Abstract: In this paper, we investigate the degree splitting graphs of $P_n$ ($n > 3$), comb graph, $(K_1^n, 1_{1,n})$, cycle graph $C_n$, and $S(K_{1,n})$ and Tadpole graph are Square difference graph (SDG).

Keywords: Square difference labeling (SDL), comb graph, tadpole graph, degree splitting graph, vertex function, induced edge labels, degree splitting graph of cycle graph, degree splitting graph of star graph, greeting label, cordial, domination in graphs.

I. INTRODUCTION

The Square difference labelling were established by Shama [8]. The degree splitting concept was introduced in [5]. P. Maya and Nicholas proved that the degree splitting of some graphs $G$ is a cordial graph [4]. Domination in degree splitting graphs were established by B. Basavangoval et al. [1]. Mean labelling on degree splitting graph of star graph was investigated in [9]. Square difference labeling of some special graphs were proved in [3]. In this paper, we use simple, finite and undirected graph and we follows notation, terminology from [2, 3, 6] prove that the degree splitting graph of various graphs are Square difference graph.

II. MAIN RESULTS

Definition 2.1.1. [8]
A graph $G = (p, q)$ is said to be a Square difference graph if it admits a bijective function $g : V(G) \rightarrow \{0, 1, 2, \ldots, p - 1\}$ such that the induced function $g^* : E(G) \rightarrow N$ given by $g^*(xy) = [g(x)]^2 - [g(y)]^2$ are all distinct, $\forall xy \in E(G)$.

Definition 2.1.2. [5]
The degree splitting graph is obtained from $G$ by adding vertices $w_1, w_2, \ldots, w_t$ and joining to each vertex of $S_i$, $1 \leq i \leq t$ which is a set of vertices having at least two vertices of the same degree and is denoted by DS($G$).

Definition 2.1.3 [7]
A tadpole $T(n, m)$ is the graph procured by appending a path $P_t$ to cycle $C_n$.

Theorem 2.1.
The graph DS($P_n$) is Square difference graph.

Proof:
Consider DS($P_n$) ($n > 3$) be the graph with $V = \{u_1, u_2, \ldots, u_n\}$ and $E = \{u_{i-1}u_i, u_{i+1}u_i, w_{2i}, w_{2i+1} | 1 \leq i \leq n\} \cup \{w_{2i} | 2 \leq i \leq n - 1\}$. Clearly, $|V(G)| = n + 2$, and $|E(G)| = 2n - 1$
Now, define the vertex function as $f : V \rightarrow \{0, 1, \ldots, n + 1\}$ as $f(u) = i - 1$, $1 \leq i \leq n$

Then, the induced edge labels $f^*$ are given below:

- $f^*(u_iu_{i+1}) = 2i - 1$
- $f^*(w_{2i}u_i) = n^2$
- $f^*(w_{2i}u_i) = 2n - 1$

Thus, the entire $2n - 1$ edge labeling are all distinct. Hence the theorem.

Example 2.1.
The SDG of DS($P_n$)

Figure 1. SDG of DS($P_n$)

Theorem 2.2.
The Degree splitting graph of $P_m \circ K_1$ admits Square difference labeling.

Proof:
Let $G = DS (P_m \circ K_1)$ with the vertex set $V(G) = \{u_1, u_2, \ldots, u_{2m}, w_1, w_2, \ldots, w_m | 1 \leq i \leq m\}$ and $E(G) = \{u_{2i-1}u_{2i} | 1 \leq i \leq m - 1\} \cup \{u_{2i}v_i | 1 \leq i \leq m\} \cup \{w_{2i}v_i, w_{2i+1} | i = 2, 3, \ldots, m - 1\} \cup \{w_{2m}, u_n, w_{2m}\}$

It is clear that, $|V(G)| = 2m + 3$ and $|E(G)| = 4m - 1$.

Now, the vertex valued function $f$ as:

- $f(u_i) = 2(i - 1)$
- $f(v_i) = 2i - 1$
- $f(w_i) = 2m$
- $f(w_{2i}) = 2m + 1$
- $f(w_{2i}) = 2m + 2$

Consider, the edge labeling $f^*$ as:

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\[ f^*(u_i u_{i+1}) = 8i - 4 \]
\[ f^*(u_i v_j) = 4i - 3 \]
\[ f^*(w_i v_j) = \begin{cases} (7 \mod 8), n \text{ is even} \\ (3 \mod 8), n \text{ is odd} \end{cases} \]
\[ f^*(w_i u_j) = \begin{cases} (5 \mod 8), i \text{ is even} \\ (1 \mod 8), i \text{ is odd} \end{cases} \]
\[ f^*(w_i u_j) \equiv 0 \mod (8) \]

Therefore, both the vertex and edge labeling are satisfies the SD labeling. Hence the theorem.

Example: 2.2.

SDG of \( P_7 \circ K_5 \)

![Figure 2.2. P_7 \circ K_5](image)

**Theorem 2.3.**

The graph \( DS(K_{1,n}^{(1)} \circ K_{1,n}^{(2)}) \) is Square difference graph.

**Proof:**

Consider the graph \( DS(K_{1,n}^{(1)} \circ K_{1,n}^{(2)}) \) with
\[ V[DS(K_{1,n}^{(1)} \circ K_{1,n}^{(2)})] = \{x_i, y_i | 1 \leq i \leq n\} \cup \{v, y\} \cup \{w_i, w_2, w_3\} \]
and \( E[DS(K_{1,n}^{(1)} \circ K_{1,n}^{(2)})] = \{x_i y_j | 1 \leq i \leq n\} \cup \{w_i x_i, w_2 x_i\} \cup \{w_2 y, w_3 y\} \)

The number of vertices and edges are denoted as \( 2n + 4 \) and \( 4n + 4 \) respectively.

Let the vertex labeling \( f: V \rightarrow \{0, 1, \ldots , 2n + 3\} \) is given below:
\[ f(w_1) = 0 \]
\[ f(w_2) = 2n + 3 \]
\[ f(w_3) = 2n + 4 \]
\[ f(x) = 2 \]
\[ f(y) = 1 \]
\[ f(x_i) = i + 2 \]
\[ f(y_i) = n + i + 2 \]

and the induced edge labels are
\[ f^*(x x_i) = i^2 + 4i \]
\[ f^*(y y_i) = (n + i + 2)^2 - 1 \]
\[ f^*(w_i x_i) = (i + 1)^2 \]
\[ f^*(w_2 y) = (n + i + 2)^2 \]
\[ f^*(w_3 y) = (2n + 3)^2 - 4 \]
\[ f^*(w_3 y) = (2n + 4)^2 - 1 \]

Thus, no edge labeling are same. Therefore, the theorem is proved.

Example: 2.3.

Square difference labeling for \( DS(K_{1,n}^{(1)} \circ K_{1,n}^{(2)}) \).

![Figure 2.3. DS(K_{1,n}^{(1)} \circ K_{1,n}^{(2)})](image)

**Theorem 2.4.**

\( DS(C_n \circ K_n) \) admits SDL.

**Proof:**

Consider the vertex and edge set of the degree splitting graph of \( C_n \circ K_n \) as
\[ V[DS(C_n \circ K_n)] = V_1 \cup V_2 \cup V_3, \]
where
\[ V_1 = \{v_i | 1 \leq i \leq m\} \]
\[ V_2 = \{v_j^{(r)} | 1 \leq j \leq n, 1 \leq r \leq t\} \]
\[ V_3 = \{x, y\} \]

And \( E[DS(C_n \circ K_n)] = E_1 \cup E_2 \cup E_3, \)
where
\[ E_1 = \{v_i v_{i+1} | 1 \leq i \leq m - 1\} \]
\[ E_2 = \{v_j^{(r)} | 1 \leq i \leq n, 1 \leq r \leq t\} \]
\[ E_3 = \{x v_i, y v_i\} \]

Now, the bijective function \( f \) on \( V \) is defined as:
\[ f(v_i) = i + 1 \]
\[ f(v_j^{(r)}) = m + j + 1 + (r - 1) n \]
\[ f(x) = 0 \]
\[ f(y) = 1 \]

The induced function \( f^* \) for the above vertex labeling is given below:
\[ f^*(v_1 v_{i+1}) = 2i + 3 \]
\[ f^*(v_j^{(r)}) = (m + 1)^2 - 4 \]
\[ f^*(v_j^{(r)}) = [(i + 1)^2 - (m + j + 1 + (r - 1)n)^2) \]
\[ f^*(x v_j^{(r)}) = [m + j + 1 + (r - 1)n]^2 \]
\[ f^*(y v_j^{(r)}) = (i + 1)^2 \]

Clearly, the induced function \( f^* \) are all distinct. Hence, the theorem.

**Example 2.4.**

The degree splitting graph of \( C_n \circ K_n \).

![Image](image)
Theorem 2.5.
The degree splitting graph of Tadpole $T(n, m)$ admits square difference labeling.

Proof: Let $G = DS[T(n, m)]$ with $V(G) = \{v_j | 1 \leq j \leq n\} \cup \{x\}$ and $E(G) = \{v_jv_{j+1} | 1 \leq j \leq m + n - 2\} \cup \{v_{n+m-1}v_m\} \cup \{xv_{n+m}\}$

It is seen clear that the number of vertices and number of edges are $n + m$ and $2(m + n) - 4$ respectively. Let the vertex valued function $g: V \rightarrow \{0, 1, \ldots, n + m - 1\}$ be defined as follows:

$g(v_j) = j - 1$

$g(x) = n + m - 1$

and the induced function $g^*: E(G) \rightarrow \mathbb{N}$ satisfies the condition of SD Labeling. Thus the edge labels are defined as

$g^*(v_jv_{j+1}) = 2j - 1$

$g^*(v_{n+m-1}v_m) = g^*(v_{n+m-1})^2 - [g(v_m)]^2$

$g^*(xv_{n+m}) = (n + m - 1)^2 - [2j - 1]^2$

Thus, the entire edge labeling are distinct. Therefore, $DS[T(n,m)]$ is SDG.

Example 2.5.
SDG of Degree splitting graph of $T(4, 6)$.

Theorem 2.6.
The degree splitting of subdivision of $K_{1,n}$ admits Square difference graph.

Proof: Consider the graph $DS(S(K_{1,n}))$ with $V = \{u_j, w_j, x, y, z | 1 \leq j \leq n\}$ and $E = \{u_jw_j, xu_j, yw_j, zu_j | 1 \leq j \leq n\}$

Now, define the function $f$ as

$f(z) = 0$

$f(y) = 1$

$f(x) = 2$

$f(u_j) = 2j + 2$

$f(w_j) = 2j + 4$

and the induced edge function $f^*$ receive labeling as:

$f^*(u_jw_j) = 4j + 3$

$f^*(xu_j) = [2j + 2]^2 - 4$

$f^*(yw_j) = [2j + 2]^2 - 1$

$f^*(zu_j) = [2j + 2]^2 - 3$

Hence $f^*(e_j) \neq f^*(e_i), \forall e_j, e_i \in E(G)$. Thus, all the edge labeling are not same. Therefore, the degree splitting graph of subdivision of $K_{1,n}$ is square difference graph.

Example: 2.6.
Square Difference Graph of $S(K_{1,5})$.

III. CONCLUSION

In this work, we investigated that the degree splitting of some graphs are square difference graph.

REFERENCES
[9]. Siliviya Francis, V. Balaji, “Mean labeling on degree splitting graph

Figure 2.4. $DS(C_4 \square K_2)$

Figure 2.5. $DS[T(4, 6)]$

Figure 2.6. $DS(K_{1,s})$

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