Predicting Solar and Wind Based Computation Using Square Difference Labelling Technique

P. Jagadeeswari

Abstract: In this paper we have shown that the graph $D_b_n$, $C(L_n)$, $T(n, m)$, $K_1 + K_{1,m}$, balloon of the triangular snake, $DHF(n)$, bull graph $(C_t)$, Duplication of the pendant vertex by the edge of bull graph $(C_3)$ and one point union of $(bull (C_3))^2$ is a square difference graph.

Keywords: Square Difference, Circular Ladder, Balloon Graph, Bull Graph

AMS classification: 05C78

I. INTRODUCTION

In a thorough manner, we utilize simple, undirected and finite graph and we follow [2,3,6] square difference labeling-are studied in [4,8]. In [7] Sankari proved umbrella and tadpole graph for odd-even graceful labeling. Jayasekaran investigated umbrella, dumb bell and circular ladder for one edge trimagic labelling. In this work, we prove some various graphs for Square Difference Labeling.

We present some definitions, which are helpful for our work.

Definition 1.1
If a graph ‘$G$’ admits a bijective function $g : V \rightarrow \{0, 1, 2, \ldots, k\}$ such that the induced function $g^* : E(G) \rightarrow \mathbb{N}$ given by $g^*(x, y) = |g(x)|^2 - |g(y)|^2$ are all distinct, $\forall xy \in E(G)$ is called Square Difference graph.

Definition 1.2
The Dumb Bell graph $D_b_n$ is obtained by two disconnected cycles $C_2$ joined by an edge.

Definition 1.3
A Circular Ladder $C(L_n)$ in the union of an outer cycle $c_i = u_1, u_2, \ldots, u_m$ and an inner cycle $c_j = v_1, v_2, \ldots, v_m$ with additional edge $u_iv_j$, $i = 1, 2, \ldots, n$ called spokes.

Definition 1.4
A tadpole $(n, m)$ in the graph obtained by attaching a path $P_m$ to cycle $C_n$.

II. MAIN RESULTS

Theorem 2.1.
$D_b_n$ admits Square difference labeling.

Proof:
Let $G = D_b_n$ be the dumb bell graph with vertices $u_1, u_2, \ldots, u_m$ & $v_1, v_2, \ldots, v_m$ and the edges $u_1v_1, u_1v_2, \ldots, u_mv_m$ and $v_nv_n$ for $i = 1, 2, \ldots, n$.

Clearly, $|V(G)| = 2m$ and $|E(G)| = 2m+1$.

Now, define the vertex function $f; 0, 1, \ldots, 2m$ as

$$f(u_i) = 2(i-1)$$

$$f(v_i) = 2i - 1$$

And the induced function $f^*$ be

$$f^*(u_i u_{i+1}) = 8i - 4$$

$$f^*(v_i v_{i+1}) = 8i$$

for $1 \leq i \leq m - 1$

$$f^*(u_1 v_1) = 1$$

$$f^*(u_m u_1) = f(u_m)^2$$

$$f^*(v_1 v_n) = f(v_1)^2 - 1$$

Hence, all the edge labeling are distinct. Therefore, $D_b_n$ admits Square difference labeling.

Example 2.1:

![Fig 1. SDL of DB8](image)

Theorem 2.2.
The Circular Ladder $C(L_n)$ is SDG.

Proof:
Consider the circular ladder graph with the vertex set $u_i$ and $v_j$ for $j = 1, 2, \ldots, n$ and the edge set $E = \{u_i u_{i+1}, v_j v_{j+1}, u_1 v_1, u_n v_n, u_j v_j, u_1 v_1, u_n v_n\}$, $j = 1, 2, \ldots, n-1$.

Obviously, $|V(C(L_n))| = 2n$ and $|E(C(L_n))| = 3n$.

Now, define the vertex function $f$ & edge function $f^*$ as

$$f(u_i) = 2(j-1)$$

$$f(v_j) = 2j - 1$$

for $j = 1, 2, \ldots, n$

and
Theorem 2.3:
The Bull (C₃) graph is Square difference graph.
Proof: 
Define the bull graph (C₃) with 5 vertices and 5 edges. Let the bijective function \( f: \{0, 1, 2, \ldots, 9\} \rightarrow \{0, 1, 2, \ldots, 9\} \) as 
\[
 f(i) = i - 1 \quad \text{for} \quad 1 \leq i \leq 5 
\]
For the above labeling, we receive the edge label as:
\[
 f^{-1}(v_j v_{j+1}) = 2i - 1 \\
 f^{-1}(v_j v_{j+2}) = 8 
\]
Thus, the induced function \( f^{-1}(e_i) \neq f^{-1}(e_j) \) for all \( e_i, e_j \in E(C_3) \). Hence the bull (C₃) graph is Square difference graph.

Example 2.3:

![Fig 2. C(I_10)](image)

\[
 V(G) = \{x_1, x_2, x_3, x_4, x_5\} \quad 1 \leq j \leq 5 \quad \text{and} \\
 E(G) = \{x_jx_{j+1}, x_j x_j, x_j x_j, x_j x_j, x_jx_j, x_jx_j\} \quad 1 \leq j \leq 4 \]

Clearly, \(|V(G)| = 9 \& |E(G)| = 11\).

Define the bijective function \( f: V(G) \rightarrow \{0, 1, \ldots, 8\} \) as
\[
 f(x_1) = j - 1, \quad 1 \leq j \leq 5 \\
 f(x_2) = f(x_3) + 1 \\
 f(x_2) = 6 \\
 f(x_5) = 8 
\]
and \( f^{-1} \) yields the edge labeling as follows:
\[
 f^{-1}(x_j x_{j+1}) = 2j - 1, \quad 1 \leq j \leq 4 \\
 f^{-1}(x_j x_{j+2}) = 8 \\
 f^{-1}(x_j x_j) = 24 \\
 f^{-1}(x_j x_j) = 25 \\
 f^{-1}(x_j x_j) = 49 \\
 f^{-1}(x_j x_j) = 20 \\
 f^{-1}(x_j x_j) = 48 
\]
Thus, the labeling of edges of \( G \) is distinct and hence the theorem.

Example 2.4:

![Fig 3. Bull(C_3)](image)

Theorem 2.4:
Duplication of the pendant vertex by the edge of bull graph (C₃) in SDG.

Proof: 
Consider the graph \( G \) with 
\[
 f^{-1}(u_i u_{i+1}) = 8j - 4 \equiv 0 \pmod{4} \\
 f^{-1}(v_j v_{j+1}) = 8j = 0 \pmod{8} \\
 f^{-1}(u_i u_j) = [f(u_i)]^2 \\
 f^{-1}(v_i v_j) = [f(v_i)]^2 - 1 \\
 f^{-1}(u_i v_j) = 4j - 3 
\]

here, \( f^{-1}(u_i u_{i+1}) \neq f^{-1}(v_j v_{j+1}), j = 1, 2, \ldots, n-1 \). Thus, the entire 3n edges labels are distinct. Hence the theorem.

Example 2.2:

![Fig 4. SDL for duplication of pendant vertex of bull(C_3)](image)
Thus, the edge in $E$ have distinct labels. Therefore, the theorem is verified.

**Example 2.5**

![Fig 5. SDG of $(C_5)^5$](image)

**Theorem 2.6:**

The Tadpole graph $T(n, m)$ is SDG.

**Proof:**

Consider the tadpole graph with the vertices $v_1, v_2, \ldots, n + m - 1$ and the edges $v_i, v_{i+1}, v_{n+m-j}v_m, i = 1, 2, \ldots, n + m - 2$. Clearly the cardinality of vertex set and edge set are $n + m - 1$.

Now, define the vertex values function $'G'$ as

$$g(v_i) = i - 1 \quad \text{with} \quad 1 \leq i \leq n + m - 1$$

And the induced edge function $g^*$ for the above labeling pattern, we get,

$$g^*(v_i, v_{i+1}) = 2i - 1, \quad 1 \leq i \leq n + m - 2$$

$$g^*(v_{n+m-j}v_m) = [g(v_{n+m-j})]^2 - [g(v_m)]^2$$

Hence $g^*(e_i) \neq g^*(e_j) \quad \forall \quad e_i, e_j \in E(G)$ i.e., all the edge labeling are distinct and strictly increasing.

Thus, $T(n, m)$ is SDG.

**Example 2.6:**

![Fig 6. SDL for $T(5, 7)$](image)

**Theorem 2.7:**

The graph $K_1 + K_{1,a}$ admits SDL.

**Proof:**

Let $G = K_1 + K_{1,a}$ with $|V(G)| = n + 2$ and $|E(G)| = 2n + 1$.

The vertex set and edge set are defined as $V(G) = \{u, w, x_i \mid 1 \leq i \leq n\}$ & $E(G) = \{u x_1, u w, w x_i \mid 1 \leq i \leq n - 1\}$

Define a mapping $f$: $V(K_1 + K_{1,a}) \rightarrow \{0, 1, 2, \ldots, n-1\}$ as

$$f(x_i) = i + 1$$

$$f(u) = 0$$

$$f(w) = 1$$

Then $f$ induces an edge mapping $f^*$ receives edge labeling as,

$$f^*(u x_1) = [i + 1]^2$$

$$f^*(u w) = 1$$

$$f^*(w x_i) = [i + 1]^2 - 1$$

Thus, the entire $2n+1$ edge receives distinct labeling. Hence, the graph $K_1 + K_{1,a}$ is SDG.

**Example 2.7:**

![Fig 7. SDL for $K_1 + K_{1,a}$](image)

**Theorem 2.8:**

The balloon of the triangular snake graph $T_n(C_m)$ is square difference graph.

**Proof:**

Let $v_1, v_2, \ldots, v_n$ and $w_1, w_2, \ldots, w_{n+1}$ be the vertices of $T_n$ and $v_{n+1}, v_{n+2}, \ldots, v_{n+m}$ be the vertices of $C_m$.

Obviously,

$|V(G)| = 2n + m - 2$ and $|E(G)| = m + 3n - 3$

Let $f$ be the bijective function from $f$: $V \rightarrow \{0, 1, 2, \ldots, 2n + m - 1\}$ as

$$f(v_1) = 2(j - 1) \quad 1 \leq j \leq n$$

$$f(w_1) = 2j - 1 \quad 1 \leq j \leq n - 1$$

$$f(v_{n+1}) = f(v_n) + 1 \quad 1 \leq j \leq n - m - 1$$

For the above vertex labeling, we receive the edge label as.

$$f^*(v_1, v_{n+1}) = 8j - 4 \equiv 0 \text{mod} 4, \quad 1 \leq j \leq n - 1$$

$$f^*(v_{n+1}, v_{n+2}) = 4n + 2j - 3, \quad 0 \leq j \leq n - 2$$

$$f^*(v_1, w_1) = 4j - 3$$

$$f^*(w_1, v_{n+1}) = 4j - 1 \quad 1 \leq j \leq n - 1$$

Hence, $f^*(e_i) \neq f^*(e_j), \forall e_i, e_j \in E(G)$, Thus, all the edges receive distinct labeling.

Therefore, the theorem is verified.
Example 2.8.

Theorem 2.9:
DHF\( (n) \) admits square difference labeling.

Proof:
Let, the double headed circular fan DHF\( (n) \) be the cycle \( v_1, v_2, \ldots, v_n \) with the additional edges \( v_i u, i = 1, 2, \ldots, n-3 \) and \( v_i v, i = n-2, n-1, n \). Clearly,

\[
| V(DHF(n)) | = n + 2 \quad \text{and} \\
| E(DHF(n)) | = 2n
\]

Define, the vertex valued function \( f \) as follows:

\[
f(v_j) = j - 1, 1 \leq j \leq n \\
f(w) = 9 \\
f(u) = n + 1
\]

For, the above defined labeling, we receive \( f^* \) as follows:

\[
f^*(v_j v_{j+1}) = 2j - 1, 1 \leq j \leq n \\
f^*(u v_j) = (n + 1)^2 - (j + 1)^2, 1 \leq j \leq n - 3 \\
f^*(w v_j) = | n^2 - f(v_j)^2 |, j = n - 2, n - 1, n.
\]

Thus, all the edge labels are different. Hence, the theorem is verified.

Example 2.9.

III. CONCLUSION

In this work, we proved that some cycle related graphs admit square difference labeling.

REFERENCES


AUTHORS PROFILE

P. Jagadeeswari is an Assistant Professor, Department of Science and Humanities, BIHER, Chennai. She is currently working on Graph labeling. She published 7 papers in international journal.