Considering Square Difference Labelling for Validating Theta Graphs of Dynamic Machinaries

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Abstract: In this work, we prove that the graphs $Z - P_n$, braid graph, switching of an apex vertex of $CH_n$, $T_n$, $\theta$ $K_5$, bull graph ($C_5$), truncated tetrahedron, frucht graph are Square difference graph (SDG).

Keywords: Square Difference, braid graph, truncated tetrahedron, frucht graph, Bull Graph, AMS classification: 05C78

I. INTRODUCTION

Throughout this paper, we utilize simple, undirected and finite graph. In this paper and we follow [2,3,8]. Square difference labeling are studied in [9, 10]. N. B. Rathod proved $Z - P_n$, braid graph, triangular ladder are 4 - cordial [7]. Bull graph, shell graph flag graph for L cordial were proved in [6]. Jagadeeswari et. al. proved square difference labeling for pyramid and H graph [4,5]. Subashini proved theta graph admits square difference labeling [11]. In this work, we prove some various graphs for Square Difference Labeling.

We present some definitions, which are helpful for our work.

Definition 1.1.
A function of a graph $G = (p,q)$ is said to be a Square difference graph (SDG), if it admits a bijective function $f : V(G) \rightarrow \{0, 1, 2, ..., p-1\}$ such that the induced function $f^* : E(G) \rightarrow N$ given by $f^*(uv) = |f(u)|^2 - |f(v)|^2$, $\forall uv \in E(G)$ and the edge labels are distinct.

Definition 1.2,[1]
The graph $Z - P_n$ is obtained from the pair of paths $P_n^r$ and $P_n^l$. Let $v_i$ and $u_i$, $i = 1, 2, .. n - 1$, are the vertices of path $P_n^r$ and $P_n^l$ respectively. To find $Z - P_n$, join $i^{th}$ vertex of path $P_n^r$ with $(i + 1)^{th}$ vertex of path $P_n^l$ for all $i = 1, 2, .. n - 1$.

Definition 1.3,[1]
The braid graph $B(n)$, $(n \geq 3)$, is obtained by joining $i^{th}$ vertex of $P_n^r$ with $(i + 1)^{th}$ vertex of $P_n^l$ and $j^{th}$ vertex of $P_n^r$ with $(i + 2)^{th}$ vertex of $P_n^l$ with the new edges for all $1 \leq i \leq n - 2$.

Definition 1.4.
A closed helm $CH_n (n \geq 3)$, is the graph obtained from the helm $H_n$ and adding a edges between the pendant vertices.

Definition 1.5.
The truncated tetrahedron graph is formed with 12 vertices and 18 edges. It is a 3 regular graph and it has no trivial symmetries.

II. MAIN RESULTS

Theorem 2.1.
The graph $Z - P_n$ is Square difference graph.

Proof:
Let the graph $Z - P_n$ has $2n$ vertices and $3n - 3$ edges. Consider, $V (Z - P_n) = \{u_i, v_i / 1 \leq i \leq n\}$ and $E(Z - P_n) = \{u_i u_{i+1}, v_i v_{i+1}, v_i u_{i+1} / 1 \leq i \leq n - 1\}$

Define the mapping $f : V \rightarrow \{0, 1, 2, ..., n\}$ as follows:

$f(u_i) = 2i - 1$
$f(v_i) = 2(i - 1)$

and $f^*$ for the above labeling is mentioned as:

$f^*(u_i u_{i+1}) = 8i \equiv 0 (mod 8)$
$f^*(v_i v_{i+1}) = 8i - 4 \equiv 0 (mod 4)$
$f^*(v_i u_{i+1}) = 12i - 3$

Thus, all the edge labeling are distinct. Hence the theorem. For instance, the example mentioned below.

Fig. 1. SDL for $Z - P_5$
Theorem 2.2.

The Braid graph admits SDL.

Proof:

Let $u_1, u_2, \ldots, u_n$ be the vertices of path $P'_n$ and $v_1, v_2, \ldots, v_n$ be the vertices of path $P''_n$. Similarly, the edges of path be $u_j u_{j+1}$, $v_j v_{j+1}$, and $u_j v_{j+1}$. The bijective function of braid graph $f$ is given as:

$$f(u_j) = 2j - 1$$

$$f(v_j) = 2(j - 1) \text{ for } 1 \leq j \leq n$$

and the injective function $f^*$ is given below:

for $1 \leq j \leq n - 1$

$$f^*(u_j u_{j+1}) = 8j$$

$$f^*(v_j v_{j+1}) = 8j - 4$$

$$f^*(v_j u_{j+1}) = 20j + 5$$

$$f^*(u_j v_{j+1}) = 4j - 1$$

Thus, all the edge labeling are not repeated. Hence, the braid graph admits SDL and B(5) given below.

Fig. 2 SDL for B(5)

Theorem 2.3.

The switching of an apex vertex of $CH_n$ admits SDL.

Proof:

Consider the graph $G$ with the vertex set $V(G) = \{x_i, y_i, w_i / 1 \leq i \leq n\}$ and the edges $E(G) = \{x_i x_{i+1}, y_i y_{i+1}, x_i y_i, w_i y_i, x_n x_1, y_n y_1, w_n y_1 / 1 \leq i \leq n - 1\}$.

Clearly, $|V(G)| = 2n$ and $|E(G)| = 3n$

Now, define the vertex labeling $g$ and edge labeling $g^*$ as

$$g(x_i) = 2j$$

$$g(y_i) = 2j - 1$$

$$g(w) = 0$$

$g^*(x_i x_{i+1}) = 8j + 4 \equiv 0 \text{ (mod 4)}$  

$g^*(y_i y_{i+1}) = 8j \equiv 0 \text{ (mod 8)}$

$g^*(x_n x_1) = 4n - 4$

$g^*(y_n y_1) = (2n - 1)j - 1$

$g^*(x_i y_i) = 4j - 1$

$g^*(w_i y_i) = (2j - 1)^2$

Hence, all the edge labeling are satisfied the condition of square difference labeling. Therefore, the theorem is verified.

For instance, switching of an apex vertex of CH4 given.

Fig. 3. SDL of switching graph CH4

Theorem 2.4.

$T_{n}^{\alpha} K_2$ is SDG.

Proof:

Let $v_o, v_1, v'_1$ be the vertices and $v_1 v_{i+1}, v_o v_{i+1}, v_1 v_{i+1}, v'_1 v_{i+1}$ be the edges of the graph $T_{n}^{\alpha} K_2$ with $|V(T_{n}^{\alpha} K_2)| = 21$ and $|E(T_{n}^{\alpha} K_2)| = 32$. Now determine the mapping $f : V \rightarrow \{0, 1, 2, \ldots, n\}$

$$f(v_o) = 0$$

$$f(v_i) = i, 1 \leq i \leq 6$$

$$f(v'_i) = i + 7, 0 \leq i \leq 6$$

$$f(v_{i+1}) = i + 14, 0 \leq i \leq 6$$

and edge function $f^*$ be

$$f^*(v_1 v_{i+1}) = 2i + 1, 1 \leq i \leq 5$$

$$f^*(v_1 v'_i) = (i + 1)^2 - i^2$$

$$f^*(v_o v_{i+1}) = 35$$

$$f^*(v_o v'_i) = 16$$

Thus, the induced function $f^*(e_i) \neq f^*(e_j) \forall e_i, e_j \in E(G)$.

Hence the theorem is proved and the example given below.

Fig. 4. SDL for $T_{n}^{\alpha} K_2$

Theorem 2.5.

Bull graph (C_n) is Square difference graph.

Proof:

Consider, $G = bull$ (C_n) with the vertex set $V = \{x_i, w_1, w_2 / 1 \leq i \leq n\}$ and the edge set $E = \{x_i x_{i+1}, x_n x_1, w_1 x_i, w_2 x_1 x_{i+1} / 1 \leq i \leq n - 1\}$ and also with the cardinality of vertices and edges be $n + 2$. 

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Define, the vertex function $f$ as
\[ f(x_j) = i + 1 \]
\[ f(w_1) = 0 \]
\[ f(w_2) = 1 \]
and the edge function $f^*$ as
\[ f^*(x_i, x_{i+1}) = 2i + 3 \]
\[ f^*(x_i, x_j) = [n + 1]^2 - 4 \]
\[ f^*(w_1, x_i) = 4 \]
\[ f^*(w_2, x_i) = 8 \]
Thus, the edge in E have distinct labels. Therefore, the theorem is verified.

**Theorem 2.6.**

The truncated tetrahedron graph admits Square difference labeling.

**Proof:**

Let the truncated graph is obtained with 12 vertices and 18 edges. \( v_j \) be the vertices for \( j = 1, 2, \ldots, 12 \).

Define the bijective function \( g \)
\[ g(v_j) = j - 1 \]
and the induced function \( g^* \) as
\[ g^*(v_1, v_2) = 4 \]
\[ g^*(v_1, v_6) = 16 \]
\[ g^*(v_1, v_7) = 25 \]
\[ g^*(v_2, v_3) = 8 \]
\[ g^*(v_2, v_5) = 35 \]
\[ g^*(v_3, v_4) = 48 \]
\[ g^*(v_3, v_9) = 60 \]
\[ g^*(v_3, v_10) = 77 \]
\[ g^*(v_4, v_11) = 91 \]
\[ g^*(v_5, v_12) = 112 \]
\[ g^*(v_6, v_7) = 105 \]

Clearly, the edge labels defined above are all distinct. Hence the theorem. The truncated tetrahedron graph for SDL given below:

**Theorem 2.7.**

The Frutcht graph admits SDG.

**Proof:**

Consider the graph \( G \) be the Frutcht graph with 12 vertices and 18 edges.

Let \( u_j \) be the vertex set for \( 1 \leq j \leq 12 \).

Then label the vertex \( u_j \) as \( j - 1 \) and the edge labels as
\[ f^*(v_1, v_2) = 64 \]
\[ f^*(v_2, v_10) = 80 \]
\[ f^*(v_2, v_11) = 76 \]
\[ f^*(v_2, v_11) = 91 \]
\[ f^*(v_3, v_11) = 84 \]
\[ f^*(v_3, v_12) = 24 \]
\[ f^*(v_3, v_12) = 36 \]
\[ f^*(v_3, v_12) = 13 \]
\[ f^*(v_3, v_12) = 57 \]
\[ f^*(v_3, v_12) = 40 \]
\[ f^*(v_3, v_12) = 21 \]

For the above vertex labeling we receive the distinct edge labels. Hence the theorem.
III. CONCLUSION

In this paper we discussed some special graphs like $Z\cdot P_n$, braid graph, switching of an apex vertex of $C_{4n}$, $T_{4} \oplus K_2$, bull graph ($C_n$), truncated tetrahedron, frucht graph are Square difference graph (SDG).

REFERENCES


AUTHORS PROFILE

P. Jagadeeswari is an Assistant Professor, Department of Science and Humanities, BBHER, Chennai. She is currently working on Graph labeling. She published 7 papers in international journal.