Inverse kinematics Solution of PUMA by using ANFIS Technique

Gurjeet Singh, V. K. Banga

Abstract: In this paper Inverse kinematics solution of the PUMA 560 is solved. In robotics the main problem is to find the inverse kinematics solution. Forward kinematics is calculated with the help of D-H (Denavit-Hartenberg) parameter method. Now a day’s inverse kinematics is the area of research in robotics. In present paper, Inverse kinematics is calculated by mathematically and by ANFIS and then difference between the predicted value and deducted value is calculated. Workspace area of PUMA Robot is also shown in this paper.

Keywords: ANFIS, kinematics, Robotics arm, optimization, Robot manipulator, Neural networks, Fuzzy logic.

I. INTRODUCTION

With the advancement of technology robots play an important role in the Industry. PUMA robot used in different industries for safety reasons. It also increase the production in the industries. The efficiency and the accuracy of the industries also improve with the use of the robots[1]. Now a days, study of robotics is a new topic of research. This study of robotics is done with the help of kinematics. In robotics there are forward and inverse kinematics[11]. As the degree of freedom increase in the robot it’s very difficult to find the solution of Inverse kinematics[12]. The forward kinematics is solved with the help of Denavit-Hartenberg (D-H) parameter. PUMA 560 is a six degree of freedom robots and each link is shown in the figure[4].

![Puma 560 robot](image)

Many techniques are used to calculate the solution of inverse kinematics like algebraic, geometric and numerical method. In this paper the inverse kinematics solution is find out by the D-H parameter and code is written in the MATLAB, the same solution is find out by the applying the adaptive neuro fuzzy interface system (ANFIS)[3].

II. D-H PARAMETER

D-H parameters are nearly new to calculate the kinematics solution of the robots. There are the four parameters used in this which define the relationship between the joints[13]. The parameters are shown below in table. In forward kinematics the theta values are given and the values of x,y,z are find out. In each robot the first step is to find the forward kinematics using the table[5].

<table>
<thead>
<tr>
<th>Θi</th>
<th>αi</th>
<th>di</th>
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</tr>
</thead>
<tbody>
<tr>
<td>90</td>
<td>-90</td>
<td>0</td>
<td>-160 to +160</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>149.09</td>
<td>-225 to 45</td>
</tr>
<tr>
<td>90</td>
<td>90</td>
<td>0</td>
<td>-45 to 225</td>
</tr>
<tr>
<td>0</td>
<td>-90</td>
<td>433.07</td>
<td>-110 to 170</td>
</tr>
<tr>
<td>90</td>
<td>0</td>
<td>-100 to 100</td>
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</tr>
<tr>
<td>0</td>
<td>0</td>
<td>56.25</td>
<td>-266 to 266</td>
</tr>
</tbody>
</table>

\[ T = T_1 T_2 \cdots T_n = ^0A_1^1A_2^2A_3^3A_4^4A_5^5A_6 = \begin{bmatrix} n_x & s_x & a_x & p_x \\ n_y & s_y & a_y & p_y \\ n_z & s_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \]

\[ N_1 = C_1 \left[ C_{23} \left( \begin{array}{c} C_4 \ C_5 \ C_6 - S_4 \ S_5 \end{array} \right) - S_25 \ S_3 \ C_6 \right] - S_1 \left( \begin{array}{c} S_4 \ C_5 \ C_6 \end{array} \right) + C_1 \left( \begin{array}{c} S_4 \ C_5 \ C_6 \end{array} \right) + C_4 \ S_6 \] (1)

\[ N_2 = S_1 \left[ C_{23} \left( \begin{array}{c} C_4 \ C_5 \ C_6 - S_4 \ S_5 \end{array} \right) - S_25 \ S_3 \ C_6 \right] + C_1 \left( \begin{array}{c} S_4 \ C_5 \ C_6 \end{array} \right) + C_4 \ S_6 \] (2)

\[ N_3 = -S_{22} \left( \begin{array}{c} C_4 \ C_5 \ C_6 - S_4 \ S_5 \end{array} \right) - C_{23} \ S_5 \ C_6 \] (3)

\[ N_4 = C_3 \left[ -C_{23} \left( \begin{array}{c} C_4 \ C_5 \ C_6 + S_4 \ S_5 \end{array} \right) + S_25 \ S_3 \ C_6 \right] - S_1 \left( \begin{array}{c} S_4 \ C_5 \ C_6 \end{array} \right) + C_1 \left( \begin{array}{c} S_4 \ C_5 \ C_6 \end{array} \right) + C_4 \ S_6 \] (4)

\[ N_5 = S_3 \left[ -C_{23} \left( \begin{array}{c} C_4 \ C_5 \ C_6 + S_4 \ S_5 \end{array} \right) + S_25 \ S_3 \ C_6 \right] + C_1 \left( \begin{array}{c} S_4 \ C_5 \ C_6 \end{array} \right) + C_4 \ S_6 \] (5)

\[ N_6 = S_{22} \left( \begin{array}{c} C_4 \ C_5 \ C_6 + S_4 \ S_5 \end{array} \right) - C_{23} \ S_5 \ C_6 \] (6)

\[ a_1 = C_1 \left[ C_{23} \ C_4 \ S_5 + S_25 \ C_3 \right] - S_1 \ S_4 \ S_5 \] (7)

\[ a_2 = S_1 \left[ C_{23} \ C_4 \ S_5 + S_25 \ C_3 \right] - C_1 \ S_4 \ S_5 \] (8)

\[ a_3 = -S_{23} \ C_4 \ S_5 + C_{23} \ C_3 \] (9)

\[ p_1 = C_4 \left[ d_1 + \left( \begin{array}{c} C_{23} \ C_4 \ S_5 + S_25 \ C_3 \end{array} \right) + S_25 \ d_4 + C_{23} \ a_3 + C_2 \ a_2 \right] - S_1 \left( \begin{array}{c} d_1 \ S_4 \ S_5 \end{array} \right) + d_2 \] (10)

\[ p_2 = S_1 \left[ d_1 + \left( \begin{array}{c} C_{23} \ C_4 \ S_5 + S_25 \ C_3 \end{array} \right) + S_25 \ d_4 + C_{23} \ a_3 + C_2 \ a_2 \right] + C_1 \left( \begin{array}{c} d_1 \ S_4 \ S_5 \end{array} \right) + d_2 \] (11)

\[ p_3 = d_1 + \left( \begin{array}{c} C_{23} \ C_5 \ S_5 - S_25 \ C_4 \ S_5 \end{array} \right) + C_{23} \ d_4 - S_25 \ d_3 - S_2 \ a_2 \] (12)

Inverse Kinematics

After finding the forward kinematics the next step is to calculate the IK. The x,y,z values are given and to find out the values of the different thetas. Inverse kinematics is a
important thing in the robotics and to find the best theta values is a important thing[6][14].

\[
\theta_1 = \tan^{-1} \left( \frac{p_y d_2 - \frac{P_x}{P_y}}{p_y d_2 - \frac{P_x}{P_y}} \right)
\]

(13)

all the possible values of the arm and the elbow is shown in Table 2.

\[
\sin \theta_2 = \sin \alpha_1 \cos \beta_1 + (\text{ARM.ELBOW1}) \cos \alpha_1 \sin \beta_1
\]

(14)

\[
\cos \theta_2 = \cos \alpha_1 \cos \beta_1 - (\text{ARM.ELBOW1}) \sin \alpha_1 \sin \beta_1
\]

(15)

\[\theta_2 = \tan^{-1} \left( \frac{\sin \theta_2}{\cos \theta_2} \right)\]

(16)

\[R = \sqrt{P_x^2 + P_y^2 + P_z^2 - d_z^2}\]

(17)

\[\cos \phi = \frac{a_x^2 + (a_y^2 + a_z^2 - r^2)}{2a_x \sqrt{d_x^2 + a_z^2}}\]

(18)

\[\sin \phi = \sqrt{1 - \cos^2 \phi}\]

(19)

\[
\sin \beta = \frac{d_y}{\sqrt{d_x^2 + d_z^2}}
\]

(20)

\[
\cos \beta = \frac{a_y}{\sqrt{d_x^2 + a_z^2}}
\]

(21)

\[
\sin \theta_3 = \sin \varphi \cos \beta + \cos \varphi \sin \beta
\]

(22)

\[
\cos \theta_3 = \cos \varphi \cos \beta + \sin \varphi \sin \beta
\]

(23)

\[\theta_3 = \tan^{-1} \left( \frac{\sin \theta_3}{\cos \theta_3} \right)\]

(24)

Table 3 represent the wrist orientation.

\[
\theta_4 = \tan^{-1} \left( \frac{M (c_1 a_y - s_1 a_z)}{M(c_1 a_x c_23 + c_23 s_1 a_y - s_23 a_z)} \right)
\]

(25)

\[\theta_5 = \tan^{-1} \left( \frac{(c_1 c_2 c_3 - s_1 s_3) a_x + (c_1 c_2 c_4 + c_1 s_4) a_y - c_4 s_3 a_z}{s_1 c_23 a_x + s_1 c_{23} c_4 a_y + c_23 a_z} \right)\]

(26)

\[\theta_6 = \tan^{-1} \left( \frac{(-c_1 c_2 c_3 - s_1 s_3) a_x + (-c_1 c_2 c_4 + c_1 s_4) a_y - s_4 s_3 a_z}{-c_1 c_23 c_4 - c_1 s_4 s_3 a_x + (-c_1 c_23 s_4 + c_1 c_4) a_y - s_4 s_3 a_z} \right)\]

(27)

**III. ADAPTIVE NEURO-FUZZY INTERFACE SYSTEM (ANFIS)**

It is a rule based method also called a hybrid learning procedure; it is a amalgam of the neural network and the fuzzy logic.[7] This techniques discovered in 1990 based on Takagi–Sugeno fuzzy inference system. The fuzzy logic considered the imprecision and uncertainty while neural network give the sense of adaptability[8]. The training is done with the help of the input output datasets. The main objective of ANFIS is to build a model of robotics on which fuzzy rules are applied to get the input–output dataset[9].

Fuzzy interface model approximate the data set consist of collection of input-output. Such models have a number of membership function with adjustable parameter[10]. The neuro adaptive learning take place through input output dataset and using this database anfis construct a fuzzy interference structure whose parameters are adjusted with the backpropogation and least square method. The parameters of the membership function adjusted or changed as the learning process is going on.

**IV. SIMULATION AND RESULTS**

The methodoly used in this process is shown below. In this first the D-H parameter of the robot is used to calculate the Kinematics result of the robot. The matlab code is written for these equation and these values called the analytical values.

Now using the anfis technique the solution of inverse kinematics is find out this value is called as predicted value. For anfis we given the value of membership function ,number of epochs ,and other parameter. Then the both values are compared.

**Methodology**

```
Generate a MATLAB Code of 5 DOF Inverse Kinematics Equation
[It is called a analytical value]
```

```
Using anfis command predict the Inverse Kinematic Solution of SDOF
[It is called a Predicted value ]
```

```
Given the value of membership function , number of epochs, parameter option to display progress.
```

```
Analytical value is compared with predicted value
```

Forward and Inverse kinematics equations can be solved by the mathematical equations with the help of DH parameters and then apply the ANFIS is used to find the predicted value, the difference between the predicted and deducted value is shown in the figure 3.
Figure 3: Divergence of the predicted and analytical value

The ANFIS procedure followed is shown in figure 4.

![ANFIS Procedure Diagram](image)

In figure 5 the ANFIS procedure is shown in which first we open anfis tool and initialize the fuzzy system then set the number of iterations and the input data is taken from the workspace and then start the learning process. The data is validate in the end. The figure 5 show the training of the six theta values by ANFIS network the co-ordinates will act as a input and the different theta values will act as a output. The membership function of the model is adjust so that it will give a best theta values as output.
The work space area of PUMA 560 arm) is shown below. The red star show the resolution and in this area the movement of the robot is fine and the resolution of the end effectors is maximum.

![Figure 5: Training result of different theta(106) by ANFIS](image)

**V. CONCLUSION**

In this paper the solution of Inverse kinematics problem of PUMA robot is find out by by using ANFIS technique and joint angles are derived for the trajectory. The difference between the analytical and the predicted value is calculated and the results are acceptable. As compared to the other techniques this technique is acceptable and produces fast result of inverse kinematics. The workspace area of the end effector of a robot is also shown in a figure.

![Figure 6. Workspace Area of PUMA 560](image)

**REFERENCES**


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