Numerical Solution of Integral Equation by using New Modified Adomian Decomposition Method and Newton Raphson Methods

Najmuddin Ahmad, Balmukund Singh

Abstract: ***In this paper, we discuss the numerical solution of Adomian decomposition method and Taylor's expansion method in Volterra linear integral equation. And we apply modified Adomian decomposition method and Newton Raphson method in Volterra nonlinear integral equation with the help of example and estimated an error in MATLAB 13 versions.***

Keywords: Adomian Decomposition Method, Newton Raphson Method, Volterra Integral Equation, Taylor Expansion Methods.

I. INTRODUCTION

Many problems from mathematics and physics in other disciplines of integral equation. Several methods have been proposed for exact solution and approximate solution with the help of linear and nonlinear integral equation. One of the issues in find the solutions obtained to choices of approaches are suggest that analytical solution of linear and nonlinear integral equations [1]. In Volterra integral equation arising in various field of research, i.e diffusion problem, heat transfer problem, potential theory, Dirichlet problem, electrostatics problem, algebraic equation, differential equation etc. The Adomian decomposition method was first introduced and developed by Gorge Adomian and it has been proved to be reliable and effcient for a wide class of differential and integral equations of linear and nonlinear models. The Adomian Decomposition method are used in Volterra & Fredholm linear and nonlinear integral equation in He's polynomials. Numerical solution of integral equations by using Galerkin methods with Hermite, Chebyshev and orthogonal polynomial [2-6]. Modifed Adomian decomposition method has been applied for a long time to solve Fredholm integral equation. Recently, another modifcation of decomposition method was proposed by Wazwaz & El-Sayed . In the new modifcation, the process of dividing into two components was replaced by a Taylor series of infnite components. I recent decades, is a great deal of interest in Adomian Decomposition method [7-10]. However, the modified method was establish and based on the assumption of the two parts f_1 & f_2. A new modification Adomian Decomposition method for volterra linear and nonlinear integral equation of second kinds and this method have been used with Taylor expansion methods [11-13].

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II. PRELIMINARIES

Definition2.1. The general form of linear Volterra integral equation is defined as follows:

$$v(x).u(x) = f(x) + \lambda \int_a^x k(x,t)u(t)\,dt \quad (1)$$

Where a is constant, $f(x), v(x)$ and $k(x,t)$ are known functions, while $u(x)$ is unknown function, λ is non-zero real or complex parameter.

Definition2.2. Linear volterra integral equation (1) $v(x) = 1$. Then equation (1) is called volterra linear integral equation of second kind.

$$u(x) = f(x) + \lambda \int_a^x k(x,t)u(t)dt \quad (2)$$

Definition2.3. The nonlinear volterra integral equation of second kind is defined as follows:

$$u(x) = f(x) + \lambda \int_a^x k(x,t)F(u(t))dt \quad (3)$$

Where k(x; t) is the kernel of the integral equation, f(x) and k(x; t) are known functions, and u(x) is the unknown function that is to be determined.

III. SIGNIFICANCE OF THE STUDY

Integral equations are frequently easily too solved and more effective in other field of numerical techniques. It is not required to boundary and supplementary condition in this study. This paper we work in new modified Adomian Decomposition method with the help of Taylor expansion methods and estimated errors. This study more significance [14-15]:

(1) Integral equations are very successful and importance of differential equations.

(2) More researchers are friendly suggested and interest in rest parts of integral equations.

(3) It is given to extended the concept of integral equation in many interdisciplinary area are instead and using in different numerical techniques.

(4) Integral equation are very useful and important role of research Adomian Decomposition methods in different numerical techniques.

IV. ADOMIAN DECOMPOSITION FOR LINEAR VOLTERRA INTEGRAL EQUATION

The Adomian Decomposition methods are consists in unknown function *u(x)* of the volterra integral equation into a sum of an infinite series

$$u(x) = \sum_0^\infty u_n(x)$$

Where components $u_n(x), n \geq 0$ are recursive manner to be determined. The decomposition method concerns and finding components $u_0, u_1, u_2, \dots\dots\dots$.

The volterra integral equations are established in recurrence relations.

$$u(x) = f(x) + \lambda \int_0^x k(x,t)u(t)dt \quad (4)$$

We obtain equation (4) in series form

$$\sum_{n=0}^\infty u_n(x) = f(x) + \lambda \int_0^x k(x,t)\left(\sum_{n=0}^\infty u_n(t)\right) dt \quad (5)$$

Consequently, the component $u_n(t), n \geq 1$ of the unknown function *u(x)* has been completely determined in recurresive relations.

$$u_0 = f(x)$$

$$u_{n+1}(x) = \lambda \int_0^x k(x,t)u_n(t)\,dt, n \geq 0$$

And in the equivalent relation in above equations

$$u_0 = f(x)$$

$$u_1(x) = \lambda \int_0^x k(x,t)u_0(t)\,dt$$

$$u_2(x) = \lambda \int_0^x k(x,t)u_1(t)\,dt$$

$$u_3(x) = \lambda \int_0^x k(x,t)u_2(t)\,dt$$

V.1. Expasion of New Modified Adomian Decomposition method

The Adomian Decomposition method are apply in integral equation and we procedure following Volterra integral equation of second kinds

$$u(x) = f(x) + \lambda \int_a^x k(x,t)\{L(u(t)) + N(u(t))\}dt, \quad \lambda \neq 0 \quad (6)$$

Equation (6), *L(u(t))* and *N(u(t))* are linear and nonlinear operators of *u(x)*, kernel *k(x,t)* and *f(x)* is a real valued function.

A unknown function *u(x)* is determine in Adomian Decomposition method in series solution of (6) defined by

$$u(x) = \sum_{i=0}^\infty u_i(x) \quad (7)$$

Moreover, *N(u(x))* be the nonlinear terms in form of decomposition series

$$N(u(x)) = \sum_{n=0}^\infty A_n(x) \quad (8)$$

Where, A_n is an Adomian polynomial & it is evaluated in following form

$$A_n = \frac{1}{n!}\frac{d^n}{d\lambda^n}\left[F\left(\sum_{i=0}^n \lambda^i u_i\right)\right]_{\lambda=0}, \quad n = 0, 1, 2, \dots\dots\dots\dots \quad (9)$$

Evaluating (7) with the help of (8) and (9) then we get

$$\sum_{n=0}^\infty u_n(x) = f(x) + \lambda \int_a^x k(x,t)\left\{L\left(\sum_{n=0}^\infty u_n(t)\right) + \sum_{n=0}^\infty A_n(t)\right\}dt, \quad \lambda \neq 0 \quad (10)$$

V.2. Some New Modified Adomian Decomposition Method

Modified Decomposition Method 1: The modification is based on assumption *f(x)* into two parts $f_1(x)$ and $f_2(x)$. Then this assumption is applied we get

$$f(x) = f_1(x) + f_2(x) \quad (11)$$

Accordingly, we proposed component $u_0(x)$& $u_1(x)$. This suggestion is assumed in $f_1(x)$ in zero components $u_0(x)$. And we evaluating in remaining part $f_2(x)$ in terms of $u_1(x)$. Then equivalence relation in recursive relation

$$u_0(x) = f_1(x)$$

$$u_1(x) = f_2(x) + \lambda \int_a^x k(x,t)\{L(u_0(x) + A_0(x)\}dt$$

$$u_{n+1}(x) = \lambda \int_a^x k(x,t)\{L(u_n(x) + A_n(x)\}dt, \quad n \geq 1. \quad (12)$$

It is important noted the point in (11) is slight find $u_0(x)$ and other variation iteration are find in nonlinear integral equation.

Modified Decomposition Method 2: This Modified assumption that the function *f(x)* is express in the Taylor series and it is expanded in the following form

$$f(x) = \sum_{n=0}^\infty f_n(x)$$

This method can be genralised in following relation and it is obtained new recursive relation

$$u_0(x) = f_0(x)$$

$$u_1(x) = f_1(x) + \lambda \int_a^x k(x,t)\{L(u_0(x) + A_0(x)\}dt$$

$$u_{n+1}(x) = f_{n+1}(x) + \lambda \int_a^x k(x,t)\{L(u_n(x) + A_n(x)\}dt, \quad n \geq 1. \quad (14)$$

Having to be determined in the terms of $u_0(x), u_1(x), u_2(x), u_3(x), \dots\dots\dots$ is a solution $u(x)$ can we obtained and using (7).

New Modified Decomposition method 3:
Adomian polynomials are effective and dependent in the chosen of $f_1(x)$ and $f_2(x)$ in the famous and computational works. If in MADM 1 and MADM 2, and sometime one of the difficulties are seen in new type new modification methods, we obtain first two term

$$u_0(x) = f(x) \quad (15)$$

$$u_1(x) = \lambda \int_a^x k(x,t)\{L(u_0(x) + A_0(x)\}dt \quad (16)$$

SADM are same results and $u_1(x)$ has been sometime complicated to be continue or in analytically impossibility. Hence we are suggest that $u_1(x)$ can be suggest in Taylor series form

$u_1 = \sum_{i=0}^{\infty} u_{1i}(x)$ (17)

In the other SADM can we setting

$u_2(x) = \lambda \int_a^x k(x,t)\{L(u_1(x) + A_1(x)\}\, dt$ (18)

Where we are evaluated in (11) of Adomian Decomposition method A_1 with the help of u_0 and u_1 in (15) and (16), Also we suggest that in Taylor series expansion are act in $u_2(x)$, i.e

$u_2 = \sum_{i=0}^{\infty} u_{2i}(x)$ (19)

Then we are setting in Adomian Decomposition methods in the form of higher degree terms

$u_{n+1}(x) = \lambda \int_a^x k(x,t)\{L(u_n(x) + A_n(x)\}\, dt, \quad n \geq 3$ (20)

We are applying Taylor expansions methods are given by

$u_{n+1} = \sum_{i=0}^{\infty} u_{(n+1)i}(x), \quad n \geq 3$ (21)

This study, we have used linear and nonlinear volterra integral equations applicability and validity in new modified Adomian Decomposition methods. This method is used and discussed exact solution and approximate solution with the help of examples.

V. NONLINEAR INTEGRAL EQUATION

1. Adomian Decomposition for nonlinear Volterra Integral Equation:

The nonlinear volterra integral equation of second kinds is defined as follows:

$u(x) = f(x) + \lambda \int_a^x k(x,t) F(u(t))dt$ (22)

Adomian decomposition method defines the unknown function $u(x)$ by an infinite series

$u(x) = \sum_{n=0}^{\infty} u_n(x)$

Where the components $u_n(x)$ are usually determined in nonlinear operators $F(u)$ can be decomposed as infinite series polynomials.

$F(u) = \sum_{n=0}^{\infty} A_n$

Where A_n are called Adomian polynomials of $u_0, u_1, u_2, \dots\dots\dots u_n$. If A_n if defined as follows.

$A_n = \frac{1}{n!}\frac{d^n}{d\lambda^n}[F(\sum_{i=0}^{n} \lambda^i u_i)]_{\lambda=0}, \quad n = 0,1,2,\dots\dots\dots$ (23)

Application of Adomian Decomposition Method to (6) and using equation (7)

Where

$u_0(x) = f(x)$

$u_n(x) = \int_0^x k(x,t) A_{n-1}\, dt, \quad n \geq 1.$

In this observation firstly find A_0 depends only u_0, A_1 depends u_0 and u_1, A_2 depends on u_0, u_1 and u_2 in second and third degree nonlinear volterra integral equation.

2. Newton Raphson for nonlinear integral equation

Consider the following nonlinear Volterra integral equation of second kinds

$u(x) = f(x) + \lambda \int_a^x k(x,t) F(u(t))dt$

Adomian decomposition method defines the unknown function $u(x)$ by an infinite series

$u(x) = \sum_{n=0}^{\infty} u_n(x)$

Where the components $u_n(x)$ are usually determined in nonlinear operators $F(u)$ can be decomposed as infinite series polynomials.

$F(u(x)) = \sum_{n=0}^{\infty} A_n(x)$

Where $A_n(x)$ are called Adomian polynomials of $u_0, u_1, u_2, \dots\dots\dots u_n$. If $A_n(x)$ if defined as follows.

$A_n = \frac{1}{n!}\frac{d^n}{d\lambda^n}\left[F\left(\sum_{i=0}^{n} \lambda^i \left(u_i - \frac{F(u_i)}{F'(u_i)}\right)\right)\right]_{\lambda=0}, \quad n = 0,1,2,\dots\dots\dots$ (24)

NUMERICAL EXAMPLES

Example1. Consider the following Volterra linear integral equation of second kinds.

$u(x) = \cos x + \sin x - \int_0^x u(t)dt$

Exact Solution $= \cos x$

Solution .In this example SADM and MADM2 are effective and exact solution and MADM1 is easily finds in exact solutions. Then we can apply in new Modified methods

$u_0(x) = \cos x + \sin x$

$u_1(x) = -\int_0^x u_0(t)\, dt = -\int_0^x (\cos t + \sin t)dt = \cos x - \sin x - 1$

$u_2(x) = -\int_0^x u_1(t)\, dt = 1 + x - \sin x - \cos x$

$u_3(x) = -\int_0^x u_2(t)\, dt = 1 - x - \frac{x^2}{2!} + \sin x - \cos x$

$u_4(x) = -\int_0^x u_3(t)\, dt = -1 - x + \frac{x^2}{2!} + \frac{x^3}{3!} + \sin x + \cos x$

Thus, the approximate solution becomes

$u(x) = u_0 + u_1 + u_2 + u_3 + u_4 + \dots\dots\dots$

$u(x) = \cos x + \sin x + (-x + \frac{x^3}{3!} - \frac{x^5}{5!} + \frac{x^7}{7!} - \dots\dots\dots)$ (25)

Taylor's expansion methods in (25) then we get

$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots\dots\dots$ (26)

Applying equation (26) in (25) then solution we get

$u(x) = \cos x$

Example2. Consider the following Volterra linear integral equation of second kinds.

$u(x) = \sin x + e - e^{\cos x} - \int_0^x e^{\cos t} u(t)dt$

Exact Solution $u(x) = \sin x$

Solution: We can apply in new Modified methods

$u_0(x) = \sin x + e - e^{\cos x}$
$u_1(x) = -\int_0^x e^{\cos t} u_0(t)dt$
We set the Taylors expansion in form $u_0,\ u_1, u_3, u_4, \dots\dots\dots$
$u_1(x) = -\frac{1}{2}ex^2 - \frac{1}{6}e^2x^3 + \frac{1}{6}ex^4 + \frac{1}{12}e^2x^5 - \frac{31}{720}ex^6 -$
$u_2(x) = -\int_0^x e^{\cos t} u_1(t)dt$
$u_2(x) = \frac{1}{6}e^2x^3 + \frac{1}{24}e^3x^4 - \frac{1}{12}e^2x^5 - \frac{1}{36}e^3x^6 + \dots\dots\dots$
$u_3(x) = -\int_0^x e^{\cos t} u_2(t)\,dt$
$u_3(x) = -\frac{1}{24}e^3x^4 - \frac{1}{120}e^4x^5 + \frac{1}{36}e^3x^6 + \dots\dots\dots$
$u_4(x) = -\int_0^x e^{\cos t} u_3(t)\,dt$
$u_4(x) = \frac{1}{120}e^4x^5 + \frac{1}{720}e^5x^6 - \dots\dots\dots$
$u_5(x) = -\int_0^x e^{\cos t} u_4(t)\,dt$
$u_5(x) = -\frac{1}{720}e^5x^6 - \dots\dots\dots$
……………………………
……………………………
Thus, solution in the series form can be obtain
$u(x) = \sin x + e - e^{\cos x} + e\left(-\frac{1}{2}x^2 + \frac{1}{6}x^4 - \frac{31}{720}x^6 + \dots\dots\dots\right)$

The above equation apply in Taylor expansion then we get
$e - e^{\cos x} = e\left(\frac{1}{2}x^2 - \frac{1}{6}x^4 + \frac{31}{720}x^6 - \dots\dots\dots\right)$
Then these solutions in the closed form
$u(x) = \sin x$

Example3. Consider the following Volterra nonlinear integral equation of second kinds.

$$u(x) = x - \frac{1}{4}x^4 + \int_0^x tu^2(t)\,dt \qquad (27)$$

Exact Solution $u(x) = x$.
Solution.

Method 1. Adomian Decomposition for nonlinear Volterra Integral Equation:

In the following suggestion by using Modified Adomian Decomposition methods in two parts $f_1(x) = x$ and $f_2(x) = -\frac{1}{4}x^4$ then solution are finds exactly. Then we can use following new modified Adomian Decomposition methods. In this equation we explain Adomian polynomials of the non-linear terms $u^2(t)$ in equation (8) are in the form of non-linear integral equation and putting *i=0* in equation (23) becomes

A. A. $A_0 =$

B. A. $A_1 = 2u$

If again calculation are explain in MATLAB 13 versions software in few terms are given bellow

C. A. $A_2 = 2(u_0u_2) +$
$A_3 = 2(u_0u_3 + u_1u_2)$
$A_4 = 2(u_1u_3 + u_0u_4) + u_2^2$

Using the technique of Adomian polynomials in (23), then we obtain
$u_0(x) = x - \frac{1}{4}x^4$
$u_1(x) = -\int_0^x tA_0\,dt = -\int_0^x tu_0^2(t)\,dt$
$u_1(x) = -\frac{x^4}{4} + \frac{x^7}{14} - \frac{x^{10}}{160}$
$u_2(x) = -\int_0^x tA_1 dt = -\int_0^x t.2u_0u_1\,dt$
$u_2(x) = \frac{x^7}{14} - \frac{3}{112}x^{10} + \frac{27}{7280}x^{13} - \frac{1}{5120}x^{16}$
Subsequently, the approximate solution becomes
$u(x) = u_0(x) + u_1(x) + u_2(x) + u_3(x) + \dots\dots\dots$
$u(x) = x - \frac{1}{2}x^4 + \frac{1}{7}x^7 - \frac{5}{56}x^{10} + \frac{27}{7280}x^{13} - \frac{1}{5120}x^{16}$

Method2. Newton Raphson Nonlinear Volterra Integral Equation

The nonlinear terms $F(u(x)) = u^2(x)$ is decomposed using the formula given by (24) and these terms by used modified Adomian Decomposition polynomials are as followes:
$A_0 = \left(\frac{1}{2}\right)^2 u_0^2.$
$A_1 = \left(\frac{1}{2}\right)^2 (u_0u_1).$
$A_2 = \left(\frac{1}{2}\right)^2 (2u_0u_2 + u_1^2).$
$A_3 = \left(\frac{1}{2}\right)^2 (2u_0u_3 + 2u_1u_2).$
Using the technique of Adomian polynomials in (24), then we obtain
$u_0(x) = x - \frac{1}{4}x^4$
$u_1(x) = -\int_0^x tA_0\,dt = -\int_0^x t.\left(\frac{1}{2}\right)^2 u_0^2(t)dt$
$u_1(x) = -\frac{1}{16}x^4 + \frac{1}{56}x^7 - \frac{1}{640}x^{10}$
$u_2(x) = -\int_0^x t.A_1\,dt = -\int_0^x t.\left(\frac{1}{2}\right)^2 (u_0u_1)dt$
$u_2(x) = \frac{1}{448}x^7 - \frac{3}{3584}x^{10} + \frac{27}{232960}x^{13} - \frac{1}{174080}x^{17}$
Subsequently, the approximate solution becomes
$u(x) = u_0(x) + u_1(x) + u_2(x) + u_3(x) + \dots\dots\dots$
$u(x) = x - \frac{5}{16}x^4 + \frac{9}{448}x^7 - \frac{43}{17920}x^{10} + \frac{27}{232960}x^{13} - \frac{1}{38400}x^{16} + \dots\dots\dots$

Example4. Consider the following Volterra nonlinear integral equation of second kinds.

$$u(x) = x^2 + \frac{1}{16}x^6 - \frac{1}{2}\int_0^x tu^2(t)\,dt \qquad (28)$$

Exact Solution $u(x) = x^2$
Solution.
Method 1. Adomian Decomposition for nonlinear Volterra Integral Equation: Using the technique of Adomian polynomials in (23), then we obtain

D. A. $A_0 =$

E. A. $A_1 = 2u$

If again calculation are explain in MATLAB 13 versions software in few terms are given bellow

F. A. $A_2 = 2(u_0u_2) +$
$A_3 = 2(u_0u_3 + u_1u_2)$
$A_4 = 2(u_1u_3 + u_0u_4) + u_2^2$

Using these techniques in Adomian polynomials, we obtain u_0 and findA_0, Again we finds u_1 with the help of A_0. Then the similar prosses we can obtain
$u_0(x) = x^2 + \frac{1}{12}x^6$

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$$u_1(x) = -\frac{1}{2}\int_0^x tA_0\, dt = -\frac{1}{2}\int_0^x t u_0^2(t)\, dt$$
$$u_1(x) = -\frac{1}{12}x^6 - \frac{1}{120}x^{10} - \frac{1}{4032}x^{14}$$
$$u_2(x) = -\frac{1}{2}\int_0^x tA_1\, dt = -\frac{1}{2}\int_0^x t.2u_0u_1\, dt$$
$$u_2(x) = \frac{1}{120}x^{10} + \frac{11}{10080}x^{14} + \frac{19}{362880}x^{18} + \frac{1}{1064448}x^{22}$$

Subsequently, the approximate solution becomes
$$u(x) = u_0(x) + u_1(x) + u_2(x) + u_3(x) + \dots\dots\dots$$
$$u(x) = x^2 + \frac{17}{20160}x^{14} + \frac{19}{362880}x^{18} + \frac{1}{1064448}x^{22} + \dots\dots\dots\dots$$

Method2. Newton Raphson Nonlinear Volterra Integral Equation

Using the technique of Adomian polynomials in (24), then we obtain.The nonlinear terms $F(u(x)) = u^2(x)$ is decomposed using the formula given by (24) and these terms by used modified Adomian Decomposition polynomials are as followes:

$$A_0 = \left(\frac{1}{2}\right)^2 u_0^2.$$
$$A_1 = \left(\frac{1}{2}\right)^2 (u_0u_1).$$
$$A_2 = \left(\frac{1}{2}\right)^2 (2u_0u_2 + u_1^2).$$
$$A_3 = \left(\frac{1}{2}\right)^2 (2u_0u_3 + 2u_1u_2).$$

Using these techniques in Adomian polynomials of Newton Raphson nonlinear Volterra integral equation, we obtain u_0 and findA_0, Again we finds u_1 with the help of A_0. Then the similar prosses we can obtain

$$u_0(x) = x^2 + \frac{1}{12}x^6$$
$$u_1(x) = -\frac{1}{2}\int_0^x tA_0\, dt = -\frac{1}{2}\int_0^x t.\left(\frac{1}{2}\right)^2 u_0^2(t)\, dt$$
$$u_1(x) = -\frac{1}{48}x^6 - \frac{1}{480}x^{10} - \frac{1}{16128}x^{14}$$
$$u_2(x) = -\frac{1}{2}\int_0^x tA_1\, dt = -\frac{1}{2}\int_0^x t.\left(\frac{1}{2}\right)^2 u_0u_1\, dt$$
$$u_2(x) = \frac{1}{3840}x^{10} + \frac{11}{322560}x^{14} + \frac{19}{11612160}x^{18} + \frac{1}{34083840}x^{22}$$

Subsequently, the approximate solution becomes
$$u(x) = u_0(x) + u_1(x) + u_2(x) + u_3(x) + \dots\dots\dots$$

I: Name of the Table that justify the values

Table1. Example 3 comparison between Modified Adomian decomposition method and Newton Raphson method.

x	Exact Solution	Approximate Solution (M.A.D.M)	Absolute Errors	Approximate Solution (N.R.M)	Absolute Errors
0.0	0.0	0.00000000000	0.00000000000	0.00000000000	0.000000000000
0.1	0.1	0.09999500143	0.00000499857	0.09996875201	0.000031247990
0.2	0.2	0.19920181940	0.00079818060	0.19950025690	0.000499747431
0.3	0.3	0.29598071620	0.00401928380	0.29747312940	0.002526870600
0.4	0.4	0.38742471970	0.01257528030	0.39203266340	0.007967336600
0.5	0.5	0.46977932810	0.03022067190	0.48062336800	0.019376632000
0.6	0.6	0.53866399800	0.06133600200	0.56004800750	0.039951992500
0.7	0.7	0.58922808460	0.11077191150	0.62655756690	0.073442433100
0.8	0.8	0.61577072910	0.18422927090	0.67596101710	0.124038982900
0.9	0.9	0.61005265810	0.28994734190	0.70376535500	0.196234645000
1.0	1.0	0.55708490720	0.44291509280	0.70527959020	0.294720409800

Table2. Example 4 comparison between Modified Adomian decomposition method and Newton Raphson method.

x	Exact Solution	Approximate Solution (M.A.D.M)	Absolute Errors	Approximate Solution(N.R.M)	Absolute Errors
0.0	0.00	0.0000000000000	0.000000000000	0.0000000000	0.00000000000
0.1	0.01	0.0100000000017	$1.700000001\times 10^{-13}$	0.0100000625	$6.250000182\times 10^{-8}$
0.2	0.04	0.0400000000013	$1.381724562\times 10^{-13}$	0.0400039996	$3.999996186\times 10^{-6}$
0.3	0.09	0.0900000000403	$4.035286375\times 10^{-11}$	0.0900455517	$4.555173453\times 10^{-5}$
0.4	0.16	0.1600000023781	$2.267192364\times 10^{-9}$	0.1602558088	$2.558087785\times 10^{-4}$
0.5	0.25	0.2500000517940	$5.165758279\times 10^{-8}$	0.2509747806	$9.747806112\times 10^{-4}$
0.6	0.36	0.3600006661388	$6.661388569\times 10^{-7}$	0.3629049558	$2.904955822\times 10^{-3}$
0.7	0.49	0.4900058047722	$5.804772241\times 10^{-6}$	0.4973013830	$7.303830440\times 10^{-3}$
0.8	0.64	0.6400380368478	$3.803684789\times 10^{-5}$	0.6560655822	$1.606558215\times 10^{-2}$
0.9	0.81	0.8102008607659	$2.008607659\times 10^{-4}$	0.8425733162	$3.257331625\times 10^{-2}$
1.0	1.00	1.0008965522328	$8.965523289\times 10^{-4}$	1.0625001666	$6.250016656\times 10^{-2}$

II: Name of the Table that justify the values

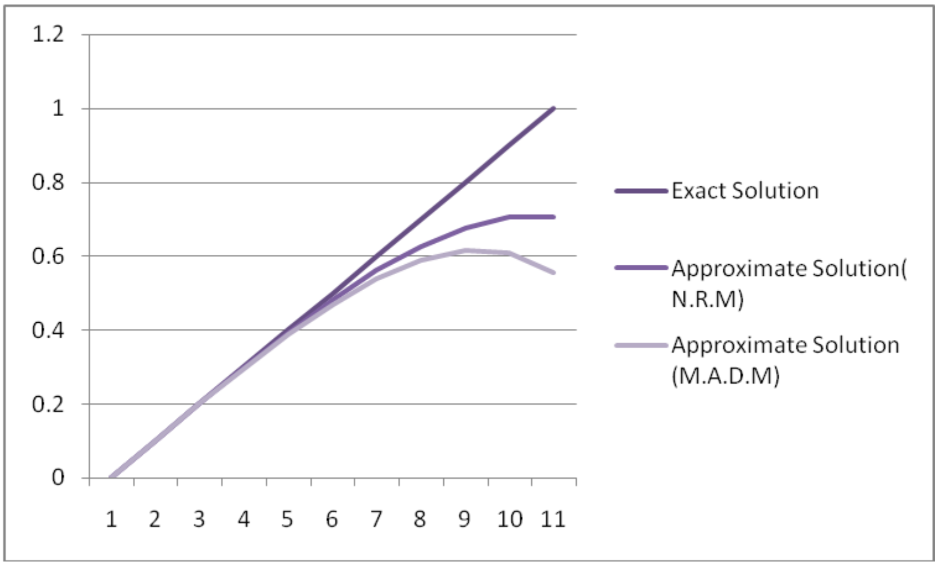

Figure1: Comparison of approximate and exact solution of example 3.

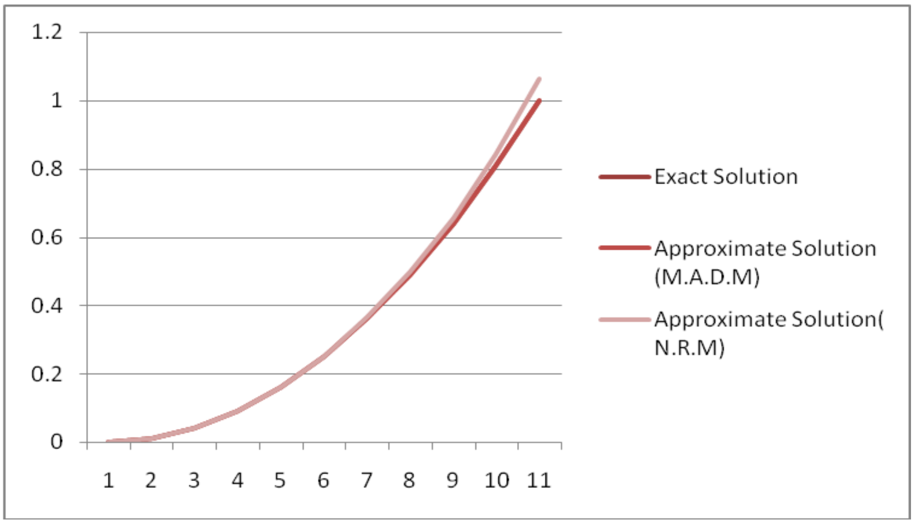

Figure2: Comparison of approximate and exact solution of example 4

VI. CONCLUSION

In this paper, we introduce a new technique to solve modified Adomian decomposition method & Newton Raphson method in nonlinear volterra integral equation and estimated errors with the help of example. We apply a modified Adomian decomposition method in Taylor's expansion and find exact solution. This is best technique in other numerical methods with volterra integral equation.

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