Simultaneous Approximation for Beta-Baskakov-Stancu Operators

Rupa Rani Sharma, Sangeeta Garg

Abstract: The paper is some proposal of advance properties of Beta-Baskakov-Stancu operators consisting hypergeo-metric series function. The central moments, Voronovskaya type asymptotic formula and error estimates applying some characteristic properties are obtained for mentioned operators.

Key words: Baskakov operators, Beta operators, Hypergeometric series, Stancu type generalization, Voronovska type Asymptotic properties,

I. INTRODUCTION

To approximate integrable function of $y$ in $[0;\infty)$, the Beta-Baskakov operators, also named as modified Beta operators, discussed in [4], [5] are given as

$$M_{n}(f, y) = \frac{1}{n} \sum_{p=0}^{n} b_{n,p}(y) \int_{0}^{\infty} p_{n,p}(t) f(t) dt, \quad \ldots (1.1)$$

where

$$b_{n,p}(y) = \frac{(n+p)!}{(n+1)! \cdot (1+y)^{n+p+1}} \cdot \frac{y^{p}}{(p+1)!},$$

and

$$p_{n,p}(t) = \frac{(n+1)! \cdot (1+y)^{n+p+1}}{(n+1)! \cdot (1+y)^{n+p+1}}.$$ 

In order to take view the modified Beta operators explained in (1.1), we can have better approximation using Hypergeometric series. Some approximation properties for these operators and related to such types of operators were studied by Gupta-Agrawal [3]. Many authors studied hypergeometric series form of several operators. Its characteristic notations can be studied in Gasper-Rahman [2], Sharma-Garg [7]. Using Pochhammer symbol $(l)_{r}$, where $(l)_{r} = l(l+1)(l+2)\ldots(l+r-1)$, we can rewrite operators (1.1) as

$$M_{n}(f, y) = \frac{1}{n} \sum_{p=0}^{n} \frac{(n+p)!}{p!} \cdot \frac{y^{p}}{(1+y)^{n+p+1}} \int_{0}^{\infty} f(t) \frac{t^{p}}{(1+t)^{n+p+1}} dt$$

Moreover

$$M_{n}(f, y) = \frac{1}{n} \sum_{p=0}^{n} \frac{(n+p)!}{p!} \cdot \frac{y^{p}}{(1+y)^{n+p+1}} \int_{0}^{\infty} f(t) \frac{t^{p}}{(1+t)^{n+p+1}} dt$$

Applying the hypergeometric series

$$\sum_{p=0}^{\infty} \frac{(c)_p (b)_p}{(a)_p} \cdot \frac{y^{p}}{p!} = (1)_v = v!,$$

We have

$$M_{n}(f, y) = \frac{1}{n} \sum_{p=0}^{n} \frac{(n+p)!}{p!} \cdot \frac{y^{p}}{(1+y)^{n+p+1}} \int_{0}^{\infty} f(t) \frac{t^{p}}{(1+t)^{n+p+1}} dt$$

Take $F_{1}(c, b; a; y) = \frac{y^{t}}{(1+y)^{n+p+1}} \int_{0}^{\infty} f(t) \frac{t^{p}}{(1+t)^{n+p+1}} dt$

$$M_{n}(f, y) = \frac{1}{n} \sum_{p=0}^{n} \frac{(n+p)!}{p!} \cdot \frac{y^{p}}{(1+y)^{n+p+1}} \int_{0}^{\infty} f(t) \frac{t^{p}}{(1+t)^{n+p+1}} dt$$

These are operators required in terms of hypergeometric series function. In [6] and [1], the Stancu type generalization of some operators made easy our study. Therefore for $0 \leq a \leq \beta$, the Stancu type generalization of (1.2) is

$$M_{n}^{\alpha,\beta}(f, y) = \frac{1}{n} \sum_{p=0}^{n} \frac{(n+p)!}{p!} \cdot \frac{y^{p}}{(1+y)^{n+p+1}} \int_{0}^{\infty} f(t) \frac{t^{p}}{(1+t)^{n+p+1}} dt$$

II. BASIC RESULTS

In this section, some particular lemmas are established to be needed to find our main objectives.

Lemma 1 For $n \in N$; $r > 0$; we have

$$M_{n}(t^{r}, y) = \frac{(n+1)!}{n! \cdot (n-r-2)!} \cdot \frac{y^{r}}{r!} \cdot \frac{(1+y)^{r}}{1+y} \ldots (2.1)$$

Moreover

$$M_{n}(t^{r}, y) = \frac{(n+1)!}{n! \cdot (n-r-2)!} \cdot \frac{y^{r}}{r!} \cdot \frac{(1+y)^{r}}{1+y} \ldots (2.2)$$

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Proof: Taking $f(t) = t^r$, $t = (1 + y)u$ and applying Pfaff-Kummer transformation in (1.2), we have

$$M_n(t^r, y) = (n - 1) \sum_{v=0}^{\infty} \frac{(n)_{v}(-n)_v}{(v)!^2} \frac{(-y)^v(1+y)^{r-n}}{(1+y)(1+u)} \times \frac{\Gamma(r+v+1)\Gamma(n-r-1)}{\Gamma(n+v)}$$

Adopting Pfaff-Kummer transformation, we get

$$M_n(t^r, y) = (n - 1) (1 + y)^{r-n} \frac{1}{y-1}$$

Lemma 2

For $0 \leq \alpha \leq \beta$, we have

$$M_n^{\alpha, \beta}(t^r, y) = y^r \frac{(n)_{v}(-n)_v}{(v)!^2} \frac{(-y)^v(1+y)^{r-n}}{(1+y)(1+u)} \times \frac{\Gamma(r+v+1)\Gamma(n-r-1)}{\Gamma(n+v)}$$

Proof: Applying Binomial theorem [B(n, p)], the aspect between operators (1.2) and (1.3) can be outlined as

$$M_n^{\alpha, \beta}(t^r, y) = \frac{(n - 1)}{n} \sum_{v=0}^{\infty} b_{n,v}(y) \int p_{n,v}(t) \left(\frac{nt + \alpha}{n + \beta} \right)^r dt$$

$$= \left(\frac{n-1}{n} \right) \sum_{v=0}^{\infty} b_{n,v}(y) \int p_{n,v}(t) r^{r-j} \left(\frac{nt + \alpha}{n + \beta} \right)^r dt$$

Lemma 3

For $m \in N^0$, if

$$T_{n,m}(y) = \sum_{n=0}^{\infty} b_{n,v}(y) \left(\frac{v}{n+1} - y \right)^m$$

then we have

$$T_{n,m+1}(y) = y(y + 1) [T_{n,m}(y) + mT_{n,m-1}(y)]$$

Consequently,

1. $T_{n,m}$ are polynomials in $y$.
2. $T_{n,m}$ is of order $O(n^{-(m+1)/2})$.

Lemma 4

Taking integer $m \geq 0$, the central moments related to the operators (1.3) are defined as

$$T_{n,m}^{\alpha, \beta}(y) = M_n^{\alpha, \beta}((t - y)^m, y)$$

then we achieve

$$T_{n,m}^{\alpha, \beta}(y) = 1, T_{n,1}^{\alpha, \beta}(y) = y$$

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and subsequent recurrence formula for \( n > m + 1, \ m \geq 1 \) is
\[
(n - m - 2) \left( \frac{n + \beta}{n} \right) T_{n,m+1}(y) = y(1 + y)\left[ T'_{n,m}(y) + m T_{n,m-1}(y) \right] + \left( \frac{\alpha}{n + \beta - y} \right) \left( \frac{n + \beta}{n} \right) \left( \frac{n + \beta - y}{n + \beta} \right) \sum_{\nu=0}^{\infty} b_{n,\nu}(y) P'_{n,\nu}(t) \int_{0}^{\infty} \frac{(nt + \alpha)^{m+1}}{(n + \beta - y)} \, dt - \left( \frac{\alpha}{n + \beta - y} \right) \left( \frac{n + \beta}{n} \right) \sum_{\nu=0}^{\infty} b_{n,\nu}(y) P'_{n,\nu}(t) \int_{0}^{\infty} \frac{(nt + \alpha)^{m}}{(n + \beta - y)} \, dt + \left( \frac{\alpha}{n + \beta - y} \right) \left( \frac{n + \beta}{n} \right) \sum_{\nu=0}^{\infty} b_{n,\nu}(y) P'_{n,\nu}(t) \int_{0}^{\infty} \frac{(nt + \alpha)^{m+2}}{(n + \beta - y)} \, dt.
\]

Proof: Obviously \( T_{n,0}(y) = 1 \). For the proof of different moments, we obtain the recurrence relation for \( m \geq 1 \). The following identities are used to prove it:

1. \( (1 + y)b_{n,v}'(y) = (v - (n + 1)y)b_{n,v}(y) \),
2. \( (1 + y)p_{n,v}'(y) = (v - ny)p_{n,v}(y) \).

Differentiating the given central moment with respect to \( y \) and using identity
\[
y(1 + y)T'_{n,m}(y) + my(1 + y)T_{n,m-1}(y) = \left( \frac{n - 1}{n} \right) \sum_{\nu=0}^{\infty} (v - (n + 1)y)b_{n,\nu}(y) \int_{0}^{\infty} p_{n,\nu}(t) \left( \frac{nt + \alpha}{n + \beta - y} \right)^{m} \, dt.
\]

Thus
\[
y(1 + y)T'_{n,m}(y) + my(1 + y)T_{n,m-1}(y) = \left( \frac{n - 1}{n} \right) \sum_{\nu=0}^{\infty} (v - ny)b_{n,\nu}(y) \int_{0}^{\infty} p_{n,\nu}(t) \left( \frac{nt + \alpha}{n + \beta - y} \right)^{m} \, dt.
\]

Integrating by parts
\[
(n - m - 1) \left( \frac{n + \beta}{n} \right) T_{n,m+1}(y) = y(1 + y)\left[ T'_{n,m}(y) + m T_{n,m-1}(y) \right] + \left( \frac{\alpha}{n + \beta - y} \right) \left( \frac{n + \beta}{n} \right) \left( \frac{n + \beta - y}{n + \beta} \right) \sum_{\nu=0}^{\infty} b_{n,\nu}(y) P'_{n,\nu}(t) \int_{0}^{\infty} \frac{(nt + \alpha)^{m+1}}{(n + \beta - y)} \, dt - \left( \frac{\alpha}{n + \beta - y} \right) \left( \frac{n + \beta}{n} \right) \sum_{\nu=0}^{\infty} b_{n,\nu}(y) P'_{n,\nu}(t) \int_{0}^{\infty} \frac{(nt + \alpha)^{m}}{(n + \beta - y)} \, dt + \left( \frac{\alpha}{n + \beta - y} \right) \left( \frac{n + \beta}{n} \right) \sum_{\nu=0}^{\infty} b_{n,\nu}(y) P'_{n,\nu}(t) \int_{0}^{\infty} \frac{(nt + \alpha)^{m+2}}{(n + \beta - y)} \, dt.
\]

III. MAIN RESULTS

We obtain certain direct results including asymptotic properties and some error estimates in simultaneous approximation here.

Theorem 1 If \( f \in R[0, \infty) \) is bounded on each finite subinterval of \([0, \infty)\) accepts the \((r + 2)^{th}\) order derivative at a fixed point \( y \in [0, \infty) \). Again, if \( f(t) = O(t^{r}) \) as \( t \to \infty \) and \( y > 0 \) then we have
\[
\lim_{n \to \infty} n^{r} \left[ M_{n,a,b}(f,y) - f^{(r)}(y) \right] = r(r + 2 - \beta) f^{(r)}(y) + \frac{(1 + r + \alpha)}{y(2r + 3 - \beta)} \times f^{(r+1)}(y) + (1 + y) f^{(r+2)}(y).
\]
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**Proof:** Through Taylor hypothesis for every function $f$, we have

$$f(t) = \sum_{j=0}^{r+2} \frac{f^{(j)}(y)}{j!} (t - y)^j + \varepsilon(t, y)(t - y)^{r+2},$$

here $\varepsilon(t, y) \to 0$ as $t \to y$ and $\varepsilon(t, y) = O((t - y)^\delta)$ as $t \to \infty$ for some $\delta > 0$, therefore we can compose

$$\lim_{n \to \infty} nM^{(r)}_{n, \alpha, \beta}(f, y) - f^{(r)}(y)$$

$$= n \left[ \sum_{j=0}^{r+2} \frac{f^{(j)}(y)}{j!} M^{(r)}_{n, \alpha, \beta}(t - y)^j - f^{(r)}(y) \right]$$

$$+ nM^{(r)}_{n, \alpha, \beta}(\varepsilon(t, y)(t - y)^{r+2}, y)$$

$$= E_1 + E_2.$$  

Lemma 2 gives

$$E_1 = n \left( \sum_{j=0}^{r+2} \frac{f^{(j)}(y)}{j!} \sum_{i=0}^{i} \binom{i}{j} (-y)^{j-i} M^{(r)}_{n, \alpha, \beta}(t^j, y) - nf^{(r)}(y) \right)$$

$$= \frac{f^{(r)}(y)}{r!} n \left[ M^{(r)}_{n, \alpha, \beta}(t^r, y) - r! \right] + \frac{f^{(r+1)}(y)}{(r+1)!} n \times$$

$$\left\{ (r + 1)(-y)M^{(r)}_{n, \alpha, \beta}(t^r, y) + M^{(r)}_{n, \alpha, \beta}(t^{r+1}, y) \right\} + \frac{f^{(r+2)}(y)}{(r+2)!} n \times$$

$$\left\{ \frac{(r+2)(r+1)}{2} y^2 M^{(r)}_{n, \alpha, \beta}(t^r, y) + (r+2)(-y) \right\} \left[ M^{(r)}_{n, \alpha, \beta}(t^{r+1}, y) + M^{(r+1)}_{n, \alpha, \beta}(t^{r+2}, y) \right]$$

$$= nf^{(r)}(y) \left[ \frac{n^r(n + r)!}{(n + \beta)^{r+1}!(n - 2)!} \right] \times$$

$$+ \frac{n^{r+1}(n + r + 1)!}{(n + \beta)^{r+2}!(n - 2)!} r! + (r+2)$$

$$\times \left\{ (r+1)(-y) \frac{n^{r+1}(n + r)!}{(n + \beta)^{r+1}!(n - 2)!} r! + (r+2) \right\}$$

$$+ \frac{n^{r+1}(n + r + 1)!}{(n + \beta)^{r+2}!(n - 2)!} r! \times$$

$$\left\{ r^{r+1}(n + r)! \frac{y}{(n + \beta)^{r+1}!(n - 2)!} r! \right\}$$

$$+ \frac{n^{r+1}(n + r)!}{(n + \beta)^{r+2}!(n - 2)!} r!$$

$$+ \frac{n^{r+2}(n + r + 2)!}{(n + \beta)^{r+3}!(n - 2)!} r! \times$$

$$\left\{ (r + 2)(r + 1) \frac{y^2}{(n + \beta)^{r+2}!(n - 2)!} r! \right\}$$

$$\left\{ (r + 2) \frac{y^2}{(n + \beta)^{r+2}!(n - 2)!} r! \right\}$$

In the above expression, coefficients of derivatives $f^{(r)}$, $f^{(r+1)}$ and $f^{(r+2)}$ are $r(r + 2 - \beta), (1 + r + \alpha) + y(2r + 3 - \beta)$ and $y(1 + y)$ respectively, after taking the limits as $n \to \infty$ and by induction on $r$. To find the entire result, it is enough to appear $E_2 \to 0$ as $n \to \infty$. Hence to estimate $E_2$, using Lemma 5, we possess

$$|E_2| \leq \sum_{\binom{i+j}{r}, i, j \geq 0} |Q_{i, j, r}(y)| |y|^{n - 1} \times$$

$$\sum_{\binom{i+j}{r}, i, j \geq 0} \left[ |v| + (n + 1) \right] \frac{\varepsilon(t, y)|y|^{n + \alpha}}{(n + \beta)^{r+2}!} \times$$

$$\int_0^\infty \frac{p_{n, \alpha}(t) \varepsilon(t, y)}{(n + \beta - y)^{r+2}} dt.$$  

For $\varepsilon > 0$, there exist $\delta > 0$ such that $|\varepsilon(t, y)| < \varepsilon$ whenever $|t - y| < \delta$, and $\varepsilon(t, y) \to 0$ as $t \to y$. for $|t - y| \geq \delta$. Also if $\alpha > \max(y, r + 2)$ is an integer, we notice a constant $K > 0$ such that

$$\varepsilon(t, y) \frac{n^{r+\alpha}}{(n + \beta)^{r+2}!} \leq K \frac{n^{r+\alpha}}{(n + \beta)^{r+2}!} - \varepsilon^{r+2}.$$  

Accordingly

$$|E_2| \leq \sum_{\binom{i+j}{r}, i, j \geq 0} |Q_{i, j, r}(y)| \frac{n - 1}{(y(1 + y))^r} \times$$

$$\left[ \sum_{\binom{i+j}{r}, i, j \geq 0} \left[ \left| \frac{\varepsilon(t, y)}{(n + \beta - y)^{r+2}} \right| \right] \right] \times$$

$$\left[ \sum_{\binom{i+j}{r}, i, j \geq 0} \left[ \left| \frac{\varepsilon(t, y)}{(n + \beta - y)^{r+2}} \right| \right] \right] \times$$

$$\leq L_1 \sum_{\binom{i+j}{r}, i, j \geq 0} (n + 1)^{r+1} \frac{n - 1}{n} \times$$
Schwarz inequality for integration and summation gives

\[ \sum_{p=0}^{\infty} |v - (n + 1)y|^{1} b_{n,p}(y) \times \left\{ \int_{|t - y| < \delta} e^{p,n,p}(t) \left| \frac{n + \alpha}{n + \beta} - y \right|^{r+2} dt \right\} + \left\{ \int_{|t - y| \geq \delta} K \left| \frac{n + \alpha}{n + \beta} - y \right|^{2} dt \right\} =: E_2 + E_4. \]

Now adopting Schwarz inequality in \( E_4 \), we get

\[ |E_4| \leq L_1 \sum_{i,j \geq 0} (n + 1)^{i+1} \times \left( \sum_{p=0}^{\infty} \frac{1}{n} b_{n,p}(y) |v - (n + 1)y|^{2j} \right)^{1/2} \times \left( \int_{0}^{n+1} p_{n,v}(t) dt \right)^{1/2} \times \left( \int_{0}^{n+1} \left| \frac{n + \alpha}{n + \beta} - y \right|^{2r+4} dt \right)^{1/2} \]

\[ \leq \varepsilon O(n^{i+1}). O(n^{j/2}). 1.0 \left( n^{-r+2}/2 \right) \leq \varepsilon O(1). \]

For arbitrary chosen \( \varepsilon \) we follow that \( \| E_3 \| = O(1) \). Now adopting Schwarz inequality in \( E_4 \), we get

\[ |E_4| \leq L_2 \sum_{i,j \geq 0} (n + 1)^{i+1} \times \left( \sum_{p=0}^{\infty} \frac{1}{n} b_{n,p}(y) |v - (n + 1)y|^{2j} \right)^{1/2} \times \left( \int_{0}^{n+1} p_{n,v}(t) dt \right)^{1/2} \times \left( \int_{0}^{n+1} \left| \frac{n + \alpha}{n + \beta} - y \right|^{2j} dt \right)^{1/2} \]

\[ \leq O(1). O(n^{j/2}). 1.0 \left( n^{-r+2}/2 \right) \leq O(1). \]

Hence from above \( E_3 \) and \( E_4 \) give \( E_2 \to 0 \) as \( n \to \infty \). Linking estimates \( E_1 \) and \( E_2 \), we obtain the complete theorem.

**Theorem 2** Let us take \( f \in R_{r}[0, \infty) \) for some \( \gamma > 0 \). If \( f^{(m)} \) exists where \( r \leq m \leq r + 2 \) continuously on interval \( (a - \eta, b + \eta) \subset [0, \infty) \), \( \eta > 0 \) then for sufficiently large \( n \)

\[ \| M_{n,a,b}^{(r)}(f, y) - f^{(r)}(y) \|_{C[a,b]} \leq A_1 n^{-1} \sum_{m=0}^{n} \| f^{(m)} \|_{C[a,b]} + O(n^{-2}) \]

Uniformly on \([a, b]\). Now, we estimate \( f_j \) as

\[ \| f_j \|_{C[a,b]} \leq A_2 n^{-1} \sum_{m=0}^{n} \| f^{(m)} \|_{C[a,b]} + O(n^{-2}) \]

where \( A_1, A_2 \) are constants which do not depend on \( n \) and \( f \). \( \omega(f, \delta) \) represents modulus of continuity of \( f \) on sub-interval \( (a - \eta, b + \eta) \) of \([0, \infty)\) such that \( \| f \|_{C[a,b]} \) is the sup-norm on \([a, b]\).

**Proof:** Taylor expansion of \( f \) gives

\[ f(t) = \sum_{i=0}^{m} \frac{f^{(i)}(y)}{i!} (t - y)^{i} \]

\[ + \frac{f^{(m)}(\xi)}{m!} (t - y)^{m} \chi(t) \]

where \( \xi < \eta \) and \( \chi(t) \) is the characteristic function defined on \((a - \eta, b + \eta)\). Therefore

\[ M_{n,a,b}^{(r)}(f, y) - f^{(r)}(y) \]

\[ = \sum_{i=0}^{m} \frac{f^{(i)}(y)}{i!} M_{n,a,b}^{(r)}((t - y)^{i}, y) - f^{(r)}(y) \]

\[ + \frac{f^{(m)}(\xi)}{m!} M_{n,a,b}^{(r)}((t - y)^{m} \chi(t), y) \]

\[ := J_1 + J_2 + J_3. \]

Using Lemma 2, we find

\[ J_1 = \sum_{i=0}^{m} \frac{f^{(i)}(y)}{i!} \sum_{j=0}^{i} \left( \int_{0}^{n} (-y)^{i-j} \frac{d^r}{d y^r} \right) \]

\[ = \sum_{i=0}^{m} \frac{f^{(i)}(y)}{i!} \sum_{j=0}^{i} \left( \int_{0}^{n} (-y)^{i-j} \frac{d^r}{d y^r} \right) \]

\[ \leq O(1). \]

Therefore

\[ \| J_1 \|_{C[a,b]} \leq A_1 n^{-1} \sum_{m=0}^{n} \| f^{(m)} \|_{C[a,b]} + O(n^{-2}) \]

\[ \| J_2 \| \leq \sum_{p=0}^{n} \frac{b_{p,n}(y)}{n} \int_{0}^{n+1} p_{n,v}(t) \times \left( \int_{0}^{n+1} \left| \frac{n + \alpha}{n + \beta} - y \right|^{2j} dt \right)^{1/2} \]

\[ \leq A_2 n^{-1} \sum_{m=0}^{n} \| f^{(m)} \|_{C[a,b]} + O(n^{-2}) \]

where \( A_1, A_2 \) are constants which do not depend on \( n \) and \( f \). \( \omega(f, \delta) \) represents modulus of continuity of \( f \) on sub-interval \( (a - \eta, b + \eta) \) of \([0, \infty)\) such that \( \| f \|_{C[a,b]} \) is the sup-norm on \([a, b]\).
Using Schwarz inequality, we get

\[
\left( \sum_{p=0}^{\infty} b_{n,p}(y) \right)^{1/2} \times \left( \sum_{p=0}^{\infty} p_{n,p}(t) \right)^{1/2} \times \left( \sum_{p=0}^{\infty} p_{n,p}(t) \right)^{1/2}
\]

\[
\leq O(n^{m/2}), O(n^{m/2})
\]

uniformly on \([a, b]\). Therefore by Lemma 5 and (3.2). It is found that

\[
\left( \sum_{p=0}^{\infty} b_{n,p}(y) \right)^{1/2} \times \left( \sum_{p=0}^{\infty} p_{n,p}(t) \right)^{1/2} \times \left( \sum_{p=0}^{\infty} p_{n,p}(t) \right)^{1/2}
\]

\[
\leq O(n^{m/2}), O(n^{m/2})
\]

IV. CONCLUSION

In the above discussion, the linear positive operators can be classified according to the properties of Taylor’s hypothesis, Schwarz inequality etc. In other words, we can say that characteristic properties for estimates in \(L^p\)-space are discussed through our proposed operators given in (1.1), (1.2) and (1.3). It always plays an essential role how well the rate of convergence can be improved in operator theory.

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