

# Optimal Age-Based Preventive Maintenance Policy for a System Subject to Cumulative Damage Degradation and Random Shocks

W. A. Akpan, A. A. Okon, E. J. Awaka-Ama

**Abstract:** This research investigates the problem of cumulative degradation and random shocks a system like a centrifugal pump may experience during normal and adverse operating conditions. An accelerated life testing method was employed to determine the degradation of the pump under cumulative damage degradation and random shocks conditions. An age- Based policy was used to determine the optimum time interval that will minimize the total expected cost of the system. The random shock increases the number of failures and hence reduces the reliability of the system. The total expected preventive maintenance cost obtained varies from ₦1700.00 (One thousand seven hundred naira) to ₦16,000.00 (sixteen thousand naira), depending on the shock and shock duration. The methodology presented is useful and thus recommended for use to study cumulative damage degradation and random shocks for similar systems.

**Keywords:** Age-Based, Degradation, Cumulative Damage, Random Shocks, Preventive Maintenance.

## I. INTRODUCTION

Preventive maintenance (PM) is the combination of all technical and associated actions intended to retain an item or system in or restore it to a state in which it can perform its required functions. Equipment or asset failures are often caused by inadequate maintenance and inability to predict problems that may occur during usage ( Akpan *et al*; 2020).

Degradation is the reduction in the performance or the reliability of the system over time. Lehman (2006) describes degradation as the irreversible accumulation of damage in the system over its lifespan which finally leads to its failure if not attended to. Degradation is a form of deterioration which leads to loss of value. A degradable state is a state of a component or system whereby that component or system continues to perform to some limits, but which are lower than the acceptable or specified values or continues to perform only some of the required functions (SS-EN 13306, 2010). A state is a condition of a component or system at a particular period of time.

There exist several states of degradation which cause the efficiency of the component or system to decrease, this type of failure is known as soft failure. Hard failure takes place instantaneously (Liao et al; 2005). Degradation manifests in the form of excessive noise, crack, vibration, erosion, change in resistance and magnetic properties of components, high temperature, particulates in lubricants. When the degradable level exceeds a particular limit the system is said to have failed. Failure is the termination of the ability of an item to perform a required function. Functional failure is the inability of an item to meet the required standard. Degradation leads to loss of physical, functional and economic value of the component or asset. Physical degradation is due to the using up or expiration of the useful life of a component or asset. This is caused by wear, and tear, exposure to stress, fatigue, creep, corrosion, erosion and similar factors. This will result in functional obsolescence of the component or system. It will manifest in the form of inefficiencies and inadequacies or low performance of the component or system compared to its normal state. It is inevitably that the component if evaluated in economic terms will have low economic value. If the degradation process is allowed to continue indefinitely the component or system at some point will fail. A degradation model normally contains two important components, namely the degradation processes of the indicators and the relationship between the indicators and the failure events.

Degradation can occur due to aging or stress. It can also occur due to sudden shock or stress.

Degradation and random shocks are dependent. There are two types of dependency between degradation and random shock. Degradation makes the system more vulnerable to random shocks. The shocks accelerate the degradation process. It can be a sudden jump or degradation rate acceleration.

Degradation models can be classified into two groups, namely normal degradation and accelerated degradation models Normal degradation models are used to estimate the degradation obtained at normal operating condition. Accelerated degradation models make inference about degradation and reliability metrics from data obtained under accelerated conditions. Normal degradation can be divided into two groups: degradation models with stress factor and degradation models without stress factors.

Degradation models without stress factor are the ones where the degradation indicator is defined at a fixed stress level. An example is the general degradation path models; Markov models; Continuous –Time stochastic processes: gamma processes and Weiner processes.

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An example of stressed factor is the accumulative damage or shock models.

The general degradation path model is considered to be a regression model with random or fixed coefficients fitted to the degradation range. The path can be linear or non-linear. Most of the degradation paths are non-linear (Gorjian et al; 2009). Markov model is a stochastic model used to describe a sequence of possible events in which the probability of a particular event is dependent only on the state achieved in the previous events. It is used to model systems that change randomly. It assumed that the future status is dependent only on the present event and not in the event that occurred before it.

Continuous- time model is a stochastic process in which the index values assume a continuous set of values, as against the discrete- time process in which the index values take only distinct values. Examples of continuous-time model are the gamma processes, compound Poisson processes and Weiner processes.

Normal degradation models with stress factor discuss the models where the degradation level of the system is a faction of defined stresses .Shock models are usually used to establish the failure and degradation mechanism of the system that are subjected to sudden shocks with random damage magnitudes. The model assumes that the system is 100% reliable in the absence of shock. In general the random shock models include: Cumulative damage/ shock model;  $\hat{\partial}$ - shock model, Extreme shock model.

Cumulative shock model is used to define the system breakdown when the cumulative shock magnitude exceeds the given threshold. Van der Weide et al; (2010) investigated the optimal reliability maintenance policy for a system degrading due to cumulative shock model. The  $\hat{\partial}$ - shock model assumes that the system failure happens if the time interval between consecutive shocks is lower than a predetermined threshold. Lam and Zhang (2004) studied an optimal maintenance policy for system subject to random shocks where the system failure mechanism was modeled by a  $\hat{\partial}$ - shock model.

In extreme shock model the system fails immediately the magnitude of an individual shock moves into a critical region. The accelerated degradation models make inference about reliability of systems at normal condition using the degradation data collected from events with increased environmental conditions. Cirillo and Husler (2009) investigated the extreme shock model introduced by Gut and Husler (2005). A triangle urn process was developed to indirectly analyse the system's load threshold when shock was introduced.

In this research, a preventive maintenance policy for a system under optimal age-based preventive replacement policy is presented and investigated under accelerated system failure condition. The objective is to determine the optimal time interval,  $t_p$  for maintenance (replacement) that will minimize the total expected cost of the system.

## II. MATERIALS AND METHODS

The materials that were used in the research include: A 15 litres storage tank, mounted with agitator, a 745W centrifugal pump, driven by a single phase electric motor, The tank has a ball valve with connecting 2.54cm hoses. A

pressure guage was provided to measure the flow rate during the pumping operation. A view of the accelerated life testing experimental setup used is shown in Plate 1.



Plate 1: Top View of Experimental Setup

An accelerated life testing method was used in this research work. The tank mounted with an agitator initially has only 1.5 litres of water and it is pumped and circulated back to the tank. The volume flow rate of the water returning into the tank was measured and recorded. Thereafter, some specified quantity of sand were introduced into the tank and the agitator put on while pumping operation was carried out and the volumetric flow rate of the return water mixed with sand was measured and recorded. This process was repeated with different quantities of sand at a constant interval and after repeated at random intervals.

## III. PREVENTIVE MAINTENANCE POLICY

An Optimal age-based preventive replacement policy is hereby presented: Under this policy, preventive replacement is performed  $t_p$  hours after continuous operation of the system without failure,  $t_p$  is finite. If there is system failure before  $t_p$  hours elapsing, carry out maintenance (replacement) at the time of failure and reschedule the PM after  $t_p$  operation hours.

In this policy, the system is assumed to be as good as new after PM replacement has been performed. This corresponds to a general overhaul.

Under this policy, preventive replacement is performed  $t_p$  hours after continuous operation of the system without failure,  $t_p$  is finite. If there is system failure before  $t_p$  hours elapsing, carry out maintenance (replacement) at the time of failure and reschedule the PM after  $t_p$  operation hours.

In this policy, the system is assumed to be as good as new after PM replacement has been performed. This corresponds to a general overhaul.



This model determines  $t_p$  that minimizes the total expected cost of PM as well as breakdown maintenance policy per unit time. The model for preventive maintenance is presented in Equation (1):

$$UEC(t_p) = \frac{EC(t_p)}{ECL} \tag{1}$$

where

$EC(t_p)$  is the total expected cost per cycle per the expected cycle length.

$ECL$  is the length of the preventive maintenance.

$EC(t_p)$  is expressed as given in Equation (2):

$$EC(t_p) = C_p R(t_p) + C_f(1 - R(t_p)) \tag{2}$$

where

$C_p$  is the cost of preventive maintenance

$C_f$  is the cost of breakdown or failure maintenance

$R(t_p)$  is the probability the system survives until age  $t_p$

$ECL$  is given in Equation (3):

$$ECL = t_p R(t_p) + M(t_p)[1 - R(t_p)] \tag{3}$$

where

$$M(t_p) = \frac{\int_{-\infty}^{t_p} t f(t) dt}{1 - R(t_p)} \tag{4}$$

Assuming the system has a time-to-failure probability density function,  $f(t)$  and it follows a uniform distribution of  $[0, m]$  hours under accelerated condition. Then  $f(t)$  is given in Equation (5);

$$f(t) = \lim_{t \rightarrow 0} \int_0^t \frac{1}{m} dt = \frac{1}{m} t \quad 0 \leq t < m \tag{5}$$

The systems reliability at time  $t$  is given in Equation (6):

$$R = 1 - f(t) = 1 - \frac{1}{m} t \tag{6}$$

The failure function  $r(t)$  is given in Equation (7) :

$$\frac{f(t)}{1 - f(t)} = \frac{f(t)}{R(t)} = \frac{\frac{1}{m}}{1 - \frac{1}{m}t} = \frac{1}{m - t} \tag{7}$$

$r(t) =$

From Equation (7) Equation 8 is derived as:

$$M(t_p) = \frac{\int_0^{t_p} t f(t) dt}{1 - R(t_p)} = \frac{\int_0^{t_p} \frac{1}{m} dt}{1 - R(t_p)} = \frac{t_p}{2} \tag{8}$$

Substituting Equation (8) into Equation (3) and combining Equation (2), Equation 9 is obtained :

$$EC(t_p) = \frac{C_p R(t_p) + C_f(1 - R(t_p))}{t_p R(t_p) + M(t_p)[1 - R(t_p)]} \tag{9}$$

The magnitude of the shock can be calculated from Equation (10):

$$s = \frac{f_2 - f_1}{f_1} \tag{10}$$

where

$f_1$  is the measured frequency when there is no sand in the system and  $f_2$  is the measured frequency when sand is introduced into the system.  $0 \leq s \leq 1$ .

A Golden section method was employed to search for  $t_p$  the minimizes the expected preventive maintenance cost for equations 9 under this policy of Optimal age-based preventive replacement policy.

The value of  $C_p$  varies from N1000.00 (one thousand naira) to N 1500.00(one thousand five hundred) and  $C_f$  varies N1000.00 (one thousand naira) to N10,000.00 (ten thousand naira). The projection was from one year to six years for age- based policy. The purchase price of the pump was N40,000.00 (forty thousand naira).

#### IV. RESULTS AND DISCUSSION

Table 1 shows the data with no damage and shock conditions.

**Table 1: Data with no damage and shock condition (2.97 hours)**

Time (hrs)	Cum time (hr)	P inlet	P outlet	of Quantity sand	Cum of sand	Flow rate	Vibration	
		$x10^5$ $N/m^2$	$x10^5$ $N/m^2$	(kg)	(kg)	$m^3/hr$	Hz	No. of failures
0.33	0.33	0.5	0.5	0	0	120	51	
0.33	0.66	0.5	0.5	0	0	120	51	
0.33	0.99	0.5	0.5	0	0	120	51	0
0.33	1.32	0.5	0.5	0	0	120	51	
0.33	1.65	0.45	0.45	0	0	120	51	
0.33	1.98	0.45	0.45	0	0	120	51	
0.33	2.31	0.45	0.45	0	0	120	51	
0.33	2.64	0.45	0.45	0	0	120	51	
0.33	2.97	0.45	0.45	0	0	120	51	

In Table 2 data with cumulative damage and random shock at 1.08 hours is presented.

**Table 2: Data with cumulative damage degradation and random shock (1.08 hours)**

Time (hrs)	Cum time (hr)	P inlet $x10^5$ $N/m^2$	P outlet $x10^5$ $N/m^2$	Quantity of sand (kg)	Cum of sand	Shock Magnitude (kg)	Flow rate $m^3/hr$	Vibration Hz	No. of failures
0.33	0.33	0.35	0.35	0.0010	0.0010	0.18	120	60.1	2
0.25	0.58	0.25	0.25	0.0020	0.0030	0.18	120	60.0	
0.17	0.75	0.25	0.20	0.0030	0.0060	0.18	112	60.0	
0.17	0.92	0.15	0.12	0.0040	0.0100	0.24	112	63.2	
.08	1.00	0.10	0.05	0.0050	0.015	0.24	112	63.3	
0.08	1.08	0.10	0.50	0.0060	0.0210	0.24	112	63.3	

Similarly data for cumulative damage degradation and random shocks in Tables 3, 4, 5, 6, 7, 8, 9, and 10 at 1.00hrs, 1.41 hours, 1.08 hours, 1.08 hours, 0.84 hours, 1.98 hours, 1.98 hours

**Table 3: Data with cumulative damage degradation and random shock (1.00 hours)**

Time (hrs)	Cum time (hr)	P inlet $x10^5$ $N/m^2$	P outlet $x10^5$ $N/m^2$	Quantity of sand (kg)	Cum of sand (kg)	Shock Magnitude	Flow rate $m^3/hr$	Vibration Hz	No. of failures
0.25	0.25	0.25	0.25	0.003	0.003	0.27	108	65.0	2
0.25	0.50	0.20	0.45	0.005	0.008	0.27	108	65.0	
0.17	0.67	0.15	0.60	0.007	0.015	0.28	100	65.4	
0.17	0.84	0.10	0.70	0.009	0.024	0.29	100	65.9	
0.08	0.92	0.00	0.70	0.01	0.035	0.29	100	65.9	
0.08	1.00	0.00	0.70	0.01	0.038	0.31	96	67.0	



**Table 4: Data with cumulative damage degradation and random shock (1.41 hours)**

Time (hrs)	Cum time (hr)	P inlet $\times 10^5$ $N/m^2$	P outlet $\times 10^5$ $N/m^2$	Quantity of sand (kg)	Cum of Sand (kg)	Shock Magnitude	Flow rate $m^3/hr$	Vibration Hz	No. of failures
0.33	0.33	0.25	0.22	0.0020	0.002	0.31	96	67.0	3
0.33	0.66	0.20	0.18	0.004	0.006	0.32	96	67.2	
0.25	0.91	0.10	0.08	0.006	0.012	0.32	96	67.5	
0.25	1.16	0.05	0.03	0.008	0.020	0.33	96	67.9	
0.17	1.33	0.05	0.00	0.010	0.030	0.37	88	70.0	
0.08	1.41	0.00	0.00	0.012	0.042	0.37	88	70.0	

**Table 5: Data with cumulative damage degradation and random shock (1.08 hours)**

Time (hrs)	Cum time (hr)	P inlet $\times 10^5$ $N/m^2$	P outlet $\times 10^5$ $N/m^2$	Quantity of sand (kg)	Cum of sand (kg)	Shock Magnitude	Flow rate $m^3/hr$	Vibration Hz
0.25	0.25	0.25	0.20	0.001	0.001	0.33	88	70.0
0.50	0.75	0.20	0.20	0.003	0.004	0.39	88	71.0
0.75	1.50	0.15	0.12	0.005	0.009	0.40	88	71.4
0.92	2.42	0.10	0.05	0.007	0.016	0.40	84	71.4
1.09	3.51	0.05	0.00	0.009	0.025	0.41	80	72.0
1.17	4.68	9.99	0.00	0.011	0.036	0.41	80	72

**Table 6: Data with cumulative damage degradation and random shock (1.08 hours)**

Time (hrs)	Cum time (hr)	P inlet $\times 10^5$ $N/m^2$	P outlet $\times 10^5$ $N/m^2$	Quantity of sand (kg)	Cum of sand (kg)	Shock Magnitude	Flow rate $m^3/hr$	Vibration Hz	No. of failures
0.25	0.25	0.2	0.2	0.002	0.002	0.47	72	75.1	
0.17	0.42	0.2	0.15	0.003	0.005	0.48	72	75.3	
0.17	0.59	0.15	0.15	0.004	0.009	0.48	60	75.3	
0.17	0.76	0.1	0.05	0.005	0.014	0.48	60	75.7	
0.08	0.84	0.05	0	0.006	0.02	0.49	60	76	
0.08	0.92	0	0	0.007	0.027	0.49	60	76	

**Table 7: Data with cumulative damage degradation and random shock (0.84 hours)**

Time (hrs)	Cum time (hr)	P inlet $\times 10^5$ $N/m^2$	P outlet $\times 10^5$ $N/m^2$	Quantity of sand (kg)	Cum of sand (kg)	Shock Magnitude	Flow rate $m^3/hr$	Vibration Hz	No. of failures
0.17	0.17	0.15	0.15	0.4	0.4	0.51	60	77	2
0.17	0.34	0.15	0.15	0.8	1.2	0.57	60	80	
0.17	0.51	0.10	0.10	1.2	2.4	0.61	60	82	
0.17	0.68	0.00	0.00	1.6	4.0	0.63	0	83	
0.08	0.76	0.00	0.00	2.0	6.0	0.67	0	85	
0.08	0.84	0.00	0.00	2.4	8.4	$\theta$	0	-	

(-): The dash indicates that the pump stopped working or seized.  $\theta$  indicates a very high shock value

**Table 8: Data with cumulative damage degradation and random shock (1.98 hours)**

Time (hrs)	Cum time (hr)	P inlet $\times 10^5$ $N/m^2$	P outlet $\times 10^5$ $N/m^2$	Quantity of sand (kg)	Cum of sand (kg)	Shock Magnitude	Flow rate $m^3/hr$	Vibration Hz
0.33	0.33	0.35	0.30	0.001	0.001	0.18	120	60.1
0.33	0.66	0.30	0.27	0.002	0.003	0.19	120	60.9
0.33	0.99	0.30	0.25	0.003	0.006	0.24	112	63.0
0.33	1.32	0.25	0.20	0.004	0.010	0.25	112	63.9
0.33	1.65	0.25	0.15	0.005	0.015	0.26	104	64.3
0.33	1.98	0.15	0.10	0.006	0.021	0.27	104	64.9

**Table 9: Data with cumulative damage degradation and random shock (1.98 hours)**

Time (hrs)	Cum time (hr)	P inlet $\times 10^5$ $N/m^2$	P outlet $\times 10^5$ $N/m^2$	Quantity of sand (kg)	Cum of sand (kg)	Shock Magnitude	Flow rate $m^3/hr$	Vibration Hz	No. of failures
0.33	0.33	0.30	0.25	0.001	0.001	0.24	112	63.0	2
0.33	0.66	0.30	0.20	0.002	0.003	0.25	112	63.6	
0.33	0.99	0.25	0.17	0.003	0.006	0.26	104	64.2	
0.33	1.32	0.25	0.15	0.004	0.010	0.27	104	65.0	
0.33	1.65	0.20	0.10	0.005	0.015	0.28	96	65.5	
0.33	1.98	0.10	0.00	0.006	0.021	0.37	96	70	

**Table 10: Data with cumulative damage degradation and random shock (1.98 hours)**

Time (hrs)	Cum time (hr)	P inlet $\times 10^5$ $N/m^2$	P outlet $\times 10^5$ $N/m^2$	Quantity of sand (kg)	Cum of sand (kg)	Shock Magnitude	Flow rate $m^3/hr$	Vibration Hz
0.33	0.33	0.25	0.21	0.001	0.001	0.31	96	67.0
0.33	0.66	0.20	0.13	0.002	0.003	0.33	96	68.0
0.33	0.99	0.20	0.10	0.003	0.010	0.35	88	68.8
0.33	1.32	0.15	0.08	0.004	0.006	0.36	80	69.6
0.33	1.65	0.10	0.05	0.005	0.015	0.39	80	70.8
0.33	1.98	0.05	0.00	0.006	0.021	0.40	72	71.6

**Table 11: Cumulative Data set for centrifugal Pump**

S/N	TBF	No. of failure	Cum Time (TBF)	Cum Failure	Down time (hrs)	Replacement	Maintenance Time (hrs)	Remarks
1	2.97	2	2.97	2	0	0	0	WTS
2	1.08	2	4.05	4	3	0	1	WS
3	1.00	2	5.05	6	1.15	0	1	WS
4	1.41	3	5.46	9	6	1	1	WS
5	1.17	2	6.63	11	0.3	0	0.5	WS
6	0.92	2	7.55	13	1.00	0	1	WS
7	0.84	2	8.39	15	0	0	0	WS

WTS: Without sand; WS: With sand

**Table 12: Summary of Random Readings**

S/N	Time before Failure TBF (hrs)	Number of Failures (N)	Failure rate ( $\lambda$ ) per hour
1	1.08	2	0.54
2	1.00	2	0.50
3	1.41	3	0.38
4	1.17	2	0.59
5	0.92	2	0.45
6	0.84	2	0.42

**V. DISCUSSION OF RESULTS**

From Table 1, the time interval of measuring the flow parameters was 0.33 hours. The inlet and outlet pressures were the same and remained constant. The volume flow rate of  $120m^3/hr$  was constant at a frequency of vibration of  $51Hz$ .

From Table 2 to Table 7 random time interval was used where a specified quantity of sharp sand was introduced into the tank and pumping was initiated and various parameters measured.

In Table 2, when 0.0010kg of sharp sand was introduced into the system within 0.33 hours, the inlet pressure was  $0.35 \times 10^5 N/m^2$  and the outlet pressure was of the same value, while the volume flow rate stood at  $120 \times 10^5 N/m^2$  and the vibration of the pump increased from 51Hz to 60.1 Hz. This resulted in a shock of 0.18. At the inlet and outlet pressures of  $0.15 \times 10^5 N/m^2$  the shock was 0.24 and the vibration of the pump was 63.2 Hz. At the inlet and outlet pressures of  $0.10 \times 10^5 N/m^2$  the shock was still 0.24 and the vibration of the pump increased 63.3 Hz. The system failed two times. Similar trends were obtained in Table 3 with random addition of sharp sand at the span of 1.00 hour. The highest flow rate was  $108 \times 10^5 N/m^2$  at a vibration frequency of 65Hz and the lowest flow rate of

$96 \times 10^5 N/m^2$  and vibration frequency of 67Hz. In Table 4 the highest shock magnitude was 0.37 at a vibration frequency of 70Hz at a span of 1.41hours with 3 failures.

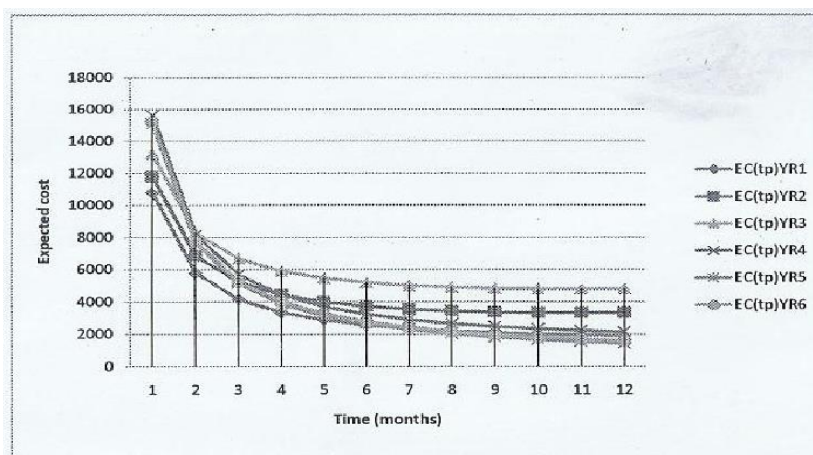
In Table 5 and Table 6 respectively the lowest flow rate was  $80m^3/hr$  at a vibration frequency of 72Hz and  $60m^3/hr$  at a vibration frequency of 76Hz and the spans of 1.08 hours and at a shock magnitude of 0.41 and 0.49 respectively.

Similar results are present for pressures, quantity of sand, shock magnitude, vibration frequency and number of failures for 1.98 hours duration under uniform time variation (steady reading).

Table 11 presents cumulative data set for the centrifugal pump without sand (WTS) and with sand (WT).

The summary of random readings for each interval of pump operation is presented in Table 12. The failure rate under each condition is also presented. The failure rate data is used in the computation of the reliability of the pump from one to twelve months as required in equation 9 during the search for  $t_p$  (the optimum interval).

The failure rate data was used in the computation of the reliability of the pump from one to twelve months as required in equation 8 during the search for  $t_p$  (the optimum interval). From Figure 1, the total expected cost varies from N1700.00 (one thousand seven hundred naira) to N16,000.00 (sixteen thousand naira).



**Fig 1: Expected Cost Versus Time**

When the pump operates under a normal working condition, the shock magnitude is small and hence the failure rate is small, thus increasing the reliability of the

system.

A high reliability will reduce the total expected cost of the system. It is a figure of merit and hence desirable for the system.

## VI. CONCLUSION

The degradation or cumulative damage for a system operating under a normal condition is different from the one under random shocks conditions. The quantity of sharp sand introduced into the system affects the inlet and outlet pressures. This is reflected in the magnitude of the shock and vibration frequency. A system operating under shock conditions will have many number of failures and this will affect the reliability of the system and hence the total expected cost of the system will increase compared to a system operating under a normal operating condition. A high reliability will reduce the total expected cost of the system. It is a figure of merit and hence desirable for the system.

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