardinality of vertices and edges of MSCL for the above graph.

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<td>$2k-1$</td>
<td>$2k-2$</td>
</tr>
</tbody>
</table>
Then the induced edge labeling is as follows

\[
\begin{align*}
\ f(u_i) &= 0, i \text{ varies from } 1 \text{ to } (k+1)/2 \\
& \quad, 1, i \text{ varies from } (k+3)/2 \text{ to } k \\
\ f(v_i) &= 0, i \text{ varies from } 1 \text{ to } (k-1)/2 \\
& \quad, 1, i \text{ varies from } (k+1)/2 \text{ to } k \\
\ f(w_i) &= 0, i \text{ varies from } 1 \text{ to } (k-1)/2 \\
& \quad, 1, i \text{ varies from } (k+1)/2 \text{ to } k
\end{align*}
\]

The following table expresses the cardinality of vertices and edges of MSCL for the above graph.

Hence subdivision of a pentagonal snake S(PS\text{k} \text{)} admits mean square cordial labeling ∀ k ≥ 2.

Illustration:

![Illustration](image)

**Figure 3:** Mean square cordial labeling of pentagonal snake PS5

**Theorem:** Mean square cordial labeling of an alternate pentagonal snake A (PS_2k),

\[ k ≥ 2 \text{ where the pentagon starts from } u_1 \text{ and ends with } u_{2k}. \]

Proof: Let \( P \) be the path \( u_1, u_2, ..., u_{2k}. \) Let \( V(A(PS_k)) = V(P_1) \cup \{v_i, w_i, x_i : i \text{ varies from } 1 \text{ to } k-1 \} \) and

\[ E(A(PS_k)) = [(u_{2i}, v_i): i \text{ varies from } 1 \text{ to } k-1] \cup [(u_{2i}, w_{i}): i \text{ varies from } 1 \text{ to } k-1] \cup [(v_{i}, x_i): i \text{ varies from } 1 \text{ to } k-1] \cup [(v_{i}, v_{i}): i \text{ varies from } 1 \text{ to } k-1] \cup [(v_{i}, u_{i+1}): i \text{ varies from } 1 \text{ to } k-1]. \]

Then \( |V| = 5k \text{ and } |E| = 6k - 1. \)

Define \( f \) maps \( V(A(PS_k)) \) to \( \{0, 1\}. \)

**Case (i) k is odd**

\[
\begin{align*}
\ f(u_i) &= 0, i \text{ varies from } 1 \text{ to } (2k)/2 \\
& \quad, 1, i \text{ varies from } (2k+2)/2 \text{ to } 2k \\
\ f(v_i) &= 0, i \text{ varies from } 1 \text{ to } (k+1)/2 \\
& \quad, 1, i \text{ varies from } (k+3)/2 \text{ to } k \\
\ f(w_i) &= 0, i \text{ varies from } 1 \text{ to } (k-1)/2 \\
& \quad, 1, i \text{ varies from } (k+1)/2 \text{ to } k
\end{align*}
\]

**Case (ii) k is even**

\[
\begin{align*}
\ f(u_i) &= 0, i \text{ varies from } 1 \text{ to } (k+1)/2 \\
& \quad, 1, i \text{ varies from } (k+1)/2 \text{ to } k \\
\ f(v_i) &= 0, i \text{ varies from } 1 \text{ to } (k+1)/2 \\
& \quad, 1, i \text{ varies from } (k+1)/2 \text{ to } k \\
\ f(w_i) &= 0, i \text{ varies from } 1 \text{ to } (k-1)/2 \\
& \quad, 1, i \text{ varies from } (k+1)/2 \text{ to } k
\end{align*}
\]
Then the induced edge labeling is as follows
\[ f(u_{i+1}) = 0, i \text{ varies from } 1 \text{ to } (2k)/2 \]
\[ 1, i \text{ varies from } (2k + 2)/2 \text{ to } 2k - 1 \]

Then the induced edge labeling is as follows
\[ f(u_{i+1}) = 0, i \text{ varies from } 1 \text{ to } (2k)/2 \]
\[ 1, i \text{ varies from } (2k + 2)/2 \text{ to } 2k - 1 \]

Then the induced edge labeling is as follows
\[ f(v_i) = 0, i \text{ varies from } 1 \text{ to } (k + 1)/2 \]
\[ 1, i \text{ varies from } (k + 3)/2 \text{ to } k \]

Then the induced edge labeling is as follows
\[ f(v_i) = 0, i \text{ varies from } 1 \text{ to } (k + 1)/2 \]
\[ 1, i \text{ varies from } (k + 3)/2 \text{ to } k \]

Then the induced edge labeling is as follows
\[ f(w_i) = 0, i \text{ varies from } 1 \text{ to } (k + 1)/2 \]
\[ 1, i \text{ varies from } (k + 3)/2 \text{ to } k \]

Then the induced edge labeling is as follows
\[ f(w_i) = 0, i \text{ varies from } 1 \text{ to } (k + 1)/2 \]
\[ 1, i \text{ varies from } (k + 3)/2 \text{ to } k \]

Then the induced edge labeling is as follows
\[ f(w_i) = 0, i \text{ varies from } 1 \text{ to } (k + 1)/2 \]
\[ 1, i \text{ varies from } (k + 3)/2 \text{ to } k \]

Then the induced edge labeling is as follows
\[ f(x_i) = 0, i \text{ varies from } 1 \text{ to } (k + 1)/2 \]
\[ 1, i \text{ varies from } (k + 3)/2 \text{ to } k \]

Then the induced edge labeling is as follows
\[ f(x_i) = 0, i \text{ varies from } 1 \text{ to } (k + 1)/2 \]
\[ 1, i \text{ varies from } (k + 3)/2 \text{ to } k \]

Then the induced edge labeling is as follows
\[ f(U_{k+1}) = 0, i \text{ varies from } 1 \text{ to } (k + 1)/2 \]
\[ 1, i \text{ varies from } (k + 3)/2 \text{ to } k \]

Then the induced edge labeling is as follows
\[ f(U_{k+1}) = 0, i \text{ varies from } 1 \text{ to } (k + 1)/2 \]
\[ 1, i \text{ varies from } (k + 3)/2 \text{ to } k \]

Then the induced edge labeling is as follows
\[ f(U_{k+1}) = 0, i \text{ varies from } 1 \text{ to } (k + 1)/2 \]
\[ 1, i \text{ varies from } (k + 3)/2 \text{ to } k \]

Then the induced edge labeling is as follows
\[ f(U_{k+1}) = 0, i \text{ varies from } 1 \text{ to } (k + 1)/2 \]
\[ 1, i \text{ varies from } (k + 3)/2 \text{ to } k \]

The following table expresses the cardinality of vertices and edges of MSCL for the above graph.

| \(|v_f(T)| \) | 0 | 1 |
|-------------|---|---|
| \(|v_f(T)| | 5k/2 | 5k/2 |

| \(|e_f(T)| \) | 3k-1 | 3k |

Hence subdivision of a pentagonal snake \( S \) \( (PS_k) \) admits mean square cordial labeling \( \forall k \geq 2 \).

Illustration: Mean square cordial labeling of pentagonal snake:

**Figure 4:** Mean square cordial labeling of pentagonal snake \( (PS_6) \)

**Figure 5:** Mean square cordial labeling of pentagonal snake \( (PS_4) \)
IV. CONCLUSION

In this section mean square cordial labeling is investigated for pentagonal snake graphs. It can be further investigated by the researcher for some more snake related graphs like alternate triangular snake graphs, double triangular snake graphs, alternate quadrilateral snake graphs, double quadrilateral graphs etc.

FUTURE SCOPE

Graph operations like union, intersection, corona of two graphs etc., can also be discussed for mean square cordial labeling in future.

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REFERENCES


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