New Family of Parity Combination Cordial Labeling of Graph

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Abstract: Let G be a (p, q) graph. Let f be an injective map from V(G) to {1, 2, ..., p}. For each edge xy, assign the label \( \begin{cases} \frac{x}{y} & \text{if } x > y \text{ or } y > x \end{cases} \) or \( \begin{cases} y & \text{if } x = y \end{cases} \) according as \( x > y \) or \( y > x \). f is called a parity combination cordial labeling (PCC-labeling) if f is a one to one map and \( |e_f(0) - e_f(1)| \leq 1 \) where \( e_f(0) \) and \( e_f(1) \) denote the number of edges labeled with an even number and odd number respectively. A graph with a parity combination cordial labeling is called a parity combination cordial graph (PCC-graph).

I. INTRODUCTION

In this paper we consider only finite, undirected and simple graphs. For standard notations and terminology related to Graph theory, we refer Harary [2]. Graph labeling, we refer Gallian[1]. The concept of Parity combination cordial labeling is introduced by Ponraj et al [3]. We present the Parity combination cordial of the graph \( G' \), which is obtained by identifying a vertex \( v_k \) in \( G \) and a vertex of degree \( n \) in \( H_n \) where \( G \) is a PCC graph with \( p \) vertices and \( q \) edges under f with \( f(v_k) = 1 \).

II. BASIC RESULTS AND DEFINITIONS

A. Definition 2.1

Let G be a (p, q) graph. Let f be an injective map from V(G) to {1, 2, ..., p}. For each edge xy, assign the label \( \begin{cases} \frac{x}{y} & \text{if } x > y \text{ or } y > x \end{cases} \) or \( \begin{cases} y & \text{if } x = y \end{cases} \) according as \( x > y \) or \( y > x \). f is called a parity combination cordial labeling (PCC-labeling) if f is a one to one map and

\[ |e_f(0) - e_f(1)| \leq 1 \]

where \( e_f(0) \) and \( e_f(1) \) denote the number of edges labeled with an even number and odd number respectively. A graph with a parity combination cordial labeling is called a parity combination cordial graph (PCC-graph).

B. Result 2.1

\[ \binom{n}{n-1} = \binom{n}{1} \]

is even if \( n \) is even and odd if \( n \) is odd.

C. Result 2.2

\[ \binom{n}{2} \]

is even if \( n = 0, 1 \pmod{4} \) and odd if \( n = 2, 3 \pmod{4} \).

D. Result 2.3

\[ \binom{n}{k} \]

is even when \( n \) is even and \( k \) is odd.

III. MAIN RESULT

A. Theorem 3.1

G is a PCC graph with \( p \) vertices and \( q \) edges under f with \( f(v_k) = 1 \), then the graph is obtained by identifying a vertex \( v_k \) in \( G \) and a vertex of degree \( n \) in \( H_n \) admits PCC labeling for \( n \geq 3 \).

Proof:

G is a PCC graph with \( v_1, v_2, ..., v_p \) vertices and \( e_1, e_2, ..., e_q \) edges and f is PCC labeling of G. Then f : V(G) \rightarrow \{1, 2, ..., p\} with \( f(v_k) = 1 \) and \( |e_f(0) - e_f(1)| \leq 1 \).

Let \( H_n \) be a helm graph. Let \( w, w_1, w_2, ..., w_{2n} \) be the vertices and \( e_{11}, e_{12}, ..., e_{1n}, e_{21}, e_{22}, ..., e_{2n}, e_{31}, e_{32}, ..., e_{3n} \) be the edges of \( H_n \). The graph obtained by identifying a vertex \( v_k \) in \( G \) and a vertex \( w \) of degree \( n \) in \( H_n \). The resultant graph is denoted by \( G' \). Here \( |V(G')| = p+2n \) and \( |E(G')| = q+3n \).

Now define g : V(G') \rightarrow \{1, 2, ..., p+2n\} as follows:

\[ g(v_k) = f(v_k), \]

for all \( v_k \in V(G) \).

Consider g(v_k) = g(w) = 1. Since \( v_k \) is identified with \( w \) in \( G' \).

Case (1) : p and q are even and \( n = 3 \).

\[ g(w_i) = p+2i+1, \quad \text{for } i = 1, 2, \]

\[ g(w_i) = p+6, \]

\[ g(w_{i1}) = p+2i, \quad \text{for } i = 1, 2, \]

\[ g(w_e) = p+1. \]
Thus for the induced edge labeling we get $g'(e_i) = f'(e_i)$, for all $e_i \in E(G)$.

For $q$ is even, then $e_q(1) = e_q(0) = \frac{q}{2}$. Thus $e_q(1) = e_q(0) = \frac{q}{2}$ for $G$ in $G'$.

When $p = 0(\text{mod } 4)$, then $e_q(1) = \frac{q}{2}$ for $G$ in $G'$+4 for $H_a$ in $G'$.

Case (2) : $p$ and $q$ are even and $n > 3$.

For $G$ in $G'$+4 for $H_a$ in $G'$ and $|e_q(0)-e_q(1)| = 1$.

For $q$ is even, then $e_q(1) = e_q(0) = \frac{q}{2}$. Thus $e_q(1) = e_q(0) = \frac{q}{2}$ for $G$ in $G'$.

When $p = 0(\text{mod } 4)$, then $e_q(1) = \frac{q}{2}$ for $G$ in $G'$+4 for $H_a$ in $G'$.

Thus for the induced edge labeling we get $g'(e_i) = f'(e_i)$, for all $e_i \in E(G)$.

When $p = 0(\text{mod } 4)$, then $e_q(1) = \frac{q}{2}$ for $G$ in $G'$+4 for $H_a$ in $G'$.

When $p = 2(\text{mod } 4)$, then $e_q(1) = \frac{q}{2}$ for $G$ in $G'$+4 for $H_a$ in $G'$.

For $q$ is even, then $e_q(1) = e_q(0) = \frac{q}{2}$. Thus $e_q(1) = e_q(0) = \frac{q}{2}$ for $G$ in $G'$.

When $p = 0(\text{mod } 4)$, then $e_q(1) = \frac{q}{2}$ for $G$ in $G'$+4 for $H_a$ in $G'$.

When $p = 2(\text{mod } 4)$, then $e_q(1) = \frac{q}{2}$ for $G$ in $G'$+4 for $H_a$ in $G'$.

Thus for the induced edge labeling we get $g'(e_i) = f'(e_i)$, for all $e_i \in E(G)$.

When $p = 0(\text{mod } 4)$, $q$ is even and $n$ is odd, then $e_q(1) = \frac{q}{2}$ for $G$ in $G'$+4 for $H_a$ in $G'$.

For $G$ in $G'$+4 for $H_a$ in $G'$, $e_q(0) = \frac{q}{2}$ for $G$ in $G'$+4 for $H_a$ in $G'$.

Thus for the induced edge labeling we get $g'(e_i) = f'(e_i)$, for all $e_i \in E(G)$.

When $p = 0(\text{mod } 4)$, $q$ is even and $n$ is odd, then $e_q(1) = \frac{q}{2}$ for $G$ in $G'$+4 for $H_a$ in $G'$.

When $p = 2(\text{mod } 4)$, $q$ is even and $n$ is odd, then $e_q(1) = \frac{q}{2}$ for $G$ in $G'$+4 for $H_a$ in $G'$.

Thus for the induced edge labeling we get $g'(e_i) = f'(e_i)$, for all $e_i \in E(G)$.

When $p = 0(\text{mod } 4)$, $q$ is even and $n$ is odd, then $e_q(1) = \frac{q}{2}$ for $G$ in $G'$+4 for $H_a$ in $G'$.

When $p = 2(\text{mod } 4)$, $q$ is even and $n$ is odd, then $e_q(1) = \frac{q}{2}$ for $G$ in $G'$+4 for $H_a$ in $G'$.

Thus for the induced edge labeling we get $g'(e_i) = f'(e_i)$, for all $e_i \in E(G)$.

When $p = 0(\text{mod } 4)$, $q$ is even and $n$ is odd, then $e_q(1) = \frac{q}{2}$ for $G$ in $G'$+4 for $H_a$ in $G'$.

When $p = 2(\text{mod } 4)$, $q$ is even and $n$ is odd, then $e_q(1) = \frac{q}{2}$ for $G$ in $G'$+4 for $H_a$ in $G'$.

Thus for the induced edge labeling we get $g'(e_i) = f'(e_i)$, for all $e_i \in E(G)$.

When $p = 0(\text{mod } 4)$, $q$ is even and $n$ is odd, then $e_q(1) = \frac{q}{2}$ for $G$ in $G'$+4 for $H_a$ in $G'$.

When $p = 2(\text{mod } 4)$, $q$ is even and $n$ is odd, then $e_q(1) = \frac{q}{2}$ for $G$ in $G'$+4 for $H_a$ in $G'$.
+ \frac{3n}{2} \text{ for } H_n \text{ in } G' \text{ and } |e_0(0) - e_0(1)| = 1.

When n is even, p is odd, q is odd with \(e_0(1) = \frac{q-1}{2}\) and \(e_0(0) = \frac{q+1}{2}\).

\(e_0(1) = \frac{q+1}{2}\), then \(e_0(1) = \frac{q-1}{2}\) for G in \(G' + \frac{3n}{2}\) for \(H_n\) in \(G'\),
\(e_0(0) = \frac{q+1}{2}\) for G in \(G' + \frac{3n}{2}\) for \(H_n\) in \(G'\) and \(|e_0(0) - e_0(1)| = 1\).

Case (4) : \(p = 1 + (mod\ 4)\), q is odd with \(e_0(1) = \frac{q+1}{2}\) and \(e_0(0) = \frac{q+1}{2}\). Thus \(e_0(1) = -\frac{q-1}{2}\) and \(e_0(0) = \frac{q+1}{2}\) for G in \(G'\).

\(g'(w_{w_1}) = g'(e_1) = 1;\)
\(g'(w_{w_i}) = g'(e_i) = 0, \quad \text{for } i = 2, 3, ..., n.
\(g'(w_{w_{w_1}}) = g'(e_{11}) = 0,
\(g'(w_{w_{w_i}}) = g'(e_{i+1}) = 0,
\(g'(w_{w_{w_{w_{w_1}}}}) = g'(e_{i+2}) = 0,
\(g'(w_{w_{w_{w_{w_{w_{w_{w_{w_1}}}}}}}}) = g'(e_{i+3}) = 1, \quad \text{for } i = 3, 4, ..., n.

For q is odd with \(e_0(1) = \frac{q+1}{2}\) and \(e_0(0) = \frac{q-1}{2}\). Thus \(e_0(1) = -\frac{q-1}{2}\) and \(e_0(0) = \frac{q+1}{2}\) for G in \(G'\).

\(g'(w_{w_1}) = g'(e_{11}) = 1;\)
\(g'(w_{w_i}) = g'(e_{i}) = 0, \quad \text{for } i = 2, 3, ..., n.
\(g'(w_{w_{w_1}}) = g'(e_{21}) = 0,
\(g'(w_{w_{w_i}}) = g'(e_{2i}) = 0,
\(g'(w_{w_{w_{w_{w_1}}}}) = g'(e_{31}) = 1, \quad \text{for } i = 3, 4, ..., n.

For q is odd with \(e_0(1) = \frac{q+1}{2}\) and \(e_0(0) = \frac{q-1}{2}\). Thus \(e_0(1) = -\frac{q-1}{2}\) and \(e_0(0) = \frac{q+1}{2}\) for G in \(G'\).

When n is odd, p = 1 (mod 4), q is odd with \(e_0(1) = \frac{q+1}{2}\) and \(e_0(0) = \frac{q-1}{2}\). Then \(e_0(1) = \frac{q+1}{2}\) for G in \(G' + \frac{3n+1}{2}\) for \(H_n\) in \(G'\),
\(e_0(0) = \frac{q-1}{2}\) for G in \(G' + \frac{3n+1}{2}\) for \(H_n\) in \(G'\) and
\(|e_0(0) - e_0(1)| = 0\).

Case (5) : \(p = 1 (mod\ 4)\), q is odd with \(e_0(1) = \frac{q-1}{2}\) and \(e_0(0) = \frac{q+1}{2}\), n is odd and \(n \geq 3\).

\(g'(w_{w_1}) = g'(e_{11}) = 1;\)
\(g'(w_{w_i}) = g'(e_{i}) = 0, \quad \text{for } i = 2, 3, ..., n.
\(g'(w_{w_{w_1}}) = g'(e_{21}) = 0,
\(g'(w_{w_{w_i}}) = g'(e_{2i}) = 0,
\(g'(w_{w_{w_{w_{w_1}}}}) = g'(e_{31}) = 1, \quad \text{for } i = 3, 4, ..., n.

When p = 0 (mod 4), then \(e_0(1) = \frac{q-1}{2}\) for G in \(G' + \frac{3n+1}{2}\) for \(H_n\) in \(G'\),
\(e_0(0) = \frac{q+1}{2}\) for G in \(G' + \frac{3n+1}{2}\) for \(H_n\) in \(G'\) and
\(|e_0(0) - e_0(1)| = 0\).

Case (7) : \(p = 0 (mod\ 4)\), q is odd with \(e_0(1) = \frac{q+1}{2}\) and
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\[ e_i(0) = \frac{q - 1}{2}, \text{ n is odd and } n = 3. \]

\[ g(w_i) = p + 2i - 1, \text{ for } i = 1, 2. \]
\[ g(w_{n+1}) = p + 6, \]
\[ g(w_{n+2}) = p + 2. \]
\[ g(w_{n+3}) = p + 4. \]
\[ g(w_{n+4}) = p + 1. \]

Thus for the induced edge labeling we get \( g'(e_i) = f'(e_i) \), for all \( e_i \in E(G) \).

For \( q \) is odd with \( e_1(1) = \frac{q + 1}{2} \) and \( e_0(0) = \frac{q - 1}{2} \). Thus \( e_8(1) = \frac{q + 1}{2} \) and \( e_8(0) = \frac{q - 1}{2} \) for \( G \) in \( G' \).

\[ g'(ww_i) = g'(e_i) = 1, \text{ for } i = 1, 2. \]
\[ g'(ww_{n+1}) = g'(e_{n+1}) = 0. \]
\[ g'(w_{n+2}) = g'(e_{n+2}) = 1. \]
\[ g'(w_{n+3}) = g'(e_{n+3}) = 0. \]

When \( p = 0 \text{ (mod 4)} \), then \( e_8(1) = \frac{q + 1}{2} \) for \( G \) in \( G' + 4 \) for \( H_a \) in \( G' \), \( e_8(0) = \frac{q - 1}{2} \) and for \( G \) in \( G' + 5 \) for \( H_a \) in \( G' \) and \( |e_8(0) - e_8(1)| = 0 \).

Case (8) : \( p = 0 \text{ (mod 4)} \), \( q \) is odd with \( e_1(1) = \frac{q + 1}{2} \) and \( e_0(0) = \frac{q - 1}{2} \).

When \( n \) is odd, \( p = 0 \text{ (mod 4)} \), \( q \) is odd with \( e_1(1) = \frac{q + 1}{2} \) and \( e_0(0) = \frac{q - 1}{2} \). Thus \( e_8(1) = \frac{q + 1}{2} \) and \( e_8(0) = \frac{q - 1}{2} \) for \( G \) in \( G' \).

\[ g'(ww_i) = g'(e_i) = 1, \text{ for } i = 1, 2, \ldots, n. \]
\[ g'(ww_{n+1}) = g'(e_{n+1}) = 0, \]
\[ g'(w_{n+2}) = g'(e_{n+2}) = 1, \]
\[ g'(w_{n+3}) = g'(e_{n+3}) = 0. \]

Thus for the induced edge labeling we get \( g'(e_i) = f'(e_i) \), for all \( e_i \in E(G) \).

For \( q \) is odd with \( e_1(1) = \frac{q + 1}{2} \) and \( e_0(0) = \frac{q - 1}{2} \). Thus \( e_8(1) = \frac{q + 1}{2} \) and \( e_8(0) = \frac{q - 1}{2} \) for \( G \) in \( G' \).

\[ g'(ww_{n+1}) = g'(e_{n+1}) = 1, \text{ for } i = 1, 2, \ldots, n - 1. \]
\[ g'(ww_{n+2}) = g'(e_{n+2}) = 0, \]
\[ g'(w_{n+3}) = g'(e_{n+3}) = 1, \]
\[ g'(w_{n+4}) = g'(e_{n+4}) = 0. \]

When \( n \) is odd, \( p = 0 \text{ (mod 4)} \), \( q \) is odd with \( e_1(1) = \frac{q - 1}{2} \) and \( e_0(0) = \frac{q + 1}{2} \). Thus \( e_8(1) = \frac{q - 1}{2} \) and \( e_8(0) = \frac{q + 1}{2} \).

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For \( q \) is odd with \( e(1) = \frac{q+1}{2} \) and \( e(0) = \frac{q-1}{2} \). Thus \( e_2(1) = \frac{q+1}{2} \) and \( e_2(0) = \frac{q-1}{2} \) for \( G \) in \( G' \).
\[
g'(w_0) = g'((e_1)_0) = 1,
\]
for \( i = 1, 2. 
\]
\[
g'(w_2) = g'((e_2)_0) = 1,
\]
\[
g'(w_3) = g'((e_3)_0) = 0,
\]
\[
g'(w_5) = g'((e_2)_0) = 0,
\]
\[
g'(w_6) = g'((e_3)_0) = 0,
\]
\[
g'(w_{n+1}) = g'((e_1)_0) = 1,
\]
\[
g'(w_{n+2}) = g'((e_2)_0) = 1,
\]
\[
g'(w_{n+3}) = g'((e_3)_0) = 0.
\]
When \( p = 2(mod \, 4) \), then \( e_2(1) = \frac{q+1}{2} \) for \( G \) in \( G' \) + 4 for \( H_n \) in \( G' \), \( e_2(0) = \frac{q-1}{2} \) for \( G \) in \( G' + 5 \) for \( H_n \) in \( G' \) and | \( e_2(0) - e_2(1) | = 0.

Case (11) : \( p = 2(mod \, 4) \), \( q \) is odd with \( e(1) = \frac{q-1}{2} \) and \( e(0) = \frac{q+1}{2} \).

Thus for the induced edge labeling we get \( g'(e_i) = f'(e_i) \) for all \( e_i \in E(G) \).

For \( q \) is odd with \( e(1) = \frac{q+1}{2} \) and \( e(0) = \frac{q-1}{2} \). Thus \( e_2(1) = \frac{q-1}{2} \) and \( e_2(0) = \frac{q+1}{2} \) for \( G \) in \( G' \).
\[
g'(w_0) = g'((e_1)_0) = 1,
\]
for \( i = 1, 2. 
\]
\[
g'(w_2) = g'((e_2)_0) = 1,
\]
\[
g'(w_3) = g'((e_3)_0) = 0,
\]
\[
g'(w_5) = g'((e_2)_0) = 0,
\]
\[
g'(w_6) = g'((e_3)_0) = 0,
\]
\[
g'(w_{n+1}) = g'((e_1)_0) = 1,
\]
\[
g'(w_{n+2}) = g'((e_2)_0) = 1,
\]
\[
g'(w_{n+3}) = g'((e_3)_0) = 0.
\]
When \( p = 2(mod \, 4) \), then \( e_2(1) = \frac{q-1}{2} \) for \( G \) in \( G' \) + 5 for \( H_n \) in \( G' \), \( e_2(0) = \frac{q+1}{2} \) for \( G \) in \( G' + 4 \) for \( H_n \) in \( G' \) and | \( e_2(0) - e_2(1) | = 0.

Case (12) : \( p = 2(mod \, 4) \), \( q \) is odd with \( e(1) = \frac{q+1}{2} \) and \( e(0) = \frac{q-1}{2} \).

Thus for the induced edge labeling we get \( g'(e_i) = f'(e_i) \) for all \( e_i \in E(G) \).

For \( q \) is odd with \( e(1) = \frac{q+1}{2} \) and \( e(0) = \frac{q-1}{2} \). Thus \( e_2(1) = \frac{q-1}{2} \) and \( e_2(0) = \frac{q+1}{2} \) for \( G \) in \( G' \).
\[
g'(w_0) = g'((e_1)_0) = 1,
\]
for \( i = 1, 2. 
\]
\[
g'(w_2) = g'((e_2)_0) = 1,
\]
\[
g'(w_3) = g'((e_3)_0) = 0,
\]
\[
g'(w_5) = g'((e_2)_0) = 0,
\]
\[
g'(w_6) = g'((e_3)_0) = 0,
\]
\[
g'(w_{n+1}) = g'((e_1)_0) = 1,
\]
\[
g'(w_{n+2}) = g'((e_2)_0) = 1,
\]
\[
g'(w_{n+3}) = g'((e_3)_0) = 1.
\]
Thus for the induced edge labeling we get $g'(e_i) = f'(e_i)$, for all $e_i \in E(G)$.

For $q$ is even, then $e_1(1) = e(0) = \frac{q}{2}$. Thus $e_1(1) = e(0) = \frac{q}{2}$ for $G$ in $G'$.

When $n$ is even, $p = 1\text{(mod 4)}$, $q$ is even with $e_1(1) = e(0) = \frac{q}{2}$, then $e_1(1) = \frac{q}{2}$ for $G$ in $G' + \frac{3n+1}{2}$ for $H_n$ in $G'$, $e(0) = \frac{q}{2}$ for $G$ in $G' + \frac{3n+1}{2}$ for $H_n$ in $G'$ and $|e(0) - e(1)| = 0$.

Case (14) : $p = 1\text{(mod 4)}$, $q$ is even, $n$ is even and $n \geq 4$.
g($w_1$) = p+2,
g($w_i$) = p+2i-1, for $i = 2, 3, ..., n$.
g($w_{n+i}$) = p+1,
g($w_{n+i}$) = p+2i, for $i = 2, 3, ..., n$.

Thus for the induced edge labeling we get $g'(e_i) = f'(e_i)$, for all $e_i \in E(G)$.

For $q$ is even, then $e_1(1) = e(0) = \frac{q}{2}$. Thus $e_1(1) = e(0) = \frac{q}{2}$ for $G$ in $G'$.

When $n$ is even, $p = 1\text{(mod 4)}$, $q$ is even with $e_1(1) = e(0) = \frac{q}{2}$, then $e_1(1) = \frac{q}{2}$ for $G$ in $G' + \frac{3n}{2}$ for $H_n$ in $G'$, $e(0) = \frac{q}{2}$ for $G$ in $G' + \frac{3n}{2}$ for $H_n$ in $G'$ and $|e(0) - e(1)| = 0$.

Case (16) : $p = 1\text{(mod 4)}$, $q$ is even, $n$ is odd and $n \geq 3$.
g($w_1$) = p+2,
g($w_i$) = p+2i-1, for $i = 2, 3, ..., n$.
g($w_{n+i}$) = p+1,
g($w_{n+i}$) = p+2i, for $i = 2, 3, ..., n$.

Thus for the induced edge labeling we get $g'(e_i) = f'(e_i)$, for all $e_i \in E(G)$.

For $q$ is even, then $e_1(1) = e(0) = \frac{q}{2}$. Thus $e_1(1) = e(0) = \frac{q}{2}$ for $G$ in $G'$.

$g'(w_{1,1}) = g'(e_{11}) = 1$.

$g'(w_{1,1}) = g'(e_{11}) = 0$, for $i = 2, 3, ..., n$.

$g'(w_{n+i}) = g'(e_{n+i}) = 1$, for $i = 2, 3, ..., n$.

$g'(w_{n+i}) = g'(e_{n+i}) = 0$, for $i = 2, 3, ..., n$.

$g'(w_{n+i}) = g'(e_{n+i}) = 0$, for $i = 2, 3, ..., n$.

When $n$ is even, $p = 3\text{(mod 4)}$, $q$ is even with $e_1(1) = e(0) = \frac{q}{2}$, then $e_1(1) = \frac{q}{2}$ for $G$ in $G' + \frac{3n}{2}$ for $H_n$ in $G'$, $e(0) = \frac{q}{2}$ for $G$ in $G' + \frac{3n}{2}$ for $H_n$ in $G'$ and $|e(0) - e(1)| = 0$.

Case (17) : $p = 3\text{(mod 4)}$, $q$ is even, $n$ is odd and $n \geq 3$.
g($w_1$) = p+2,
g($w_i$) = p+2i-1, for $i = 2, 3, ..., n$.
g($w_{n+i}$) = p+1,
g($w_{n+i}$) = p+2i, for $i = 2, 3, ..., n$.

Thus for the induced edge labeling we get $g'(e_i) = f'(e_i)$, for all $e_i \in E(G)$.

For $q$ is even, then $e_1(1) = e(0) = \frac{q}{2}$. Thus $e_1(1) = e(0) = \frac{q}{2}$ for $G$ in $G'$.

$g'(w_{1,1}) = g'(e_{11}) = 1$.

$g'(w_{1,1}) = g'(e_{11}) = 0$, for $i = 2, 3, ..., n$.

$g'(w_{n+i}) = g'(e_{n+i}) = 1$, for $i = 1, 2, ..., n$.

$g'(w_{n+i}) = g'(e_{n+i}) = 0$, for $i = 1, 2, ..., n$.

When $n$ is even, $p = 3\text{(mod 4)}$, $q$ is even with $e_1(1) = e(0) = \frac{q}{2}$, then $e_1(1) = \frac{q}{2}$ for $G$ in $G' + \frac{3n}{2}$ for $H_n$ in $G'$, $e(0) = \frac{q}{2}$ for $G$ in $G' + \frac{3n}{2}$ for $H_n$ in $G'$ and $|e(0) - e(1)| = 0$. 

Thus for the induced edge labeling we get $g'(e_i) = f'(e_i)$, for all $e_i \in E(G)$.
From the above all cases, we have $|e_2(0) - e_2(1)| \leq 1$. Hence $G$ is a PCC graph with $p$ vertices and $q$ edges under $f$ with $f(v_k) = 1$, then the graph is obtained by identifying a vertex $v_k$ in $G$ and a vertex of degree $n$ in $H_n$ admits PCC labeling for $n \geq 3$.

IV. CONCLUSION

In this paper we study the PCC- labeling of the graph $G'$. The graph $G'$ is obtained by identifying a vertex $v_k$ in $G$ and a vertex of degree $n$ in $H_n$, where $G$ is a PCC graph with $p$ vertices and $q$ edges under $f$ with $f(v_k) = 1$.

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