Nano M Locally Closed Sets and Maps in Nano Topological Spaces

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Abstract: The concepts of $\mathcal{MLC}$ sets, $\mathcal{MLC}$ continuous and $\mathcal{MLC}$ irresolute functions are introduced and some of its characteristics are discussed. Also nano $M$ submaximal spaces are defined and its properties are discussed.

Keywords and phrases: $\mathcal{MLC}$ sets, $\mathcal{M}$ submaximal, $\mathcal{MLC}$ continuous and $\mathcal{MLC}$ irresolute functions. AMS (2000) subject classification: 54B05.

I. INTRODUCTION AND PRELIMINARIES

Nano topology (briefly, $\mathcal{N}$) was introduced by Thivagar [7] in the year 2013. Also he introduced nano closed ($\mathcal{N}C$) sets & nano-interior (resp. closure ($\mathcal{N}C$)) in a nano topological spaces (briefly, $\mathcal{N}T$). Various forms of $\mathcal{N}T$ were discussed in [11-15]. Nano regular open (briefly, $\mathcal{N}O$) sets, nano $\theta$ (resp. nano $\delta$) interior (resp. closure) (briefly, $\mathcal{N}IN\theta$ (resp. $\mathcal{N}INT\theta$)) and also nano $\theta$ (resp. $\delta$) open (resp. closed) (briefly, $\mathcal{N}O\theta$ (resp. $\mathcal{N}C\theta$)) sets were introduced in [4, 7, 11, 12].

Nano $\delta$-pre (resp. $\delta$-semi, e, $M$, $\theta$-pre) & $\delta$-semi) open (briefly, $\mathcal{N}PO\delta$ (resp. $\mathcal{N}SO\delta$, $\mathcal{N}M\delta$, $\mathcal{N}PO\delta$)), and $\delta$-pre (resp. $\delta$-semi, e, $M\delta$-semi) interior (briefly, $\mathcal{N}INT\delta$ (resp. $\mathcal{N}INT\delta$)) were introduced in [10, 11, 12].

II. NANO M LOCALLY CLOSED SETS

In this section the three forms of $\mathcal{MLC}$ sets denoted by $\mathcal{MLC}(U, \mathcal{P})$, $\mathcal{MLC}^*(U, \mathcal{P})$ & $\mathcal{MLC}^{**}(U, \mathcal{P})$ are introduced and obtained its properties.

Definition 2.1 A subset $K$ of a $\mathcal{N}T(U, \mathcal{P})$ is called Nano $M$ locally closed (resp. closed $\delta$ closed $\theta$ closed $\delta$ closed) (briefly, $\mathcal{MLC}$ (resp. $\mathcal{MLC}$ & $\mathcal{MLC}^*$ & $\mathcal{MLC}^{**}$)) set if $K = G \cap F$. $G$ is $\mathcal{N}M\theta$ (resp. $\mathcal{N}M\delta\theta$ & $\mathcal{N}M\theta$) and $F$ is $\mathcal{N}M\delta$ (resp. $\mathcal{N}M\delta\theta$ & $\mathcal{N}M\theta$) in $(U, \mathcal{P})$.

The class of all $\mathcal{MLC}$ (resp. $\mathcal{MLC}^*$ & $\mathcal{MLC}^{**}$) sets is denoted by $\mathcal{MLC}(U, \mathcal{P})$, $\mathcal{MLC}^*(U, \mathcal{P})$ & $\mathcal{MLC}^{**}(U, \mathcal{P})$.

Theorem 2.1 Let $(U, \mathcal{P})$ and $(V, \mathcal{P}')$ be $\mathcal{N}T$ sets. Then every $\mathcal{MLC}$ (resp. $\mathcal{MLC}^*$ & $\mathcal{MLC}^{**}$) set is $\mathcal{N}T$ set, but converse is not.

Proof. Let $K = G \cap F$ be $\mathcal{MLC}$ set where $G$ is $\mathcal{N}M\theta$ & $\mathcal{N}M\delta\theta$ & $\mathcal{N}M\theta$ in $U$. Because each $\mathcal{N}T$ set is $\mathcal{N}M\theta$ set & $\mathcal{N}M\delta\theta$ & $\mathcal{N}M\theta$ set. Hence $K$ is $\mathcal{MLC}$ in $U$. The other results can be proved in the similar manner.

Example 2.1 Let $U = \{e, d, c, b, a\}$ with $U/R = \{[b, a], [c], [e, d]\}$ & $P = \{c, a\}$. The $\mathcal{N}t_{\mathcal{P}}(P) = \{U, \phi, [c], [b, a], [c, a]\}$.

The $\mathcal{N}S(e, d, b, a)$ (resp. $\mathcal{N}S(e, d, c, a)$) is $\mathcal{MLC}$ set but not $\mathcal{MLC}$ and $\mathcal{MLC}^*$ (resp. $\mathcal{MLC}^{**}$) set.

Theorem 2.2 For $K \subset U$, the conditions
1. $K \in \mathcal{MLC}^*(U, \mathcal{P})$.
2. $K = G \cap \mathcal{N}C(K)$ for some $\mathcal{N}M\theta$ set $G$.
3. $\mathcal{N}C(K) - K$ is $\mathcal{MLC}$.
4. \( K \cup (U - NcI(K)) \) is \( M \) or is equivalent.

**Proof.** (i) \( \implies \) (ii): Let \( K \in NMLC^*(U,P) \). Then \( \exists M \) set \( G \) and \( R \) set \( F \) such that \( K \subseteq G \cap F \). Since \( K \subseteq G \cap F \subseteq NcI(K) \), we have \( K \subseteq G \cap NcI(K) \). Conversely, since \( K \subseteq F \), \( NcI(K) \subseteq NcI(F) \), \( F \supseteq NcI(K) \), \( NcI(K) \cap G \subseteq F \cap G = K \), \( G \cap NcI(K) \subseteq K \). Thus \( K = G \cap NcI(K) \).

(ii) \( \implies \) (i): Let \( K = G \cap NcI(K) \) for some \( M \) set \( G \) and \( R \). Clearly \( NcI(K) \) is \( Rc \) and \( NcI(K) \subseteq NMLC^*(U,P) \). \( K \subseteq G \cap NcI(K) \).

(iii) \( \implies \) (iv): Let \( P = NcI(K) - K \). Then \( P \) is \( Rc \) by assumption. But \( U = P \subseteq U \cup (U - NcI(K)) \), \( K \cup (U - NcI(K)) = NcI(K) - K \). Therefore \( K \cup (U - NcI(K)) \).

(iv) \( \implies \) (iii): Let \( Q = K \cup (U - NcI(K)) \). Then \( Q \) is \( Rc \) and \( Q \subseteq U - Q \). Therefore \( K \cup (U - NcI(K)) \).

Theorem 2.3 Let \( K \subseteq U \) and \( K \subseteq UMLC^*(U,P) \), then \( K = G \cap NcI(K) \) for some \( M \) set \( G \).

**Proof.** Let \( K \in UMLC^*(U,P) \). Then \( \exists M \) set \( G \) and \( R \) set \( F \) such that \( K \subseteq G \cap F \). Since \( K \subseteq G \cap F \subseteq NcI(K) \), \( K \subseteq G \cap NcI(K) \). Conversely, if \( x \in G \cap NcI(K) \), then \( x \in G \cap NcI(K) \). Thus \( K = G \cap NcI(K) \).

Theorem 2.4 Let \( K \subseteq U \) if \( K \) is \( Rc \) set then \( K \) is \( UMLC^* \) set or \( UMLC^{**} \) set.

Theorem 2.5 Let \( K \subseteq U \) and \( K \subseteq UMLC^*(U,P) \), is closed under finite intersection (briefly, f.i.). If \( K \subseteq UMLC^*(U,P) \) then \( K \subseteq UMLC^*(U,P) \).

**Proof.** Let \( K \subseteq UMLC^*(U,P) \). Then \( \exists M \) set \( P \) such that \( K \subseteq P \). Since \( K \subseteq P \), \( K \subseteq NcI(K) \). Therefore \( K \subseteq NcI(K) \).

Theorem 2.6 Let \( K \subseteq U \) and \( K \subseteq UMLC^*(U,P) \) and \( NMLC^*(U,P) \) is closed under f.i. If \( K \subseteq UMLC^*(U,P) \) then \( K \subseteq UMLC^*(U,P) \).

**Proof.** Let \( K \subseteq UMLC^*(U,P) \). Then \( \exists M \) sets \( P \) and \( Q \) such that \( K \subseteq P \) and \( Q \subseteq NcI(K) \). Therefore \( K \subseteq NcI(K) \).

Theorem 2.7 Let \( K \subseteq U \) and \( K \subseteq NMLC(U,P) \) is closed under finite intersection. If \( K \subseteq NMLC^{**}(U,P) \) then \( K \subseteq NMLC^{**}(U,P) \).

**Proof.** Let \( K \subseteq NMLC^{**}(U,P) \). Then \( \exists M \) set \( F \) such that \( K \subseteq F \). Therefore \( K \subseteq NMLC^{**}(U,P) \). Clearly \( K \subseteq NMLC^{**}(U,P) \).

Theorem 2.8 Let \( K \subseteq U \) and \( K \subseteq NMLC^{**}(U,P) \) is closed under arbitrary intersection. If \( K \subseteq NMLC^{**}(U,P) \) then \( K \subseteq NMLC^{**}(U,P) \).

**Proof.** Let \( K \subseteq NMLC^{**}(U,P) \). Then \( \exists M \) sets \( F \) such that \( K \subseteq F \). Therefore \( K \subseteq NMLC^{**}(U,P) \).

Theorem 2.9 Let \( K \subseteq U \) and \( K \subseteq NMLC^{**}(U,P) \) is closed under arbitrary intersection. If \( K \subseteq NMLC^{**}(U,P) \) then \( K \subseteq NMLC^{**}(U,P) \).

**Definition 2.2** A \( Rc \) of a \( TsU \) is called nano \( M \) dense if \( NcI(K) = U \).

**Definition 2.3** A \( TsU \) is said to be nano \( M \) submaximal (briefly, \( NMLC^{submax} \)) if every Nano \( M \) dense subset of \( (U, \tau_R(P)) \) is \( M \) in \( (U, \tau_R(P)) \).

**Theorem 2.10** Every \( NMLC^{submax} \) space is \( NMLC^{submax} \), but not conversely.

**Proof.** Let \( U \) be a \( NMLC^{submax} \) space. Then \( \exists M \) set \( F \) such that \( K \subseteq F \). Therefore \( K \subseteq NMLC^{submax} \).

Example 2.2 In Example 2.1, the \( NMLC^{submax} \) is not \( NMLC^{submax} \).
Theorem 2.11 A \( \mathcal{R} \)-sets \( (U, \tau_{R}(P)) \) is \( R \)\( \mathcal{M} \)-submax iff \( R\mathcal{MLC}^{*}(U, P) = P(U) \).

Proof. Suppose \( K \subset P(U) \) and let \( G = K \cup (U - R\mathcal{K}(K)) \). Then \( R\mathcal{K}(G) = R\mathcal{K}(K \cup (U - R\mathcal{K}(K))) = U \). Hence \( R\mathcal{M} \). By Theorem 2.2, \( K \in R\mathcal{MLC}^{*}(U, P) \) and hence \( R\mathcal{MLC}^{*}(U, P) = P(U) \).

Conversely, consider \( K \) to be \( R \)\( \mathcal{M} \)-submax of \( U \). Let \( R\mathcal{MLC}^{*}(U, P) = P(U) \). Then by hypothesis, \( K \cup (U - R\mathcal{K}(K)) = K \cup \emptyset = K \). By Theorem 2.2, \( K \in R\mathcal{M} \) in \( U \) in \( \mathcal{MLC}^{*}(U, P) \). Hence \( U \) is \( R\mathcal{M} \)-submax.

III. \( R \)\( \mathcal{M} \) CONTINUOUS MAPS

Definition 3.1 A map \( f: (U, \tau_{R}(P)) \rightarrow (V, \sigma_{R}(Q)) \) is called \( R\mathcal{M} \) (resp. \( R\mathcal{MLC}^{*} \& R\mathcal{ML}^{*} \)-continuous (briefly, \( R\mathcal{MLC}^{*}\&R\mathcal{ML}^{*} \)-continuous) function if the inverse image of every \( R\mathcal{M} \) set in \( V \) is \( R\mathcal{MLC}^{*} \) (resp. \( R\mathcal{MLC}^{*}\&R\mathcal{ML}^{*} \)).

Definition 3.2 A map \( f: (U, \tau_{R}(P)) \rightarrow (V, \sigma_{R}(Q)) \) is called \( R\mathcal{M} \)-irresolute (resp. \( R\mathcal{MLC}^{*}\&R\mathcal{ML}^{*} \)-irresolute (briefly, \( R\mathcal{MLC}^{*}\&R\mathcal{ML}^{*} \)-irresolute)) function if the inverse image of every \( R\mathcal{M} \) (resp. \( R\mathcal{MLC}^{*}\&R\mathcal{ML}^{*} \)) set in \( V \) is \( R\mathcal{MLC}^{*} \) (resp. \( R\mathcal{MLC}^{*}\&R\mathcal{ML}^{*} \)).

Theorem 3.3 Let \( f \) be a function \& if \( f \) is \( R\mathcal{MLC}^{*}\&R\mathcal{ML}^{*} \)-irresolute (briefly, \( R\mathcal{MLC}^{*}\&R\mathcal{ML}^{*} \)-irresolute) then \( f \) is \( R\mathcal{MLC}^{*}\&R\mathcal{ML}^{*} \)-irresolute.

Proof. Let \( G \subset R\mathcal{M} \) set in \( V \). Because every \( R\mathcal{M} \) set is \( R\mathcal{MLC}^{*} \) set \([2] \) \& by Theorem 2.1(i), every \( R\mathcal{M} \) set is \( R\mathcal{MLC}^{*} \) set. Since \( f \) is \( R\mathcal{MLC}^{*}\&R\mathcal{ML}^{*} \)-irresolute, \( f^{-1}(G) \) is \( R\mathcal{MLC}^{*}\&R\mathcal{ML}^{*} \). Hence \( f \) is \( R\mathcal{MLC}^{*}\&R\mathcal{ML}^{*} \)-irresolute.

Definition 3.3 A \( \mathcal{R} \)-sets \( (U, \tau_{R}(P)) \) is called \( R \)\( \mathcal{M} \) door space if each subset of \( U \) is either \( R\mathcal{M} \) or \( R\mathcal{M} \) in \( U \).

Theorem 3.4 Any function defined from a \( R \)\( \mathcal{M} \) door space into a \( R \)\( \mathcal{M} \)-set is \( R \)\( \mathcal{M} \)-irresolute.

Proof. Let \( U \) be \( R \)\( \mathcal{M} \) door space \& \( V \) be \( R \)\( \mathcal{M} \)-sets. Let \( f: (U, \tau_{R}(P)) \rightarrow (V, \sigma_{R}(Q)) \) be a function. Let \( K \) be \( R\mathcal{M} \) in \( V \). Then \( f^{-1}(K) \) is \( R\mathcal{M} \) in \( U \). Since every \( R\mathcal{M} \) set is \( R\mathcal{MLC}^{*} \) set \([2] \) \& by Theorem 2.1(i), every \( R\mathcal{M} \) set is \( R\mathcal{MLC}^{*} \) set. In both cases, \( f^{-1}(K) \) is \( R\mathcal{MLC}^{*} \). Hence \( f \) is \( R\mathcal{MLC}^{*}\&R\mathcal{ML}^{*} \)-irresolute.

Theorem 3.5 If \( f: (U, \tau_{R}(P)) \rightarrow (V, \sigma_{R}(Q)) \) is \( R\mathcal{MLC}^{*}\&R\mathcal{ML}^{*} \)-irresolute (resp. \( R\mathcal{MLC}^{*}\&R\mathcal{ML}^{*} \)-irresolute) then \( g \circ f: (U, \tau_{R}(P)) \rightarrow (W, \mu_{R}(R)) \) is \( R\mathcal{MLC}^{*}\&R\mathcal{ML}^{*} \)-irresolute.

IV. CONCLUSION

In our paper, the concepts of \( R\mathcal{M} \) sets, \( R\mathcal{MLC}^{*}\&R\mathcal{ML}^{*} \)-continuous \& \( R\mathcal{MLC}^{*}\&R\mathcal{ML}^{*} \)-irresolute functions are introduced and some of its characteristics are discussed.

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