A Two Phase Mathematical Model of Fluid Flow through Bell Shaped Stenotic Artery

Madan Lal, Yantish Dev Jha, Haris Alam Zuberi

Abstract: The present research paper concerns with a two phase fluid flow, consists an acentric plasma layer region free from red cells and a central core region represented by Herschel – Bulkley fluid through a bell shaped stenosed artery. Mathematical expressions for characteristics of blood flow namely core velocity ($u_c$), peripheral velocity ($u_p$), shear stress at wall ($\tau_w$) and total volumetric fluid flow rate ($Q$) have been estimated and depicted graphically. The effect of shape parameter peripheral layer viscosity, on these characteristics has been depicted with graphs. It has been noticed that the fluid flow rate ($Q$) and shear stress at wall ($\tau_w$) decreases as the increases of peripheral layer viscosity.

Keywords: Blood flow, bell shaped stenosis, flow rate, plasma layer, shear stress at wall.

I. INTRODUCTION

World Health Organisation report manifested that approximately 17 million people are dying yearly due to suffer of cardiovascular related diseases. Also it is predicted that 23.3 million deaths yearly will happen due to cardiovascular related diseases throughout the world by 2023 and it will be a most focusing subject of scientific research (World Health Organisation, 2013)[10]. Some reasons of the related deaths are: first is coronary heart diseases which are associated with the blood vessels and blood circulation to the heart muscles. Second is cerebrovascular diseases associated with affecting of blood vessels and blood circulation to the brain. Brain stroke and heart attack happens when circulation of blood disturbed to the brain and heart respectively. Third one is peripheral arterial diseases which affect blood circulation in arms and legs. The exact factor of construction of atherosclerosis in blood vessels is not confirmed. It is taken into account that the deposition of fatty substances and cholesterol etc. at interior wall of artery are main factors of stenosis in an artery which is major cause of heart disease like heart attack and stroke. Construction of atherosclerosis in the blood flow tubes, flow characteristics of fluid are disturbed. Several researchers prepared a number of models to explore the different characteristics of blood flow in constricted artery including Smith, Pullan, & Hunter [9] and Shukla, Parihar & Gupta[7]. A mathematical model has been prepared by Ponalagusamy [3] in which core velocity has been expressed theoretically and experimentally slip variable and variable plasma layer considering Newtonian blood. Sharma & Yadav [5] studied the influence of permeability, thickness of plasma layer, yield stress and constriction shape on fluid flow characteristics. Shukla et al. [6]1980 Singh , Joshi & Joshi, 2014 [8] and Joshi & Gadkari [1] taken stenosis shaped as cosine, trapezium and triangular respectively and investigated plasma layer viscosity effect on flow characteristics. Sankar & Gunakala [4] have studied the fluid flow through small composite constricted vessels using a two – layered model applying uniform transverse external magnetic field and found that flux of blood and velocity both reduced with increasing of intensity of applied magnetic field. Recently Neeraja et al. [2] investigated the peripheral layer viscosity effect on fluid flow characteristics, through the stenotic blood vessels considering fluid as Herschel-Bulkley fluid for cosine shaped geometry.

Present model consist two layer of fluid namely red blood cells layer in core region and plasma layer fluid as Hershel- Bulkley fluid. Mathematical expressions are established for core and peripheral velocity ($u_c$ and $u_p$), flow rate ($Q$) and shear stress at wall ($\tau_w$). The influence of yield stress, shape parameter of constriction and plasma layer viscosity on velocity, shear stress at wall and fluid flow rate is represented as graphically and discussed briefly.

II. MATHEMATICAL FORMULATION

Let the blood flow be steady, laminar and incompressible in z - direction. Figure (1) shows geometry of stenosis.

Mathematically geometry of stenosis can be represented as

$$R(z) = R_0 \left[ 1 - \frac{\delta}{R_0} \exp \left( -\frac{m^2 \epsilon^2 z^2}{R_0^2} \right) \right].$$

(1)

where

$$R_0$$ and $$R(z)$$ represent the radius of non-constricted and constricted portion of artery respectively, $$\delta$$ is maximum depth of stenosis with parametric constant $$m$$ and $$\epsilon = \frac{R_0}{\lambda_0}$$ is relative half length of stenosed part of artery.

$$\frac{R(z)}{R_0} = \left[ 1 - a \exp \left( -bz^2 \right) \right].$$

(2)

Where

$$a = \frac{\delta}{R_0}, \quad b = \frac{m^2 \epsilon^2}{R_0^2}.$$ 

The viscosity functions are given as

$$\mu_c$$ is central layer viscosity and $$\mu_p$$ peripheral layer viscosity.

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\[ \mu = \mu_c \quad 0 \leq r \leq R_1(z) \]
\[ \mu = \mu_p \quad R_1(z) \leq r \leq R(z). \]

\[ \frac{d\tau}{dr} = \frac{1}{\mu} (\tau - \tau_0)^n \]

Figure 1. Bell shaped stenosed artery.

\( \mu_c \) is central layer viscosity and \( \mu_p \) peripheral layer viscosity.

The central layer geometry can be taken as
\[ R_1(z) = R_0 \left[ 1 - \delta \frac{z^2}{R_0^2} \right]. \]

The governing equation of blood flow in the arterial segment is given by
\[ \frac{d\tau}{dz} = \frac{1}{\mu} \frac{d}{dr} \left( r \frac{d\tau}{dr} \right) = 0 \]
or
\[ \frac{d\tau}{dz} = \frac{1}{\mu} \frac{d}{dr} \left( r \frac{d\tau}{dr} \right). \]

where \( r \) and \( z \) be the radial and axial coordinates respectively, \( p \) represent pressure and \( \tau \) be the shear stress. Consider the blood as a Hershel Bulkley type fluid, for which the constitutive equations for shear stress and shear rate is
\[ \tau = \mu \left( \frac{du}{dr} \right)^n + \tau_0. \]

Above equations may be written as
\[ \frac{d\tau}{dz} = \frac{1}{\mu} \left( \tau - \tau_0 \right)^n, \quad \tau \geq \tau_0. \]

And boundary conditions are
\[ (1) \quad u_c = 0 \quad \text{at} \quad r = R_1(z), \quad \text{core velocity} \quad \left( u_c \right). \]
\[ (2) \quad u_p = 0 \quad \text{at} \quad r = R(z), \quad \text{peripheral velocity} \quad \left( u_p \right). \]

\[ \tau_p = \tau_{R} \quad \text{at} \quad r = R_1(z), \quad \frac{du_c}{dr} = 0 \quad \text{at} \quad r = 0. \]

where \( u_c \) is core velocity of fluid and \( u_p \) is peripheral velocity of fluid.

III. SOLUTION OF THE PROBLEM

On integrating equation (5)
\[ \frac{d\tau}{dz} = \frac{1}{\mu} \frac{d}{dr} \left( r \frac{d\tau}{dr} \right) \]

Using equation (12) in above equation

\[ \tau = \mu \left( \frac{du}{dr} \right)^n \]

On integrating and using boundary conditions (10) and (11)
\[ \frac{d\tau}{dz} = \frac{1}{\mu} \left( \frac{R_p}{2} - \tau_0 \right)^{n+1} - \left( \frac{R_p}{2} - \tau_0 \right)^{n+1} \]

Flow rate \( Q \) is given by
\[ Q = \int_0^R u_c \, 2\pi rdr. \]

Using equation (16) and (17) we obtained
\[ Q = 2\pi \left( \frac{1}{\mu_c (n+1)} \left( \frac{R_p}{2} - \tau_0 \right)^{n+1} - \left( \frac{R_p}{2} - \tau_0 \right)^{n+1} \right) \]

IV. RESULT AND DISCUSSION

In this paper a two layer model of fluid flow through a bell shaped constricted artery has been developed. Two layers exist in blood flow in artery. It is considered that when blood flows in artery there exist two layers namely peripheral plasma layer and central core layer. Core layer surrounded by peripheral layer. Both layers consist distinct viscosities. The flow characteristics of blood flow are explained by various analytical expressions and depicted graphically as shown below. The values of parameters are taken as \( \tau_0 = 0.05, 0.10, 0.15 \), \( \delta = 0.10, 0.15, 0.20, 0.25, 0.30, 0.35 \) and \( \mu_p = \mu_c / 10, \mu_c / 5, \mu_c / 13 \).
Fig. 2. and fig. 3. shows the variation of core velocity and peripheral velocity for different values of $\tau_0$. It is found that core velocity of the blood and peripheral velocity of the blood both decreases up to the zero value of $z$ and then increase. Both velocity decrease with increasing of $\tau_0$.

Fig. 2. Core velocity profile for $\tau_0 = 0.05,0.10,0.15$.

Fig. 3. Peripheral velocity profile for $\tau_0 = 0.05,0.10,0.15$.

Fig. 4. Flow rate with stenosis size for $\tau_0 = 0.05,0.10,0.15$.

Fig. 5. Flow rate with stenosis size for $\mu_p = \mu_c/10, \mu_c/5, \mu_c/13$. 

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The flow rate $Q$ of fluid is obtained by equation (19). Fig. 4. and fig. 5. describe the changes in flow rate of fluid against height of stenosis for distinct value of yield stress ($\tau_0$) and viscosity ratio ($\mu_p$) respectively. It is noticed that $Q$ decreases rapidly as shape parameter increases. This is due to decrease of cross sectional area in the stenosed arterial segment. Also the flow flux rate ($Q$) decaying with increasing value of yield stress ($\tau_0$). In case of viscosity ratio same changes has seen as displayed in figure 5.

Fig. 6. explicit the effect of shear stress at wall corresponding to axial distance ($z$) for distinct viscosity ratios. The variation in shear stress at wall against axial distance ($z$) can be seen at any point in arterial segment from the graph. It is clear that shear stress ($\tau_w$) at wall of fluid increases as the axial distance increase (from $z = -1$ to $z = 0$) and after that it reduces gradually from (from $z = 0$ to $z = 1$). The reason of this variation is intensity of the stenosis which significantly affects the shear stress at wall of blood flow. The maximum shear stress occur at the center at equal to ($z = 0$) of the stenosis.

V. CONCLUSION

An effort has been made to examine various aspects of fluid flow via a bell shaped stenosed artery. The fluid velocity (core velocity $u_c$ and plasma layer velocity $u_p$), flow rate ($Q$) and shear stress at wall ($\tau_w$) are examine graphically. From the above discussions important outcomes are summarized as

1. The core velocity and peripheral velocity decreases as the yield stress increases.
2. Flow rate $Q$ reduced as value of yield stress $\tau_0$ and viscosity ratio increased.
3. Shear stress at wall ($\tau_w$) decreases when viscosity ratio increases.

Therefore the mathematical expressions and above discussion may useful to explain the flow of blood in stenosed artery.

REFERENCES


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Dr. Madan Lal has completed M.Sc. degree in Mathematics in the year 1997 from M.J.P.Rohilkhand University, Bareilly (U.P.) India. and cleared NET (CSIR JRF) examination in December 2001. Ph.D. degree was awarded to him in 2003 (Mathematics) on the topic “Some Problems on Hydrodynamics and MHD flows” and his more than 20 research papers on “Magnetohydrodynamics” has been published in reputed National/International journals. He has participated more than 14 National/International conferences/ Workshops/seminars etc. Dr. Lal also chaired the session in various National and International conferences. He is also the life member of Indian Science Congress Association, Indian Mathematical Society etc. Currently he is working as Head of Applied Mathematics Department, M.J.P.Rohilkhand University Bareilly (U.P.) India.

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