

# Optimal Control of PMSG Based Wind Energy Conversion System using State-Dependent DRE

## Haneesh K. M



Abstract: Generation of electricity from wind is becoming more economical and popular with improved system design with modern control techniques. To capture energy from the inherently variable wind source and converting it into good quality electricity need to use advanced techniques in equipment and control. Since all the subsystems involved in the generation of electricity from wind are highly nonlinear, optimal control using linear models and linear techniques will not be effective. This paper presents a closed loop optimal control for a PMSG based wind energy conversion system using State Dependent Differential Riccati Equation. A suboptimal control is obtained for the non-linear system through differential Riccati equation, which is solved by converting in to linear Lyapunov equation by change of variables in the finite-horizon. The effectiveness of the technique is verified by simulating on MATLAB platform.

Keywords: Wind Energy Conversion System, PMSG, Nonlinear Optimal control, state dependent riccati equation, finite-horizon regulation

#### I. INTRODUCTION

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Generating electricity using Wind Energy Conversion Systems (WECS) are a growing industry and by the year 2020 it is expected to deliver around 12% of the global electricity demand [1]. WECS is a renewable technology with comparatively less impact on environment and it can be

Revised Manuscript Received on March 30, 2020.

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implemented with different sizes onshore or offshore. It is important for any generation technique to become economical and WECS is attaining it through better system component design and applying advanced control strategies [2]. Application of optimal control in WECS offers maximum power extraction from the wind under various air movement conditions under the partial load conditions [3]. Optimal control offer maximum power extraction with less mechanical stress and fatigue of the aerodynamic system [4].

Linearized model for the subsystems were used earlier to get a linear control for maximum power extraction from the wind, but these were very inefficient not suitable for a complex nonlinear system. There were a few techniques used to control the WECS by considering the nonlinear dynamics of the system like PI control, sliding mode control, feedback linearization control and QFT robust control [1], but all these techniques were not considering the nonlinearity of the system as such. Linear Quadratic Regulator (LQR) technique applied to pitch control of wind turbines above rated speed, offer less over shoot and hence minimize the pitch variations [5]. In [6] an exact linearization method is used and LQR is applied on the linearized model in the neighbourhood of the operating point, but this method can be applied only for a small range. Since the WECS is highly nonlinear, a linear or linearized model may not be apt for the controller design [7]. State Dependent Riccati Equation (SDRE) is a promising technique to design controllers for complex nonlinear systems [8]. The SDRE Technique offer tracking or regulation controller design for various infinite-horizon nonlinear systems [9].

An optimal design a finite-horizon regulation for nonlinear WECS using SDRE is presented in this paper. Here the Differential Riccati Equation is converted in to a linear differential Lyapunov equation by using a change of variable technique described in [11]. The solution to the Lyapunov equation is found for each time step to obtain the optimal control, so the obtained control will be suboptimal than a perfect optimal control.

The paper is structured as follows; Section I give introduction, Section II discuss modelling of a PMSG based WECS, Section III explain the formulation of the optimal problem, Section IV describes the application of SDRE technique for the solution problem, Section VI presents simulation results and conclusions in Section VII.

#### II. MODELLING OF A PMSG BASED WECS

A WECS has mainly four subsystems; an aerodynamic subsystem consisting of wind turbine which converts the wind power in to mechanical rotation,

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a drive train to drive the generator at a higher speed, an electromagnetic sub system to generate electricity from the mechanical power available on the shaft and an electrical subsystem of grid and connections to it. A wind turbine extracts power from the available wind, but the as the velocity changes the wind power vary. At wind speeds lower than cut- in speed, power extraction is not possible, but from cut in speed to the rated speed, maximum power is extracted, this region is known as partial load operation. At speed above rated speed, it is difficult to capture the full power from the wind, during that regime the system designated to capture only the rated power [12]. Speeds above cut out speed the WECS will not be trying to capture power due to excess mechanical stress on the components. Figure 1 shows the various operational regions of WECS.

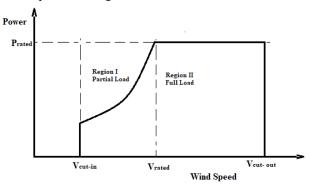


Fig 1. Operational regions of WECS under various wind speeds

The aerodynamic subsystem of the WECS gives the aerodynamic torque  $T_r$  at a rotor angular velocity of  $\omega_r$ from the incoming wind velocity of V m/s. The developed aerodynamic torque varies with the wind velocity V, air density  $\rho$ , and radius of the wind turbine blades R. The coefficient of torque is a function of pitch angle  $\beta$  and the tip speed ratio <sup>1</sup>.

Aerodynamic torque on the turbine rotor
$$T_r = \frac{1}{2} C_Q \rho \pi R^3 V^2 \qquad \dots (1)$$

Where  $^{C_{Q}}$  is the torque coefficient, which depends on  $^{\beta}$ and  $\lambda$ 

$$\lambda = \frac{\omega_r R}{v} \qquad \dots (2)$$

In the partial load regime, in order to maximise the power extraction only tip speed ratio is adjusted by controlling the generator speed maintaining the pitch angle constant. In this paper only partial load regime is considered for the maximum power capture, so the torque coefficient can be expressed as

$$\begin{array}{l} C_Q(\lambda) = a_0 + a_1 \lambda + a_2 \lambda^2 + a_3 \lambda^3 + a_4 \lambda^4 + \\ a_5 \lambda^5 + a_6 \lambda^6 & .... (3) \end{array}$$

Gear train block has a gear box to drive the generator shaft at a higher speed than the speed available at the turbine shaft. Figure 2 shows the details of a flexible gear train which translate the power available in the turbine rotor to the generator shaft. The gear is assumed to be having a gear ratio of i with an efficiency of  $\eta$ .

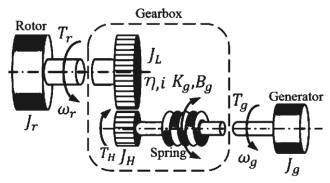


Fig 2. A flexible gear train

From fig 2, the dynamics of the gear train can be written as

$$\dot{\omega}_{r} = -\frac{i}{\eta J_{r}} T_{H} - \frac{1}{J_{r}} T_{r} \qquad ....(4)$$

$$\dot{\omega}_{g} = \frac{1}{J_{g}} T_{H} - \frac{1}{J_{g}} T_{g} \qquad ....(5)$$

$$\dot{T}_{H} = i K_{g} \omega_{r} - K_{g} \omega_{g} - B_{g} \left( \frac{1}{J_{g}} + \frac{i^{2}}{\eta J_{r}} \right) T_{H} + \frac{i B_{g}}{J_{r}} T_{r} + \frac{B_{g}}{J_{g}} T_{g} \qquad ....(6)$$

$$....(6)$$

Where  $\omega_r$  and  $\omega_g$  are the turbine shaft and generator angular velocities,  $I_r$  and  $I_g$  are the inertia of wind rotor and generator respectively.  $K_g$  and  $B_g$  are the stiffness and damping coefficient of the generator shaft.

The electromagnetic subsystem or generators output electromagnetic torque, which can be modelled in terms of flux, voltage and current. There are different kinds of electric generators used in WECS like Permanent Magnet Synchronous Generator (PMSG), induction generators, synchronous generators etc. [1]. In this paper a PMSG is based WECS is considered, whose dynamics in terms of direct and quadrature axes quantities are given by

$$\dot{i_{d}} = -\frac{R_{s}}{L_{d}}i_{d} + \frac{pL_{q}}{L_{d}}i_{q}\omega_{g} - \frac{1}{L_{d}}u_{d} \qquad ....(7)$$

$$\dot{i_{q}} = -\frac{R_{s}}{L_{q}}i_{q} + \frac{p}{L_{q}}(L_{d}i_{d} - \varphi_{m})\omega_{g} - \frac{1}{L_{q}}u_{q} \qquad ....(8)$$

$$T_{g} = p\varphi_{m}i_{q} \qquad ...(9)$$

Where  $i_d$ ,  $i_q$ ,  $u_d$ ,  $u_q$ ,  $L_d$  and  $L_q$  are the d and q axis stator current, stator voltage and stator inductance respectively. Also R<sub>s</sub> is the stator resistance, P is the pole pair and  $\Phi_{\mathbf{m}}$  is the flux linkage of the generator.

#### III. PROBLEM FORMULATION

From the equations (1) - (9) it is evident that dynamics of PMSG based WECS is nonlinear, which can be represented

$$\dot{x}(t) = f(x) + g(x)u(t), \qquad \dots$$

$$(10)$$

$$y(t) = h(x) \qquad \dots (11)$$

These nonlinear equations can be written as

$$\dot{x}(t) = A(x)x(t) + B(x)u(t), \qquad \dots (12)$$





$$y(t) = C(x)x(t)$$
 .... (13)  
This is the linear-like form with  $f(x) = A(x)x(t)$   $B(x) = g(x)$ 

$$h(x) = C(x)x(t)$$

$$h(x) = C(x)x(t)$$

The optimization problem is to minimize by obtaining a state feedback control

$$u^{*}(x) = -Kx(t) \qquad \dots (14)$$

$$J(x,u) = \frac{1}{2}x'(t_{f})Fx(t_{f}) + \frac{1}{2}\int_{t_{0}}^{t_{f}} \{x'(t)Qx(t) + u'(x)Ru(x)\}dt$$

Where error weighted matrix Q and terminal cost weighted matrix F are non-negative and weighting R is a positive matrix. The term  $x'(t_f)Fx(t_f)$  determine the terminal cost term, x'(t)Qx(t) calculate the error and u'(x)Ru(x) measures the control-effort.

#### IV. SOLUTION FOR A FINITE-HORIZON SDDRE REGULATOR

The state feedback control, which minimize the cost function can be written as

$$u^*(x) = -Kx(t) = -R^{-1}B'P(x,t)x(t) \qquad \dots (16)$$
Where  $P(x,t)$  is a solution of state dependent DRE
$$-\dot{P}(x) = P(x)A(x) + A'(x)P(x) - P(x)B(x)R^{-1}B'(x)P(x) + Q \qquad \dots (17)$$

$$P(x,t)$$
 can be found by applying the final value  $P(x,t_f) = F$   
This control will result the state variation  $\dot{x}(t) = [A(x) - B(x)R^{-1}B'(x)P(x)]x(t)$  ... (18)

$$\dot{x}(t) = [A(x) - B(x)R^{-1}B'(x)P(x)]x(t) \dots (18)$$

This trajectory depends on the present state values, which are not known and the present values of the state cannot be calculated from the SDDRE given in (17) by integrating from initial value as the state dependent coefficients are not known in advance. The method proposed in [14] is to convert the nonlinear SDDRE in to a linear Lyapunov equation by changing the parameters. This is an approximate method as the linear Lyapunov equation can be solved for the particular time step. So the solution obtained will be suboptimal than optimal solution.

The steps involved in solving the SDDRE can be summarized as in Table I

#### V. METHODOLOGY

The main aim of the optimal control is to obtain the maximum power out of the wind under the rated wind speed. The system dynamics is prepared based on the models of wind, wind turbine, gear train and the generator. The power generated by the generator drive by the wind turbine has to be pushed to the grid at constant voltage and frequency. The dynamics is formulated in the state space model and the coefficients of the state space equation contains state variable. So it is impossible to solve the equations using the conventional methods using to solve linear equations.

Table- I: The steps involved in solving the SDDRE

Step	Description	
1	Initiate $t = t_0$ , $x(0) = x(t_0)$	
2	Calculate the system matrices $A(x)$ , $B(x)$	
3	Solve Algebraic Riccati Equation (ARE) $P_{ss}(x)A(x) + A'(x)P_{ss}(x) - P_{ss}(x)B(x)R^{-1}(x)B'(x)P_{ss}(x) + Q(x) = 0$ To find $P_{ss}(x)$	
4	Apply change of variable $K(x,t) = [P(x,t) - P_{ss}(x)]^{-1}$	
5	Calculate $A_{\varepsilon l}$ using, $A_{\varepsilon l}(x) = A(x) - B(x)R^{-1}(x)B'(x)P_{ss}(x)$	
6	Solve the linear Lyapunov equation to obtain $D$ $A_{cl}(x)D + DA_{cl}'(x) - B(x)R^{-1}(x)B'(x)$	
7	Solve the Differential Lyapunov equation $ \dot{K}(x,t) = K(x,t)A_{cl}'(x) + A_{cl}(x)K(x,t) - B(x)R^{-1}(x)B'(x) $ The solution is in the form $ K(x,t) = e^{A_{cl}(x)(t-t_f)} \Big( K\Big(x,t_f\Big) - De^{A_{cl}'(x)(t-t_f)} \Big) + D $	
8	Calculate $P(x,t)$ by using the change of variables $P(x,t) = K^{-1}(x,t) + P_{ss}(x)$	
9	Find the optimal feedback control $u(x,t) = -R^{-1}(x)B'(x)P(x,t)x(t)$	
10	Continue from step 1 updating the values of $t_0$ and $x(0) = x(t_0)$ in steps $t = t_0 + \Delta t$ until $t = t_f$	

In this paper the non-linear dynamics of the system is solved by using State Dependent Riccati equation method, where the dynamics is solved piecewise by using the state values calculated and hence calculating the matrix coefficients. This method need to calculate the state values in advance, and here the Lyapunov method is used to calculate it from the known final steady state values expected. This method will give a sub-optimal solution for the nonlinear optimal problems. This method gives flexibility in control as well as in implementation. For the solutions the state dependent parameterisation of coefficients gives lot of flexibility to select different coefficients without losing the controllability and stability. The problem is solved for a known system using MATLAB platform and validated the results for the optimal performance.

#### VI. SIMULATIONS

A WECS with a PMSG is modelled as per the equations (1)-(9). The parameters selected for the WECS is given in the Table II. The matrices in the dynamic equations were found with the parameters from the table, but which are nonlinear and state dependent. The initial values were assumed as  $\omega_r = 2 \ rad/s$ ,  $\omega_r = 1 \ rad/s$ , generator shaft torque  $T_H = 0 Nm_{, direct axis}$  and quadrature axis current of the generator as  ${}^{4}$   ${}^{A}$  and  ${}^{3}$   ${}^{A}$  respectively. The simulations done in a MATLAB platform and results are shown in figures 4. The results show the closed loop feedback is giving optimal solutions after few time steps.

Table- II: Values of parameters used [15]

Table- II. Values of parameters used [13]			
Parameter	Symbol	value	
ρ	Air density	$1.25 \ kg/m^3$	
R	Blade length	2.5 m	
i	Gear ratio	6	
η	Gear box efficiency	1	
$J_r$	Wind rotor inertia	$2.88 \ kg.m^2$	
$I_g$	Generator inertia	$0.22 \ kg.m^2$	
$K_g$	Stiffness coefficient of generator shaft	75 Nm/rad	
$B_g$	Damping coefficient of generator shaft	$0.3 \ kg. m^2/s$	
p	Number of pole pairs	3	
$R_s$	Stator resistance of generator	3.3 Ω	
$\phi_m$	Flux linkage of the generator	0.4382 Wb	
$L_d$	Direct axis inductance of generator	41.56 mH	
$L_q$	Quadrature axis inductance of generator	41.56 mH	

The direct and quadrature axes current of the PMSG obtained through the simulations are given below. The result shows that the currents transients are settling down in quick time with this control.

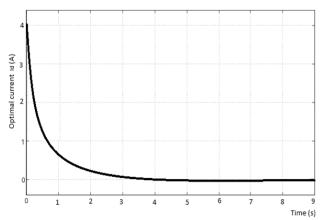


Fig 3. Optimal control of direct axis current

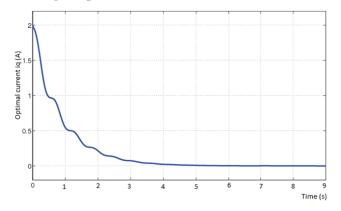


Fig 4. Optimal control of quadrature axis current

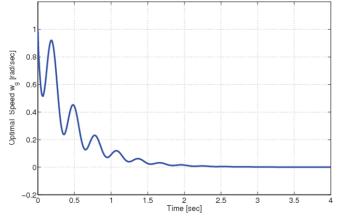


Fig 5. Optimal control of speed Performace Index 0 1.5 5.5 3 3.5 4.5 X1 5 0 0 X 2 6 0 2 U2 X<sub>2</sub> 72 0 [ 0 [ 0 0 U3 XI3 13 0 E 0 E 0 2 0 2 U4 X4 2 0 [ 0 U5 U<sub>2</sub> 0 [ 0 2 0 2 6 4 6

Fig 6. Variation of cost function and various input and state variables obtained through simulation

## VII. CONCLUSIONS

WECS is a highly nonlinear system with many complex subsystems, in order to obtain an optimal control linearised models and linear control techniques are not sufficient. This paper tried present a finite-horizon state dependent differential Riccati equation technique to obtain a closed loop optimal control. The SDDRE is solved by converting it into a linear Lyapunov equation by changing variable and which is solved in each time step. The dynamics of the method suggested was simulated on a MATLAB platform to verify the performance of the technique.

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#### **ACKNOWLEDGMENT**

The author would like to thank E-mobility R&D center in CHRIST (Deemed to be University), Bangalore for the laboratory facilities and to the support.

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