International Journal of Innovative Technology and Exploring Engineering (IJITEE)
ISSN: 2278-3075 (Online), Volume-11 Issue-3, January 2022

Analysis of Variable Viscosity and Thermal Conductivity of MHD flow of Mixed Convection over a Nonlinear Vertical Stretching Sheet

Geeti Gogoi

Abstract: In this present study we examine analytically the MHD flow problem close by a stagnation point on a stretching sheet which is nonlinear. Similarity transformation is used to modify the fluid flow governing equations into a system of ordinary differential equations .The modified equations are resolved with the assist of MATLAB bvp4c. The influence of viscosity parameter, thermal conductivity parameter, suction parameter and magnetic parameter on velocity and temperature are computed and presented through graphs.

Keywords: Bvp4c, MHD flow, Stretching Sheet, Thermal Conductivity, Variable Viscosity.

I. INTRODUCTION

The MHD problem of stagnation –point flow and transfer of heat on top of a stretching sheet has many practical implementations in the area of engineering as well as industry, such as extrusion of polymer and electronic devices cooling process, storage device of thermal energy, production of glass fiber etc. Due to the many practical application in real life, many researchers are interest to research the MHD flow and transfer of heat on top of a stretching sheet.

A seady, MHD and two dimensional stagnation point flow over a stretching sheet is investigated by Ishak et al. [1].Khan et al.[2] examined the viscous dissipation influence on a MHD stagnation point flow over a stretching sheet. The effect of viscous dissipation on a stagnation point flow on a stretching surface is observed by Shateyi et al.[3].Also Shateyi et al.[4] studied the analysis of MHD flow close to a stagnation point of porous body. Makinde [5] analyzed the transfer of heat and mass by Magnetohydrodynamics stagnation point flow over a vertical porous plate. The mixed convection point flow over a stretching sheet with magnetic field is observed by Ali et al. [6]. The bouncy effects of MHD flow and transfer of heat towards a stretching sheet is studied by Makinde et al.[7].The order of chemical reaction and effects of convective boundary condition on a micropolar fluid on a stretching surface is observed by Matta et al.[8]. Medikare et al.[9] examined the Magnetohydrodynamics stagnation point flow of casson fluid over stretching sheet. Nandeppanavar et al.[10] studied the influence of non-uniform heat source due to nonlinear moving surface. The generation of entropy of magnetic flow of fluid which is micropolar is analyzed by Dey et al.[11]. P.D.W. et al [12] analyzed the stability analysis of MHD flow on top of a shrinking sheet.The influence of variable viscosity on transfer of heat and mass in the direction of vertical surface is analyzed by Lai F.C. et al.[13].also Thakur et al.[14] analyzed the consequence of variable viscosity as well as thermal conductivity on MHD flow of micropolar fluid over a infinite upright surface. Recently, Hazarika et al. [15] studied the influence of variable viscosity and thermal conductivity of micropolar fluid over a upright cone. Kalita et al.[16] examined the use of CNTs in a upright medium with porosity for human being blood flow. In this present paper we enlarge the work of Shen et al. [17] to examine the influence of variable viscosity and thermal conductivity. In [17] the viscosity as well as thermal conductivity are assumed as stable but these properties of flow vary with the variation in temperature. We suppose viscosity and thermal conductivity as inverse linear functions of temperature. The nonlinear governing equations are converted to ordinary differential equations with the help of similarity transformation. The modified equations with boundary conditions are work out using bvp4c technique. The influence of viscosity parameter, thermal conductivity parameter, suction parameter and magnetic parameter upon fluid velocity and temperature are computed and displayed through graphs.

II. MATHEMATICAL FORMULATION

We analyze a MHD, steady, incompressible and mixed convection flow close by a stagnation point flow over a stretching sheet. The flow is considered in the region $y \geq 0$ .Here, y coordinate is measured upright to the stretching sheet. A uniform magnetic field is implemented in the upright direction to the surface y=0.The model of the fluid flow associated with this problem are shown in fig1.We assumed the velocity of stretching sheet is $u_w(x) = cx^m$ and outward velocity is $u_e = ax^m$, c and a are positive constant and m is parameter of non linearity.

Manuscript received on January 20, 2022.
Revised Manuscript received on January 27, 2022.
Manuscript published on February 28, 2022.
* Correspondence Author
Geeti Gogoi*, Department of Mathematics, Assam University, Assam, India.

In case of linear $m=1$ and $m \neq 1$ in case of nonlinear.
With the help of above approximation of boundary layer, the fluid flow governing equations which describe the flow are given by

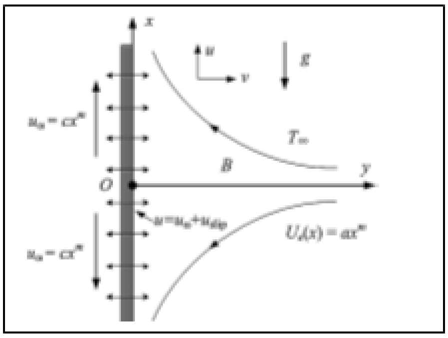

Fig1: Flow model of problem

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial x}=0 \qquad (1)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=u_e\frac{du_e}{dx}+\frac{\partial}{\partial y}\left(\nu\frac{\partial u}{\partial y}\right)+\left(\frac{\sigma B^2(x)}{\rho}\right) \qquad (2)$$
$$(u_e-u)+g\beta(T-T_\infty)$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{1}{\rho C_p}\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right) \qquad (3)$$

Where
u and v are velocity component in x and y directions respectively, ν is kinematic viscosity, ρ is the density of the fluid, σ is electrical conductivity, g is acceleration due to gravity, $B(x)$ is transverse magnetic field, β is coefficient of thermal expansion , T is temperature of the fluid, k is thermal conductivity.
Lai and Kulacki (1990) assumed viscosity as

$$\frac{1}{\mu}=\frac{1}{\mu_\infty}\left[1+\delta(T-T_\infty)\right] \quad \text{or} \quad \frac{1}{\mu}=a(T-T_c) \quad \text{where}$$
$$a=\frac{\delta}{\mu_\infty} \text{ and } T_c=T_\infty-\frac{1}{\delta}$$

Lai and Kulacki(1990) and Hazarika et al.(2020) assumed thermal conductivity as

$$\frac{1}{k}=\frac{1}{k_\infty}\left[1+\xi(T-T_\infty)\right] \quad \text{or} \quad \frac{1}{k}=b(T-T_r) \quad \text{where}$$
$$b=\frac{\xi}{k_\infty} \text{ and } T_r=T_\infty-\frac{1}{\xi}$$

Here T_∞ is free stream temperature, a,b,T_c,T_r are constants, k,k_∞ are thermal conductivities at T,T_∞ respectively.
The boundary conditions are

$$u=u_w(x)+\frac{2-\sigma_v}{\sigma_v}\lambda_0\frac{\partial u}{\partial y},\; v=v_w(x),\; \frac{\partial T}{\partial y}=-\frac{q_w(x)}{k},$$
$$\text{at } y=0 \qquad (4)$$
$$u\to u_e(x), T\to T_\infty \text{ as } y\to\infty \qquad (5)$$

Where σ_v is coefficient of momentum accommodation in tangential direction, λ_0 is mean free path, $v_w(x)$ is suction(injection) velocity, k is thermal conductivity and $q_w(x)$ is surface heat flux.
The similarity transformations used in this problem are:

$$\eta=\sqrt{\frac{a}{\upsilon}}yx^{\frac{(m-1)}{2}},\psi=\sqrt{a\nu x}^{\frac{(m-1)}{2}}f(\eta),\theta=\sqrt{\frac{a}{\nu}}\frac{k(T-T_\infty)}{q_0x^{2m-1}} \qquad (6)$$

Where ψ is the stream function.
The equation (1) is satisfied by introducing ψ such that

$$u=\frac{\partial\psi}{\partial y},v=-\frac{\partial\psi}{\partial x}$$

With the help of similarity transformations (6) the velocity components in x and y directions are

$$u=ax^m f'(\eta) \qquad \text{and}$$
$$v=-\sqrt{a\nu x}^{(m-1)/2}\left[\frac{(m+1)}{2}f(\eta)+\frac{(m-1)}{2}\eta f'(\eta)\right] \quad (7)$$

To get similarity solutions, we have taken

$$B(x)=B_0x^{(m-1)/2},v_w=-\frac{\sqrt{a\nu}(m+1)}{2}x^{(m-1)/2}S, \qquad (8)$$
$$q_w(x)=q_0x^{(5m-3)/2}$$

Where B_0,S and q_0 are constants. In the injection case the value of $S>0$ and in suction $S<0$.
With the help of equation (6) the equation (2) and (3) transformed into

$$\frac{\theta_r}{(\theta-\theta_r)^2}f''-\frac{\theta_r}{\theta-\theta_r}f'''+m(1-f'^2)+\lambda\theta+\left(\frac{m+1}{2}\right)ff''$$
$$+M(1-f')=0 \qquad (9)$$

$$\left(\frac{\theta_C}{\theta-\theta_C}\right)\theta''-\frac{\theta_c}{(\theta-\theta_c)^2}(\theta')+\Pr(2m-1)f'\theta \qquad (10)$$
$$-\Pr\left(\frac{m+1}{2}\right)f\theta'=0$$

And the boundary conditions (4) and (6) converted into

$$f(0)=S,\; f'(0)=\varepsilon+\delta f''(0),\theta'(0)=-1 \qquad (11)$$
$$f'(\infty)=1,\theta(\infty)=0 \qquad (12)$$

Here $M=\frac{\sigma B_0^2}{a\rho}$ is the magnetic parameter,

$$\lambda=\frac{g\beta q_0\sqrt{\upsilon}}{ka^{5/2}}=\frac{Gr_x}{\mathrm{Re}_x^{5/2}}$$ is mixed convection parameter,

Published By:
Blue Eyes Intelligence Engineering and Sciences Publication (BEIESP)

$\Pr = \frac{\upsilon}{\alpha}$ is the Prandtl number, $Gr_x = g\beta q x^4 / k\nu^2$ is the local Grashof number, $\mathrm{Re}_x = u_e x / \nu$ is Reynolds Number, $\varepsilon = c/a$ is the velocity ratio parameter and $\delta = (2-\sigma_v) Kn_x \mathrm{Re}_x^{1/2} / \sigma_v$ is the velocity slip parameter with Knudsen number $Kn_x = \lambda_0 / \sqrt{\varepsilon}\, x$.

The coefficient of skin friction is defined as

$$Cf = \frac{\tau_w(x)}{\rho u_e^2}$$, Here $\tau_w(x) = \mu(\partial u/\partial y)_{y=0}$ and with the help of similarity transformation we get

$$Cf(\mathrm{Re}_x^{\frac{1}{2}}) = \frac{\theta_r}{1-\theta_r} f''(0)$$

The coefficient of heat transfer rate is defined as

$$Nu = \frac{x q_w}{k_\infty (T_w - T_\infty)}$$, Here $q_w = q_0 x^{\frac{5m-3}{2}}$ and with the help of similarity transformation we get

$$Nu(\mathrm{Re}_x^{-\frac{1}{2}}) = \frac{\theta_c}{1-\theta_c} \frac{1}{\theta(0)}$$

III. RESULT AND DISCUSSION

The equation (9) and (10) with boundary conditions (11) and (12) are solved with the help of MATLAB bvp4c method. The obtained results are displayed through graphs for velocity and temperature profile. Here use the values of parameters as $\theta_r = -8, \theta_c = -8, M = 2, \lambda = -1, m = .2, \Pr = .71, S = .1,$ $\varepsilon = .1, \delta = .1$

Fig 2 and 3 show the effect of magnetic parameter on velocity as well as temperature for both the two cases $\varepsilon < 1$ and $\varepsilon > 1$.Velocity of the fluid decline for $\varepsilon > 1$ and it rises when $\varepsilon < 1$ .It is due to the fact that rise in magnetic parameter, the Lorentz force associated with this magnetic force higher and it makes the layer of boundary thinner. The temperature profile increases with the increase of magnetic parameter for $\varepsilon > 1$ and it shows opposite reaction for $\varepsilon < 1$

In fig 4 and 5 we observe that fluid velocity decline in the case of $\varepsilon < 1$ and it increases for $\varepsilon > 1$ with the increase of viscosity parameter. It is due to fact that with the increase of viscous force it creates frictional force between fluid particles and the velocity decline inside the boundary layer and it shows retards property outside. Increase in viscous force the heat energy increases so the fluid temperature rises with the rise in viscosity parameter within boundary layer and opposite in outside this boundary.

Velocity of the fluid increases with the increase thermal stratification parameter(S) for $\varepsilon < 1$ and decline for $\varepsilon > 1$ and it is displayed in fig 6 and 7. For both two cases the fluid temperature decline. It is due to the change in surface temperature and ambient temperature.

In fig 8 and 9, we observe the effect of thermal conductivity parameter (θ_c) on velocity as well as temperature. For both two cases the velocity rises as well as temperature decline with the rise in thermal conductivity parameter. It is due to the reason that increases in thermal conductivity parameter, it generates heat and transports the particles of the fluid and as a result fluid velocity rises.

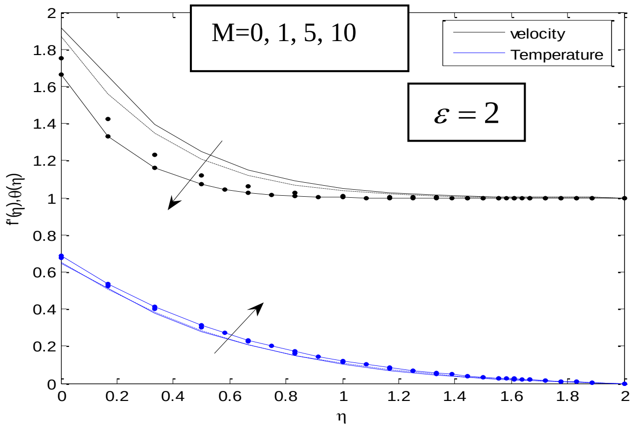

Fig 2: Velocity, Temperature profile for various values of M

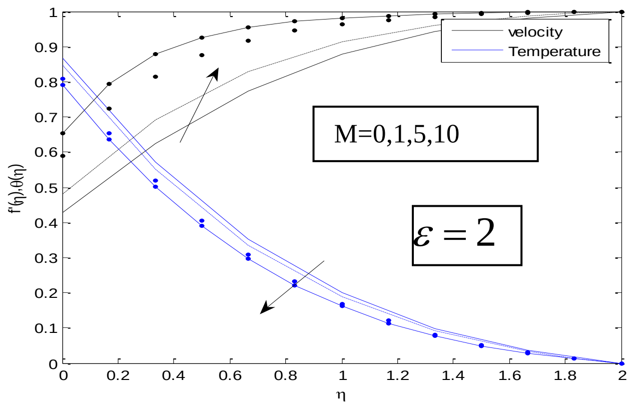

Fig 3: Velocity, Temperature profile for various values of M

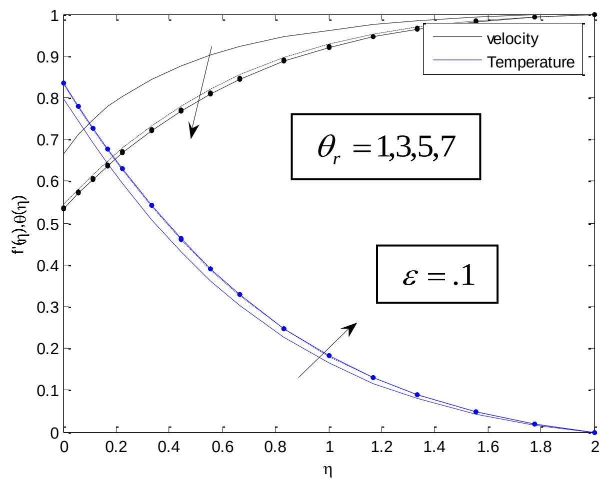

Fig 4: Velocity, Temperature profile for various values of θ_r

Published By:
Blue Eyes Intelligence Engineering
and Sciences Publication (BEIESP)

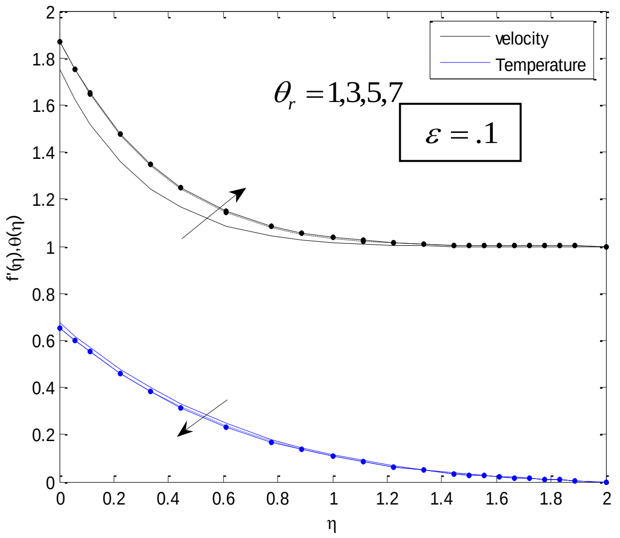

Fig 5: Velocity, Temperature profile for various values of θ_r

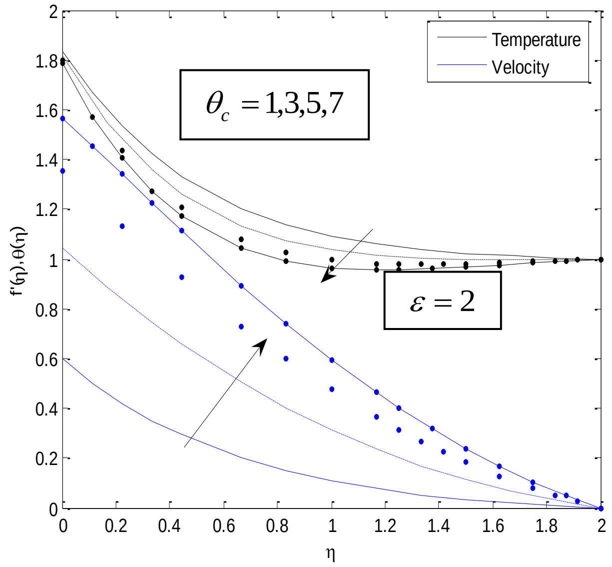

Fig 8: Velocity, Temperature profile for various values of θ_c

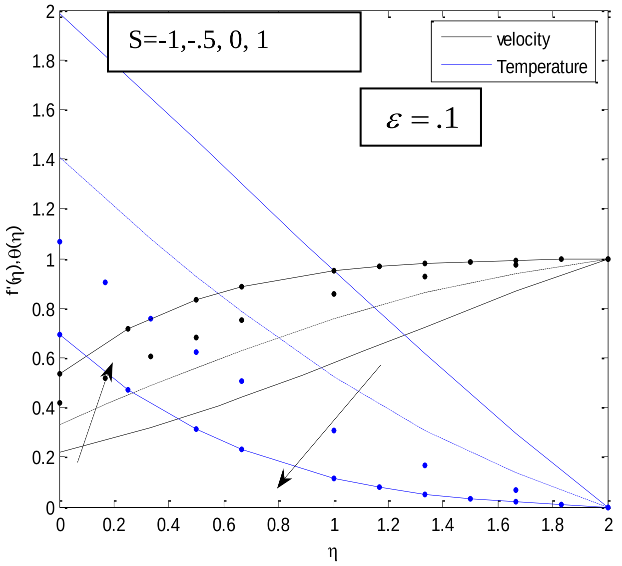

Fig 6: Velocity, Temperature profile for various values of S

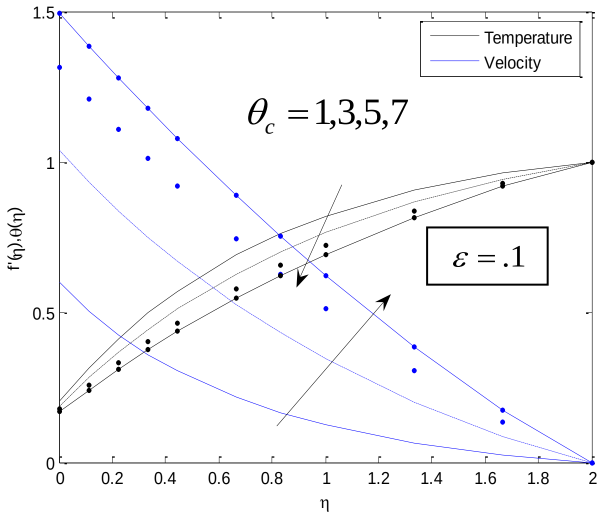

Fig 9: Velocity, Temperature profile for various values of θ_c

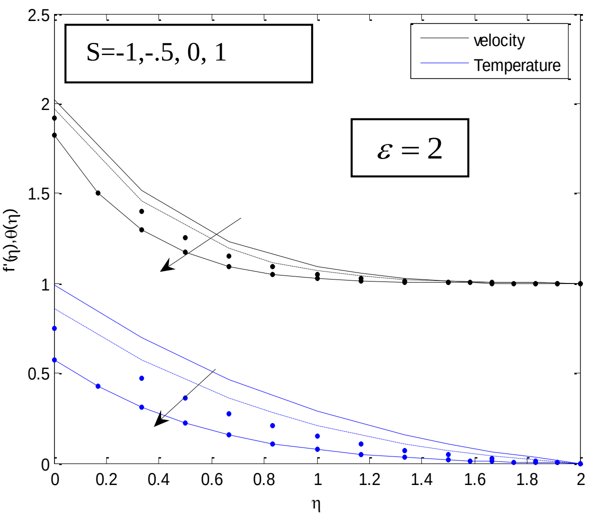

Fig 7: Velocity, Temperature profile for various values of S

Table 1 to 4 shows the influence of Cf, Nu for different values of parameters. Here we use the parameters as $\theta_r = -8, \theta_c = -8, M = 2, \lambda = -1, m = .2, \Pr = .71, S = .1, \varepsilon = .1, \delta = .1$

Retrieval Number: 100.1/ijitee.C97430111322
DOI: 10.35940/ijitee.C9743.0111322
Journal Website: www.ijitee.org

Published By:
Blue Eyes Intelligence Engineering
and Sciences Publication (BEIESP)

Table1: Variant of M and θ_c on Cf and Nu

M	.1		1		2	
θ_c	Cf	Nu	Cf	Nu	Cf	Nu
-5	.085233	-.541339	-.371215	-.523846	-.706064	-.512535
-4	.105267	-.513077	-.354532	-.495120	-.691813	-.483455
-3	.131609	-.473534	-.332096	-.455044	-.672275	-.442955
-2	.163599	-.414215	-.303415	-.395319	-.646485	-.382829
.5	-.727484	-4.374844	-.717039	-4.27249	-.977831	-4.264560
1.5	-.511350	-4.062447	-.825688	-4.059152	-.960489	-4.057340
3.5	-.272296	-1.265768	-.647740	-1.257923	-.930507	-1.253054
5	-.198858	-1.036961	-.592731	-1.027269	-.886939	-1.021219

Table2: Variant of M and θ_r on Cf and Nu

M	.1		1		2	
θ_r	Cf	Nu	Cf	Nu	Cf	Nu
-5	1.733776	-1.305874	1.803685	-1.289325	1.868093	-1.274513
-4	1.736491	-1.305476	1.807583	-1.288641	1.872924	-1.273619
-3	1.741552	-1.304641	1.814723	-1.287353	1.881526	-1.272005
-1	1.806400	-1.292340	1.890408	-1.272450	1.964900	-1.255515
1	3.036293	-1.254760	3.077800	-1.248683	3.117974	-1.242912
3	2.667764	-1.303193	2.751495	-1.290368	2.830561	-1.278537
5	2.656248	-1.305369	2.746672	-1.291488	2.831538	-1.278776
7	2.653992	-1.305922	2.747110	-1.291622	2.834289	-1.278567

Table3: Variant of S and θ_c on Cf and Nu

S	.1		1		1.5	
θ_c	Cf	Nu	Cf	Nu	Cf	Nu
-5	-.112620	-.329650	-.706064	-.512535	-.949693	-.633763
-4	-.070210	-.306764	-.691813	-.483455	-.940792	-.600009
-3	-.008631	-.275567	-.672275	-.442955	-.928596	-.552466
-2	-1.085675	-.230631	-.646485	-.382829	-.912749	-.480906
.5	-1.016914	-3.667490	-.977831	-4.234195	-1.191192	-4.411925
1.5	-0.854831	-3.564075	-.957489	-4.057340	-1.198529	-4.399812
3.5	-0.636208	-0.990325	-.930507	-1.253054	-1.094439	-1.438251
5	-0.553121	-0.776497	-.886939	-1.021219	-1.064935	-1.192113

Table4: Variant of S and θ_r on Cf and Nu

S	.1		1		1.5	
θ_r	Cf	Nu	Cf	Nu	Cf	Nu
-5	1.66680	-1.17007	1.81712	-1.49285	1.89849	-1.69782
-4	1.67013	-1.16950	1.81913	-1.49264	1.89989	-1.69774
-3	1.67647	-1.16835	1.82311	-1.49214	1.90276	-1.69750
-1	1.75008	-1.15309	1.87588	-1.48337	1.94424	-1.69170
1	3.19246	-1.08782	3.03118	-1.45902	3.08756	-1.67347
3	2.56293	-1.16777	2.79838	-1.48986	2.92568	-1.69468
5	2.54948	-1.17025	2.78866	-1.49170	2.91726	-1.69627
7	2.54711	-1.17082	2.78650	-1.49223	2.91517	-1.69676

In Table 1, we observe that rise in M and θ_c the value of Cf rises but Nu decline. Table 2 also shows same behavior. In table 3 and 4, Cf and Nu rises with rise in θ_c and θ_r.

IV. CONCLUSION

From the above analysis we conclude the following points:

1. Velocity rises with the rise in magnetic parameter and suction parameter for $\varepsilon < 1$ and decline gradually for $\varepsilon > 1$.
2. Temperature always rise with the rise in thermal conductivity parameter but it decline with the rise in suction parameter.
3. Fluid temperature decline with the rise in stratification parameter (S).
4. With the rise in viscosity parameter, velocity decline for $\varepsilon < 1$ and it increases for $\varepsilon > 1$.
5. Coefficient of skin friction and Nusselt Number rises with rise in thermal conductivity parameter and viscosity parameter.

REFERENCES

1. Ishak, A, Jafar, K, Nazar, R, Pop, I 2009 ," MHD stagnation point flow towards a stretching sheet", Physica A 388(13), 3377-3383.
2. Khan, ZH, Khan, WA, Qasim, M, Shah, IA 2014 ,"MHD stagnation point ferrofluid flow and heat transfer toward a stretching sheet", IEEE Trans. Nanotechnol. 13(1), 35-40.
3. Shateyi, S, Makinde, OD 2013, "Hydromagnetic stagnation-point flow towards a radially stretching convectively heated disk", Math. Probl. Eng. Article ID 616947 (2013)
4. Shateyi, S, Marewo, GT 2014, " Numerical analysis of unsteady MHD flow near a stagnation point of a two-dimensional porous body with heat and mass transfer, thermal transfer, and chemical reaction", Bound. Value Probl. 2014, 218(2014)
5. 5.Makinde, OD 2012, " Heat and mass transfer by MHD mixed convection stagnation point flow toward a vertical plate embedded in a highly porous medium with radiation and internal heat generation", Meccanica 47, 1173-1184.
6. Ali, FM, Nazar, R, Arifin, NM, Pop, I 2014, "Mixed convection stagnation-point flow on vertical stretching sheet with external magnetic field", Appl. Math. Mech. 35(2), 155-166.
7. Makinde, OD, Khan, AH, Khan, ZH 2013, " Buoyancy effects on MHD stagnation point flow and heat transfer of a nanofluid past a convectively heated stretching/shrinking sheet", Int. J. Heat Mass Transf. 62, 526-533.
8. Matta A. and Gajjela N. 2018, " Order of chemical reaction and convective boundary condition effects on micropolar fluid flow over a stretching sheet", AIP Advances 8, 115212.
9. Medikare M., Joga S. and Chidem K.K. 2016, " MHD stagnation point flow of a casson fluid over a nonlinearly stretching sheet with viscous dissipation", American Journal of Computational Mathematics, vol.6, No.1, pp.10.4234/ajcm,61005.
10. Nandeppanavar M.M., Kemparaju M.C. and Shakunthala S.2018, " MHD stagnation point slip flow due to a nonlinearly moving surface with effect of non-uniform heat source", Nonlinear Engineering, Modelling and Applications, DE- Gruyter, vol.8, No.1.
11. D. Dey and M. Hazarika 2020, "Entropy Generation of Hydro-Magnetic Stagnation Point Flow of Micropolar Fluid With Energy Transfer Under the Effect of Uniform Suction / Injection", vol. 50, no. 3, pp. 209–214.
12. P. D. W. and A. I. I.S. Awaludin 2016, "Stability Analysis of Stagnation-Point Flow over a Stretching/Shrinking Sheet", in AIP ADVANCES, p. 045308.
13. Lai F.C. and Kulacki, F.A.1990, "The effect of variable viscosity on convective heat and mass transfer along a vertical surface in saturated porous medium",Int. J. Heat Mass Tran., Vol. 33, No. 5, 1028-1031.
14. Thakur, P.M., Hazarika, G. C. 2015, " Effects of variable viscosity and thermal conductivity on the MHD flow of micropolar fluid past an accelerated infinite vertical insulated plate", Int. J. of Heat and Tech. USA., Vol. 33, pp. 73-78.
15. Hazarika, G.C., Phukan, B., and Ahmed, S. 2020, "The effect of variable viscosity and thermal conductivity on unsteady free convective flow of a micropolar fluid past a vertical cone", Journal of Engineering Physics and Thermophysics, Vol. 93, pp. 184-191.
16. Kalita D, Hazarika S, Ahmed S 2020 "Applications of CNTs in a vertical channel of porous medium for human blood flow: a rheological model", JP Journal of Heat and Mass Transfer 20 (2): 105-120.
17. Shen M., Wang F. and Chen H. 2015 "MHD Mixed Convection slip flow near stagnation-point on a nonlinearly vertical stretching sheet", Boundary value problem. Springer open Journal, No.1, 78.
18. Jafar, K., Ishak, A. and Nazar, R 2013 "MHD stagnation point flow over a nonlinearly stretching/shinking sheet", J. Aeospace Eng., 26, pp. 829-834.
19. Kasmani, R.M., Sivasankaran, S., Bhuvaneswari, M.and Siri, Z 2016 "Effect of chemical reaction on convectiveheat transfer of boundary layer ow in nanouid over a wedg with heat generation/absorption and suction",J. Appl. Fluid Mech., 9, pp. 379-388.
20. Mozayyeni, H.R. and Rahimi, A.B. 2014 "Similarity solutions of axisymmetric stagnation-point flow and heat transfer of a viscous, Boussinesq-related density fluid on a moving at plate", Scientia Iranica, 21(4), pp.1440-1450.
21. Hazarika G.C., Borah Joydeep and Konch Jadav.,2018, "Effects of variable viscosity and thermal conductivity on hydromagnetic transient flow over a hot vertical plate",International Journal of Scientific Research ,Vol. 7.

AUTHOR PROFILE

Geeti Gogoi, is currently a Research Scholar in Department of Mathematics, Assam University, Assam and India. She pursued her Bachelor's (Honours) from J.B College, Jorhat and M.Sc and M.Phil from Dibrugarh University. Her area of interest is fluid dynamics, MHD and Heat and Mass transfer. Formerly she was working as an Assistant Professor in Jorhat Institute of Science and Technology, Jorhat, Assam for 2.5 years.