Model of Functioning Data-Transfer Systems Special Purposes Taking into Account the Influence of Cyber Attack

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Abstract: The article proposes an analytical model of the functioning of the data transmission system, taking into account the influence of short-term interruptions in messages and the recovery of information distorted by cyber-attacks. The model uses the Laplace-Stieltjes transform and the Lagrange multiplier method. This helps determine the performance of switching nodes, the throughput of communication channels and their total cost in a communication system.

Keywords: model, data transmission system, cyber attack, mathematical expectation, throughput, communication channels.

I. INTRODUCTION

Designing a data transmission system (DTS) for special purposes requires preliminary studies necessary to justify the technical and economic characteristics of the system, which ensures the processing and transmission of information with a given time cycle. One of the ways to determine the necessary technical and economic assessments of the system is the use of mathematical modeling methods. Currently, to obtain various technical and economic assessments, a sufficiently large number of analytical models for the operation of DTS has been developed [1–6]. However, these models do not fully take into account the possibility of distortion of the transmitted information through the influence of short-term failures (cyber-failures), which are self-eliminating. To eliminate this drawback, there is a need to develop an analytical model of DTS functioning, with the help of which it is possible to determine the main technical and economic characteristics of the system, taking into account DTS functioning algorithms under the influence of short-term cyber-failures on transmitted information.

II. PRESENTATION OF THE MAIN RESEARCH MATERIAL

To solve this problem, we will represent m elements of type A - switching nodes and n elements of type B - data channels in the form of an open stochastic network (Fig. 1).

![Fig. 1. Network model of DTS](image)

Each element of type A (a1, …, am) is characterized by the performance μa1, μa2, …, μam, and the elements of type B (b1, …, bn) are the capacity of ρb1, ρb2, …, ρbn. Type B elements provide the transmission of messages of medium length L̄, and type A elements distribute these messages between subscribers and control the functioning of the elements of the DTS. In the process of processing one message, each element of type A needs to perform, on average θa1, θa2, …, θam operations.

On the elements a1, …, am, b1, …, bn affect Poisson flows of cyber crashes according to average densities, λa1, λa2, …, λam and λb1, λb2, …, λbn [7]. It is assumed that the recalculation time of the segment of the program, which is between the moment of the last control and the moment of occurrence of cyber failure in the i-th element (i = 1, m) of type A, is subject to the exponential law with parameter γa. The time of retransmitting the part of the message between the moment of the last control and the moment i of occurrence of a cyber-failure in the j-th element (j = 1, n) type B obeys exponential law with parameter γb [7].

The input of the i-th element of type A (i = 1, m) and j-th element of type B (j = 1, n) messages arrive with intensity λai and λbj.

It is assumed that the value of Cai i-th and Cbj j-th elements of type A and B varies according to a linear law depending on μai, 1/ρbj [8].

\[ C_{ai} = K_{ai}μ_{ai} + K_{ai}'. \] (1)
\[ C_{ij} = K_{ij} \rho_{ij} + K'_{ij}, \]  

where \( K_{ai} \) and \( K'B_{ai} \) are proportionality coefficients reflecting the value of the type \( A \) and \( B \) elements, depending on the performance of \( \mu_{ai} \) and the throughput of \( \rho_{ij} \). \( K'_{ai} \) and \( K'_{bj} \) are proportionality coefficients, depending on the cost of the optional network equipment. The task of determining the required performance of type \( A \) elements and the capacity of type \( B \) elements is formulated as follows. Given \( m \) elements of type \( A \) and \( n \) elements of type \( B \), presented in the form of a network model. The cycle of the data transmission system is \( \Delta T \). For each \( i-th \) element type \( A \) \( (i = I, m) \) known: \( \lambda_{ai}, \theta_{ai}, \gamma_{ai}, K_{ai}, K'_{ai}, \) and for each \( j-th \) element of type \( B \) \( (j = I, n) \) known: \( \lambda_{bj}, L_{jo}, \lambda_{bj}, \gamma_{bj}, K_{bj}, K'_{bj}, \)

It is necessary to determine the optimal values \( \mu_{ai}(i = I, m) \) and \( \rho_{ij} \) \( (j = I, n) \), for which it is true:

\[ M\left[ \tau_{a} \right] \leq \Delta T; \]

\[ C_{a} = \min_{\mu_{ai} \rho_{ij}} \left( \sum_{i=1}^{m} C_{ai} + \sum_{j=1}^{n} C_{bj} \right), \]

where \( M[\tau_{a}] \) is the mathematical expectation of the processing and transmission of messages in DTS; \( C_{a} \) is total cost of the network.

The solution to this problem is reduced to the following steps [4, 7]:

Step 1. Using the Laplace-Stieltjes method, the mathematical expectation of the processing time and transmission of messages \( M[\tau_{a}] \) by an element of type \( A \) and an element of type \( B \) is calculated taking into account the influence of short-term interruptions caused by the action of cyber-failures.

Step 2. The analytical dependence of \( M[\tau_{a}] \) on \( \mu_{ai} \) and \( \rho_{ij} \) \( (j = I, n) \) is determined.

Step 3. Using the method of Lagrange undetermined multipliers, optimal values \( \mu_{ai} \) and \( \rho_{ij} \) are calculated.

The implementation of steps 1-3 is carried out on the basis of the integrated use of methods for calculating communication networks for computers and taking into account the preconditions given in the works [4, 7, 8-10]:

- Elements of type \( A \) and \( B \) are considered as independently functioning queueing systems (QS) with an unlimited queue and discipline of service in the order of receipt.
- The configuration of the data network is represented as an open exponential stochastic network.
- At the input of each \( i-th \) and each \( j-th \) QS of type \( A \) and \( B \), a stream of requests with intensities \( \lambda_{ai} \) and \( \lambda_{bj} \) is received;
- The network operates continuously.

- In each \( i-th \) (\( j-th \)) QS, the Poisson flow of interruptions (cyber-failures) with an average density \( \lambda^{crash}_{ai} \) (\( \lambda^{crash}_{bj} \)) influences the request servicing process.
- After each interruption, the service time for requests in the \( i-th \) (\( j-th \)) QS is subject to the exponential law with the parameters \( \xi_{ai} \) (\( \xi_{bj} \)).

The average time \( (T^0_{ai} \) and \( T^0_{bj} \) for servicing the QS request \( a_i \) and \( b_j \) is determined by the formulas [5, 8]:

\[ T^0_{ai} = \left( \frac{\mu_{ai}}{\theta_{ai}} - \lambda_{ai} \right)^{-1} \]

\[ T^0_{bj} = \left( \frac{\rho_{bj}}{L_{jo}} - \lambda_{bj} \right)^{-1} \]

If we assume that when servicing a request in the \( i-th \) QS \( a_i \) and in the \( j-th \) QS \( b_j \), there were \( Z_{ai} \) and \( Z_{bj} \) interruptions, then, respectively, the service time of the request \( (T^Z_{ai}, T^Z_{bj}) \) in each QS is calculated as follows:

\[ T^Z_{ai} = \sum_{n=1}^{Z_{ai}+1} T^0_{ai} + \sum_{n=1}^{Z_{ai}} + T^0_{ai}, \]

\[ T^Z_{bj} = \sum_{n=1}^{Z_{bj}+1} T^0_{bj} + \sum_{n=1}^{Z_{bj}} + T^0_{bj}, \]

where \( T^0_{ai}, T^0_{bj} \) — the time of service of the request in the \( i-th \) and \( j-th \) QS \( a_i, b_j \) between the \((n-1)\)-th and \( n\)-th interrupts;

\( T^Z_{ai}, T^Z_{bj} \) — the time of service of the request in the \( i-th \) and \( j-th \) QS \( a_i, b_j \) after the \( n\)-th interruption.

The total time \( (T_{a}) \) for servicing a request with two QS \( a_i \) and \( b_j \) is determined by the formula:

\[ T_{a} = T^Z_{ai} + T^Z_{bj} \]

Using the Laplace-Stieltjes transforms, we calculate the mathematical expectation \( M[\tau_{a}] \) under the following conditions [7]:

\[ \sum_{n=1}^{Z_{ai}+1} T^0_{ai} = T^0_{ai}, \sum_{n=1}^{Z_{bj}+1} T^0_{bj} = T^0_{bj}, Z_{ai} = Z_1, Z_{bj} = Z_2 \]
\[ M \left( e^{-P_T_{ai}}, e^{-P_T_{bj}}, z_{ai}, z_{bj} \right) = M \left( \frac{e^{P_T_{ai}}}{T_{ai}^{0}}, \frac{e^{P_T_{bj}}}{T_{bj}^{0}}, z_{ai}, z_{bj} \right) = M \left[ T_{ai}^{0}, T_{bj}^{0}, \gamma'_{ai}, \gamma'_{bj} \right] = M \left[ \frac{P_T_{ai}^{0}}{T_{ai}^{0}}, \frac{P_T_{bj}^{0}}{T_{bj}^{0}}, \gamma'_{ai}, \gamma'_{bj} \right] = e^{-P_T_{ai}} \left( L_i \left[ T_{ai}^{0}, P \right] \right)^{z_{ai}} + e^{-P_T_{bj}} \left( L_j \left[ T_{bj}^{0}, P \right] \right)^{z_{bj}}, \]

where \( L_i \left[ T_{ai}^{0}, P \right] \) and \( L_j \left[ T_{bj}^{0}, P \right] \) are Laplace-Stieltjes transforms of distribution functions \( F_i(t) \) and \( F_j(t) \) of random variables \( T_{ai} \) and \( T_{bj} \):

\[ F_i(t) = \begin{cases} 1 - e^{-\lambda_{ai} t}, & t > 0 \\ 0, & t \leq 0 \end{cases} \]

\[ F_j(t) = \begin{cases} 1 - e^{-\lambda_{bj} t}, & t > 0 \\ 0, & t \leq 0 \end{cases} \]

Given the fact that \( a_i \) and \( b_j \) are Poisson streams of interrupt of request service in QS with parameters \( \lambda_{ai} \) and \( \lambda_{bj} \), we remove the restrictions on \( Z_{ai} \) and \( Z_{bj} \) [7]:

\[ M \left[ \tau_i \right] = M \left[ \left( \frac{P_T_{ai}^{0}}{T_{ai}^{0}}, \frac{P_T_{bj}^{0}}{T_{bj}^{0}}, \gamma'_{ai}, \gamma'_{bj} \right) \right] = M \left[ T_{ai}^{0}, T_{bj}^{0}, \gamma'_{ai}, \gamma'_{bj} \right] = e^{-\lambda_{ai} \tau_i} \left( \frac{P_T_{ai}^{0}}{T_{ai}^{0}} \right)^{\tau_i} e^{-\lambda_{bj} \tau_i} \left( \frac{P_T_{bj}^{0}}{T_{bj}^{0}} \right)^{\tau_i}, \]

The optimal values \( \mu_{ai} \) and \( \rho_{bj} \) are determined using the method of Lagrange undetermined multipliers under the following conditions:

\[ M \left[ \tau_i \right] = \sum_{i=1}^{m} \left( \frac{\mu_{ai}}{\theta_{ai}} - \lambda_{ai} \right)^{-1} \left( I + \gamma_{ai} \right) G_{ai} + \sum_{j=1}^{n} \left( \frac{\rho_{bj}}{\theta_{bj}} - \lambda_{bj} \right)^{-1} \left( I + \gamma_{bj} \right) G_{bj} \]

The mathematical expectation \( M[\tau_i] \) of the QS request service \( a_i \) and \( b_j \) is determined as follows:
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\[
\begin{align*}
\sum_{i=1}^{m} \left( \frac{\mu_{ai}}{\theta_{ai}} - \lambda_{ai} \right)^{-1} (I + \frac{\lambda_{ai}}{\gamma_{ai}}) G_{ai} + \\
+ \sum_{j=1}^{n} \left( \frac{\rho_{bj}}{L_{b}} - \lambda_{bj} \right)^{-1} (I + \frac{\gamma_{bj}}{\gamma_{bj}}) G_{bj} = T
\end{align*}
\]

The Lagrange function (I) has the following form:

\[
I = \sum_{i=1}^{m} (K_{ai} \mu_{ai} + K'_{ai}) + \sum_{j=1}^{n} (K_{bj} \rho_{bj} + K'_{bj}) + \\
+ \sum_{i=1}^{m} G_{ai} \left( \frac{\mu_{ai}}{\theta_{ai}} - \lambda_{ai} \right)^{-1} (I + \frac{\lambda_{ai}}{\gamma_{ai}}) + \\
+ \sum_{j=1}^{n} G_{bj} \left( \frac{\rho_{bj}}{L_{b}} - \lambda_{bj} \right)^{-1} (I + \frac{\gamma_{bj}}{\gamma_{bj}})
\]

where \( \chi \) is the undetermined multipliers.

Equating the product \( \partial I / \partial \mu_{ai} \) and \( \partial I / \partial \rho_{bj} \) to zero, we get:

\[
K_{ai} - \chi \frac{\mu_{ai}}{\gamma_{ai}} I \theta_{ai} = 0; \quad i = 1, m
\]

\[
K_{bj} - \chi \frac{\rho_{bj}}{L_{b}} I L_{b} = 0; \quad j = 1, n
\]

where

\[
\mu_{ai} = \lambda_{ai} \theta_{ai} + \sqrt{\frac{\chi_{i}}{K_{ai}}} \sqrt{G_{ai} \theta_{ai} \left( I + \frac{\lambda_{ai}}{\gamma_{ai}} \right)}
\]

\[
\rho_{bj} = \lambda_{bj} \theta_{bj} + \sqrt{\frac{\chi_{j}}{K_{bj}}} \sqrt{G_{bj} L_{bj} \left( I + \frac{\gamma_{bj}}{\gamma_{bj}} \right)}
\]

The values of \( T_{1} \) and \( T_{2} \) are determined as follows.

Step A. The number is selected \( \delta > 0 \) is the accuracy of minimum \( C_{\Sigma} \): \( r = 1; \quad \delta = \delta; \quad T_{1} = \delta; \quad T_{2} = (T_{0} - \delta) \).

Step B. Using formulas (29-31) is calculated \( C_{\Sigma} \).

Step C. \( \delta_{r+1} = \delta; \quad T_{1,r+1} = \delta_{r}; \quad T_{2,r+1} = (T_{0} - \delta_{r+1}) \); \( C_{\Sigma,r+1} \) is determined.

Step D. If \( C_{\Sigma} > C_{\Sigma,r+1} \), then \( r = r + 1 \) and the transition to step B takes place, or the problem is already solved, that is, \( T_{1} = T_{1} \), and \( T_{2} = T_{2} \) are found for which \( C_{\Sigma} \) is minimal.

### III. CONCLUSION

One of the most important problems in developing DTS used for special purposes, for example, in security, is the determination of the necessary technical and economic assessments. These assessments are proposed to be made based on determining the mathematical expectation of information transmission time and its analytical dependence on the performance of switching nodes, as well as the throughput of communication channels.
The article considers an analytical model of work that allows the possibility of the influence of short-term failures on messages and the recovery time of information distorted under the influence of cyberattacks. This model uses the Laplace-Stieltjes transform and the Lagrange multiplier method. The analytical expressions that were obtained make it possible to calculate the cost of the DTS and its optimal parameter values, which ensure the processing and transmission of information with a given time cycle and taking into account the effect of cyber failures.

REFERENCES

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