The Influences of Slippage and Hall Currents on Peristaltic Transport of a Maxwell Fluid with Heat and Mass Transfer Through a Porous Medium

Nabil T. M. Eldabe, Amira S. A. Asar, Shaimaa F. Ramadan

***Abstract:** In this paper, the effects of slip velocity and Hall currents on peristaltic motion of a non-Newtonian fluid with heat and mass transfer through a porous medium inside a symmetric horizontal channel with flexible walls are studied. The fluid obeys Maxwell model, the ohmic and viscous dissipations are taken into account. Some of partial differential equations describe the fluid motion with the appropriate boundary conditions are written in dimensionless form and simplified by using the approximations of long wavelength and low Reynolds number. These equations are solved analytically, and the stream function, pressure rise, temperature, and concentration distributions are obtained as functions of physical parameters of the problem. The effects of the parameters of the problem on these solutions are discussed numerically and illustrated graphically through a set of figures. It is found that the physical parameters played important roles in controling the obtained solutions.*

***Keywords:** Slip boundary condition; Hall currents; Peristaltic transport; a Maxwell fluid; Viscous dissipation; Joule Heating; Heat and mass transfer.*

I. INTRODUCTION

The process of transportation of fluid in a channel or tube due to successive contractions of walls is called a peristalsis. This process has a host of well-established applications in the applied sciences, in biological systems like the urine transport from the kidney to the bladder, the movement of chyme in the gastrointestinal tract, transport of spermatozoa in the male reproductive tract, movement of ovum in fallopian tube of the female, swallowing of food through the esophagus, transport of lymph in the lymphatic vessels and the vasomotion of small blood vessels such as arterioles, and venules. Also, from an industrial point of view peristaltic flows play an important role in sanitary fluid transport, toxic liquid transport in the nuclear industry, transport of corrosive fluid, etc. Blood pumps in heart-lung machine and finger pumps also operate according to the mechanism of peristalsis. The study of peristaltic flow has been generated a lot of interest and hence the huge amount of literature on the topic is available for the non-Newtonian fluids.

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* Correspondence Author

Nabil T. M. Eldabe, Faculty, Department of Mathematics Education, Ain-Shams University, Cairo, Egypt

Amira S. A. Asar*, Faculty, Department of Mathematics Arts & Science, Prince Sattam Bin Abdulaziz University, Wadi Adwassir, Saudi Arabia, E-mail: amiraasar@yahoo.com

Shaimaa F. Ramadan, Faculty, Department of Mathematics Science (Girls), Al-Azhar University, Cairo, Egypt.

The non-Newtonian fluids motion has been an important subject in the field of biomedical, chemical and environmental engineering and science. Undoubtedly the mechanics of non-Newtonian fluids presents special challenges to engineers, physicists and mathematicians. This is because nonlinearity manifests itself in a variety of ways. The flows of non-Newtonian fluids are not only important in the interesting mathematical features presented by the equations governing the flow but also due to their technological significance. Moreover, the elastic properties of real fluids can be determined and measured. The viscoelastic fluid is a non- Newtonian fluid, which contains both viscous and elastic properties, most of the biological fluids such as chyme, blood, and food bolus are found to be viscoelastic in nature.

In addition, there are series of investigations have been carried out about the understanding of the dynamics of viscoelastic material since the contribution of James Clerk Maxwell. The dynamics of material which having the properties of viscosity and elasticity when undergoing deformation is a fundamental topic in fluid dynamics. This kind of materialreferred to as Maxwell fluid, it has attracted the attention of many researchers because of its wide technical and industrial applications. In 1867, James Clerk Maxwell proposed (Maxwell fluid) and the knowledge was popularized by James G. Oldroyd a few years after. Also, Mass transfer can be described as the movement of mass (material) through a fluid-solid interface or a fluid-fluid interface. The term "mass transfer" is commonly used in industry and in engineering for physical processes that involve convective and diffusive transport of chemical species within physical systems. Convective heat transfer plays a vital role in processes involving high temperatures such as nuclear plants, thermal energy storage, gas turbines, etc.. The ratio between the electron-cyclotron frequency and the electron-atom-collision frequency referred to as the Hall parameter; its effect is important when it is high. This happens, when the collision frequency is low or when the magnetic field is high its effect is important when it is high.

Most of recent research related to the peristaltic transport with heat and mass transfer for Newtonian and non-Newtonian fluid under the effect of different parameters. Eldabe, El-Sayed, Ghaly and Sayed [4] investigated the unsteady mixed convection peristaltic mechanism problem, the flow includes a temperature-dependent viscosity with effects of diffusion-thermo and thermal diffusion . Also, the effect of chemical reaction, heat and mass transfer on non-Newtonian fluid flow in a vertical peristaltic tube through porous medium is studied by El-Sayed, Eldabe and Ghaly [6].

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The effect of elasticity of flexible walls on peristaltic motion of a dusty fluid in a horizontal channel in the presence of chemical reaction with heat and mass transfer has been investigated under long wavelength approximation by Muthuraj, Nirmala and Srinivas [16].

A theoretical study for peristaltic flow of a MHD fluid in an asymmetric channel is presented by Sinha, Shit and Ranjit [22], effects of velocity-slip as well as thermal-slip and viscosity variation have been duly taken care of in this study, they include a heat source term which simulates either absorption or generation in the equation of energy . The problem of peristaltic transport of Maxwell fluid in an asymmetric channel with heat and mass transfer, taking into account the effect of thermal diffusion (Soret), with creeping flow developed by Musharafa Saleem and Aun Haider [15].

Mention may be made to some recent interesting studies available about peristaltic flow of non-Newtonian fluids. Eldabe, Elogail, Elshaboury and Hasan [5] introduced the peristaltic flow in a symmetric planner channel through a porous medium of an incompressible, electrically conducting Williamson fluid with mass and heat transfer. Viscous dissipation, Hall effects and Joule heating are taken into consideration. Bhattia, Ali and Rashidi [3] studied the combine effects of partial slip and Magnetohydrodynamics (MHD) on peristaltic flow of Blood Ree-Eyring fluid with wall properties. Heat and mass transfer with Joule heating on (MHD) peristaltic blood flow under the influence of Hall effect is examined by Bhattia and Rashidi [2]. Hayat, Aslam, Rafiq and Alsaadi [11] investigated the peristaltic transport of conducting Eyring Powell fluid in an inclined symmetric channel with Hall current effect. They have modeled the energy equation by taking Joule heating effect into consideration, they take into account the velocity and thermal slip conditions. Hayat, Zahir, Tanveer and Alsaedi [13] studied the peristaltic flow of mixed convective Prandtl fluid in a planar channel with compliant walls, effects of Hall current and applied magnetic field are retained, they characterize the heat transfer in fluid flow through convective boundary conditions.

Ramesha and Devakar [21] investigated the influence of heat transfer on the peristaltic transport of an incompressible magnetohydrodynamic second-grade fluid through a porous medium in an inclined asymmetric channel. Some other workers have investigated peristaltic transport of viscoelastic fluid with Maxwell model and they have discussed the effect of different parameters on the peristaltic transport. Hayat Alvi and Ali [10] studied the peristaltic flow of a Maxwell fluid in an asymmetric channel, also Hayat, Qureshi and Ali studied the problem of the peristaltic motion of a third order fluid under the effect of slip in an asymmetric channel [14]. Ali, Hayat and Asghar [1] introduced the peristaltic motion of a non-Newtonian fluid in a channel of compliant boundaries, constitutive equations for a Maxwell fluid have been used. Tripathi [23] studied the peristaltic transport of viscoelastic non-Newtonian fluids with fractional Maxwell model in a channel, the effects of friction force along one wavelength and relaxation time, fractional parameters and amplitude on the pressure difference are received and analyzed. El-Shehawy. Eldabe and El-Desoky [7] analyzed the effect of slip boundary conditions on the dynamics of the flow of a Newtonian and non-Newtonian Maxwellian fluid in an axisymmetric cylindrical tub, the flow is induced by traveling transversal waves on the tube wall through porous media. Gad [8] described the peristaltic transport under the effect of Hall currents in a channel having compliant boundaries, The effect of wall damping, wall tension, Hall parameter and Hartmann number on the mean axial velocity and reversal flow is investigated. Hayat and Hina [12] described the effects of heat and mass transfer on the magnetohydrodynamic (MHD) peristaltic flow of an incompressible Maxwell fluid in a planar channel having compliant walls.

Nadeem and Akram investigated the peristaltic flow of a linear Maxwell model through porous boundaries in a porous medium [17]. Hayat, Ali and Asghar [9] introduced the problem of peristaltic transport of an incompressible, electrically conducting Maxwell fluid through porous space in a planar channel. The effect of Hall is taken into account and permeability of porous medium is considered uniform, see also [18]-[20]. In this article we introduced a theoretical study of the slip boundary condition as well as the Hall currents effects on the peristaltic transport of a Maxwell fluid through a porous medium in a symmetric channel with heat and mass transfer taking into account viscous and ohmic dissipations.

II. THE GOVERNING EQUATION

The vector form of the equations which describe the motion of non-Newtonian fluid with heat and mass transfer under the effect of Hall current can be written as

Continuity equation

$$\nabla\cdot V=0, \qquad (1)$$

Momentum equation

$$\rho\frac{dV}{dt}=-\nabla P+\nabla\cdot S+(J\wedge B)+R, \qquad (2)$$

Energy equation

$$\rho c_p\frac{dT}{dt}=k\nabla^2T+S\cdot\nabla V+\frac{1}{\sigma}J\cdot J, \qquad (3)$$

Concentration equation

$$\frac{dC}{dt}=D_m\nabla^2C+D_mk_T\nabla^2T, \qquad (4)$$

where V is fluid velocity vector, ρ is the density of the fluid, P is the pressure, R is the Darcy's resistance for a fluid in a porous medium, $S=(S_{ij})$ is the extra stress tensor of Maxwell fluid, T is temperature, k is the thermal conductivity, C is the concentration, D_m is the coefficient of mass diffusivity, k_T is the thermal diffusion ratio, $\left(\frac{d}{dt}\right)$ denotes the material time derivative and ∇^2 is the Laplacian operator in second order. The current density including the Hall effect is given by

$$J=\sigma\left[V\wedge B-\frac{1}{e\ n_e}\ (J\wedge B)\right]; \qquad (5)$$

where J is the current density including the Hall effect, σ is the electric conductivity, e is the electric charge, n_e is the number density of electrons and $B=(0,0,B_0)$ is the uniform magnetic field. Also, in Maxwell fluid the Darcy's resistance R satisfies the following expression

$$\left(1+\lambda_1\frac{d}{d\ t}\right)R=-\frac{\mu}{K}V$$

where λ_1 is the relaxation time, K is the permeability parameter and μ is the dynamic viscosity.

The constitutive equation for Maxwell fluid is given by

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$$S_{ij} + \lambda_1 \left(\frac{dS_{ij}}{dt} - L_k^i S_{kj} - S_{ki} L_k^j\right) = 2\mu e_{ij} \qquad (6)$$

where

$$\left.\begin{aligned} \frac{dS_{ij}}{dt} &= \frac{\partial S_{ij}}{\partial t} + (V\cdot\nabla)S_{ij}, \\ L_k^i &= \frac{\partial V_i}{\partial x_k}, \qquad L_k^j = \frac{\partial V_j}{\partial x_k}, \\ e_{ij} &= (L_j^i + L_i^j). \end{aligned}\right\} \qquad (7)$$

III. MATHEMATICLAL FORMULATION

Choose a cartesian coordinates (X, Y) where X-axis is taken in motion direction while Y-axis is perpendicular on it as shown in Fig. (1).

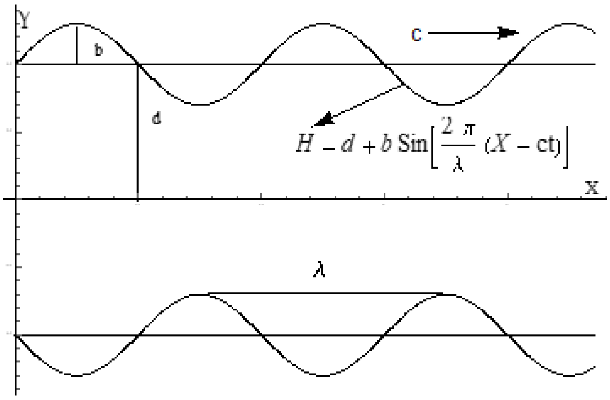

Fig.(1) The geometry of the problem.

The velocity components in X and Y directions are U and V respectively, taking $m = \frac{\sigma B_0}{e\ n_e}$ is the Hall parameter. then equations (1-6) can be written -after using equations (5, 7)- as

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0, \qquad (8)$$

$$\rho\left(\frac{\partial U}{\partial t} + U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y}\right) = -\frac{\partial P}{\partial X} + \frac{\partial S_{XX}}{\partial X} + \frac{\partial S_{XY}}{\partial Y} + \frac{\sigma B_0^2}{1+m^2}(mV - U) + R_X, \qquad (9)$$

$$\rho\left(\frac{\partial V}{\partial t} + U\frac{\partial V}{\partial X} + V\frac{\partial V}{\partial Y}\right) = -\frac{\partial P}{\partial Y} + \frac{\partial S_{YX}}{\partial X} + \frac{\partial S_{YY}}{\partial Y} - \frac{\sigma B_0^2}{1+m^2}(mU + V) + R_Y, \qquad (10)$$

$$\rho c_p\left(\frac{\partial T}{\partial t} + U\frac{\partial T}{\partial X} + V\frac{\partial T}{\partial Y}\right) = k\left(\frac{\partial^2 T}{\partial X^2} + \frac{\partial^2 T}{\partial Y^2}\right) + S_{XX}\frac{\partial U}{\partial X} + S_{XY}\frac{\partial U}{\partial Y} + S_{YX}\frac{\partial V}{\partial X} + S_{YY}\frac{\partial V}{\partial Y} + \frac{\sigma B_0^2}{1+m^2}(U^2 + V^2), \qquad (11)$$

$$\left(\frac{\partial C}{\partial t} + U\frac{\partial C}{\partial X} + V\frac{\partial C}{\partial Y}\right) = D_m\left(\frac{\partial^2 C}{\partial X^2} + \frac{\partial^2 C}{\partial Y^2}\right) + D_m k_T\left(\frac{\partial^2 T}{\partial X^2} + \frac{\partial^2 T}{\partial Y^2}\right), .5cm \qquad (12)$$

$$S_{XX} + \lambda_1\left[\frac{dS_{XX}}{dt} - 2\left(S_{XX}\frac{\partial U}{\partial X} - S_{XY}\frac{\partial U}{\partial Y}\right)\right] = 2\mu\frac{\partial U}{\partial X}, \qquad (13)$$

$$S_{XY} or S_{YX} + \lambda_1\left[\frac{dS_{XY}}{dt} - S_{XX}\frac{\partial V}{\partial X} - S_{YY}\frac{\partial U}{\partial Y}\right] = \mu\left(\frac{\partial V}{\partial X} + \frac{\partial U}{\partial Y}\right), \qquad (14)$$

$$S_{YY} + \lambda_1\left[\frac{dS_{YY}}{dt} - 2\left(S_{XY}\frac{\partial V}{\partial X} - S_{YY}\frac{\partial V}{\partial Y}\right)\right] = 2\mu\frac{\partial V}{\partial Y}. \qquad (15)$$

The equations of the walls of a symmetric channel can be written as

$$Y = \pm H = \pm d \pm a\sin\frac{2\pi}{\lambda}(X - c\ t) \qquad (16)$$

where d is the mean half width of the channel, a is the amplitudes of the waves, λ is the wave length, c is the velocity of propagation and t is the time.

Introducing a wave frame (x, y) moving with the velocity c away from the laboratory frame (X, Y) , by the transformations

$$x = X - c\ t, \quad y = Y, \quad u = U - c, \quad v = V, \quad p\ (x) = P(X, t), \qquad (17)$$

Also, the non-dimensional variables are considered

$$\bar{x} = \frac{x}{\lambda}, \quad \bar{y} = \frac{y}{d}, \quad \delta = \frac{d}{\lambda}, \quad \bar{t} = \frac{ct}{\lambda}, \quad \bar{u} = \frac{u}{c}, \quad \bar{v} = \frac{v}{\delta c}, \quad M = \sqrt{\frac{\sigma}{\mu}}B_0 d,$$

$$\bar{a} = \frac{a}{d}, \quad \bar{h} = \frac{H}{d}, \quad \bar{\psi} = \frac{\psi}{cd}, \quad \Theta = \frac{T - T_0}{T_w - T_0}, \quad \Phi = \frac{C - C_0}{C_w - C_0},$$

$$\bar{p} = \frac{d^2 p}{\mu c\lambda}, \quad \bar{\lambda}_1 = \lambda_1\frac{c}{d}, \quad \bar{K} = \frac{K}{d^2}, \quad R_e = \frac{\rho c\ d}{\mu}, \quad \bar{S} = \frac{d}{\mu c}S, \qquad (18)$$

where u and v are the fluid velocity components, p is pressure in the wave frame of references, δ is the wave number, ψ is the stream function, M and R_e are the Hartmann and Reynolds numbers respectively. Apply the above transformations equation (17) with the dimensionless equation (18) then the governing equations (9 -16) can be written as

$$\delta R_e\left(1 + \delta\bar{\lambda}_1\frac{d}{d\ \bar{t}}\right)\left(\bar{u}\frac{\partial\bar{u}}{\partial\bar{x}} + \bar{v}\frac{\partial\bar{u}}{\partial\bar{y}}\right) = \left(1 + \delta\bar{\lambda}_1\frac{d}{d\ \bar{t}}\right)\left[-\frac{\partial\bar{p}}{\partial\bar{x}} + \delta\frac{\partial\bar{S}_{xx}}{\partial\bar{x}} + \frac{\partial\bar{S}_{xy}}{\partial\bar{y}}\right] + \left(1 + \delta\bar{\lambda}_1\frac{d}{d\ \bar{t}}\right)\left[\left(\frac{M^2}{1+m^2}\right)(m\delta\bar{v} - \bar{u} - 1)\right] - \frac{\bar{u}}{\bar{K}}, \qquad (19)$$

$$\delta^3 R_e\left(1 + \delta\bar{\lambda}_1\frac{d}{d\ \bar{t}}\right)\left(\bar{u}\frac{\partial\bar{v}}{\partial\bar{x}} + \bar{v}\frac{\partial\bar{v}}{\partial\bar{y}}\right) = \left(1 + \delta\bar{\lambda}_1\frac{d}{d\ \bar{t}}\right)\left[-\frac{\partial\bar{p}}{\partial\bar{y}} + \delta^2\frac{\partial\bar{S}_{yx}}{\partial\bar{x}} + \delta\frac{\partial\bar{S}_{yy}}{\partial\bar{y}}\right] -$$

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$$\delta\left(1+\delta\bar{\lambda}_1\frac{d}{d\,\bar{t}}\right)\left[\left(\frac{M^2}{1+m^2}\right)(m(\bar{u}+1)+\delta\bar{v})\right]-\delta\frac{\bar{v}}{\bar{K}}, \tag{20}$$

$$P_rR_e\delta\left(\bar{u}\frac{\partial\Theta}{\partial\bar{x}}+\bar{v}\frac{\partial\Theta}{\partial\bar{y}}\right)=\left(\delta^2\frac{\partial^2\Theta}{\partial\bar{x}^2}+\frac{\partial^2\Theta}{\partial\bar{y}^2}\right)+B_r\left(\delta\bar{S}_{xx}\frac{\partial\bar{u}}{\partial\bar{x}}+\bar{S}_{xy}\frac{\partial\bar{u}}{\partial\bar{y}}+\delta^2\bar{S}_{yx}\frac{\partial\bar{v}}{\partial\bar{x}}+\delta\bar{S}_{yy}\frac{\partial\bar{v}}{\partial\bar{y}}\right)$$

$$+B_r\left(\frac{M^2}{1+m^2}\right)[\delta^2\bar{v}^2+(\bar{u}+1)^2], \tag{21}$$

$$R_e\delta\left(\bar{u}\frac{\partial\Phi}{\partial\bar{x}}+\bar{v}\frac{\partial\Phi}{\partial\bar{y}}\right)=\frac{1}{S_c}\left(\delta^2\frac{\partial^2\Phi}{\partial\bar{x}^2}+\frac{\partial^2\Phi}{\partial\bar{y}^2}\right)+S_r\left(\delta^2\frac{\partial^2\Theta}{\partial\bar{x}^2}+\frac{\partial^2\Theta}{\partial\bar{y}^2}\right), \tag{22}$$

$$\bar{S}_{xx}+\bar{\lambda}_1\left[\delta\frac{d\bar{S}_{xx}}{d\bar{t}}-2\left(\delta\bar{S}_{xx}\frac{\partial\bar{u}}{\partial\bar{x}}+\bar{S}_{xy}\frac{\partial\bar{u}}{\partial\bar{y}}\right)\right]=2\delta\frac{\partial\bar{u}}{\partial\bar{x}}, \tag{23}$$

$$\bar{S}_{xy}or\bar{S}_{yx}+\bar{\lambda}_1\left[\delta\frac{d\bar{S}_{xy}}{d\bar{t}}-\delta^2\bar{S}_{xx}\frac{\partial\bar{v}}{\partial\bar{x}}-S_{yy}\frac{\partial\bar{u}}{\partial\bar{y}}\right]=\left(\delta^2\frac{\partial\bar{v}}{\partial\bar{x}}+\frac{\partial\bar{u}}{\partial\bar{y}}\right), \tag{24}$$

$$\bar{S}_{yy}+\bar{\lambda}_1\left[\delta\frac{d\bar{S}_{yy}}{d\bar{t}}-2\left(\delta^2\bar{S}_{xy}\frac{\partial\bar{v}}{\partial\bar{x}}+\delta\bar{S}_{yy}\frac{\partial\bar{v}}{\partial\bar{y}}\right)\right]=2\delta\frac{\partial\bar{v}}{\partial\bar{y}}, \tag{25}$$

$$\bar{y}=\pm\bar{h}=\pm1\pm\phi\sin2\pi\bar{x}, \tag{26}$$

where $P_r=\frac{\mu C_p}{k}$ is the Prandtle number, $E_c=\frac{\mu c^2}{k\Delta T}=\frac{c^2}{c_p\Delta T}$ is the Eckert number, $B_r=P_rE_c$ is the Brinkman number, $S_c=\frac{\nu}{D_m}$ is the Schmidt number, $S_r=\frac{D_mk_T(T_1-T_0)}{\nu(C_1-C_0)}$ is the Soret number and $\phi=\frac{a}{d}$ is the amplitude ratio.

Under the assumptions of long wave length ($\delta\ll1$) and low Reynolds number and after dropping bars marks the equations (19 -25) in terms of ψ where $(u=\frac{\partial\psi}{\partial y},v=-\frac{\partial\psi}{\partial x})$ will take the form

$$\frac{\partial p}{\partial x}=\frac{\partial S_{xy}}{\partial y}-\left(\frac{M^2}{1+m^2}\right)\left[\frac{\partial\psi}{\partial y}+1\right]-\frac{1}{K}\frac{\partial\psi}{\partial y}, \tag{27}$$

$$\frac{\partial p}{\partial y}=0, \tag{28}$$

$$\frac{\partial^2\Theta}{\partial y^2}+B_r\ S_{xy}\ \frac{\partial^2\psi}{\partial y^2}+B_r\left(\frac{M^2}{1+m^2}\right)\left[\frac{\partial\psi}{\partial y}+1\right]^2=0 \tag{29}$$

$$\frac{1}{S_c}\frac{\partial^2\Phi}{\partial y^2}+S_r\frac{\partial^2\Theta}{\partial y^2}=0 \tag{30}$$

$$S_{xx}-2\lambda_1S_{xy}\ \frac{\partial^2\psi}{\partial y^2}=0 \tag{31}$$

$$S_{xy}-\lambda_1\ S_{yy}\ \frac{\partial^2\psi}{\partial y^2}=\frac{\partial^2\psi}{\partial y^2} \tag{32}$$

$$S_{yy}=0 \tag{33}$$

Using (33) we get that

$$S_{xy}=\frac{\partial^2\psi}{\partial y^2} \tag{34}$$

Also, equation (26) can be written as

$$y=\pm h=\pm1\pm\phi\sin2\pi x, \tag{35}$$

After eliminating the pressure between equations (27, 28) and then using (34) we have

$$\frac{\partial^4\psi}{\partial y^4}-N^2\frac{\partial^2\psi}{\partial y^2}=0, \tag{36}$$

$$\frac{\partial^2\Theta}{\partial y^2}+B_r\left[\left(\frac{\partial^2\psi}{\partial y^2}\right)^2+\mathcal{H}^2\left(\frac{\partial\psi}{\partial y}+1\right)^2\right]=0, \tag{37}$$

$$\frac{1}{S_c}\frac{\partial^2\Phi}{\partial y^2}+S_r\frac{\partial^2\Theta}{\partial y^2}=0 \tag{38}$$

where $N=\sqrt{\mathcal{H}^2+\frac{1}{K}}$ and $\mathcal{H}^2=\frac{M^2}{1+m^2}$.

The appropriate dimensionless boundary conditions due to the symmetry of the walls in terms of the stream function are

$$\psi=\frac{q}{2},\quad \frac{\partial\psi}{\partial y}+\beta\ \frac{\partial^2\psi}{\partial y^2}=-1,\quad \Theta=0,\quad \Phi=0\quad at\quad y=h, \tag{39}$$

$$\psi=-\frac{q}{2},\quad \frac{\partial\psi}{\partial y}-\beta\ \frac{\partial^2\psi}{\partial y^2}=-1,\quad \Theta=1,\quad \Phi=1\quad at\quad y=-h, \tag{40}$$

where β is the non-dimensional slip parameter and q is the flux between the two walls in the wave frame. q is related to the dimensionless average volume flow rate over one period $T^*=\frac{\lambda}{c}$ of the peristaltic wave in the laboratory frame $\overline{Q}$ through the relation $\overline{Q}=q+1$.

The equations (36-38) are solved subjected to the boundary conditions (39 and 40) and the stream function ψ, the longitudinal velocity $u=\frac{\partial\psi}{\partial y}$, the temperature distribution Θ and the concentration distribution Φ are obtained as functions of the physical parameters of the problem where

$$\psi=\frac{y((\beta qN^2+2)\sinh(hN)+Nq\cosh(hN))-(2h+q)\sinh(Ny)}{2(\beta hN^2-1)\sinh(hN)+2hN\cosh(hN)},$$

$$u=\frac{(\beta qN^2+2)\sinh(hN)+Nq\cosh(hN)-N(2h+q)\cosh(Ny)}{2(\beta hN^2-1)\sinh(hN)+2hN\cosh(hN)},$$

$$\Theta=l_0(h-y)+l_1(l_2+F(h,y))$$

and

$$\Phi=m_0(m_1(h-y)-m_2(l_2+G(h,y)))$$

Also, by integrating the axial pressure gradient $\frac{dp}{dx}$ over one wavelength λ, the pressure rise is given by the relation

$$\Delta p=\int_0^1\frac{dp}{dx}dx \tag{41}$$

where

$$l_0=\frac{1}{2h},\quad l_1=\frac{B_r(2h+q)^2}{32\big((\beta hN^2-1)\sinh(hN)+hN\cosh(hN)\big)^2},\quad l_2=A_1-A_2-8\mathcal{H}^2,$$

$$m_0=\frac{1}{32h\big((\beta hn^2-1)\sinh(hn)+hn\cosh(hn)\big)^2},\quad m_1=16\big((\beta hn^2-1)\sinh(hn)+hn\cosh(hn)\big)^2,\quad m_2=hS_cS_rB_r(2h+q)^2,$$

$A_1 = (N^2 - 7\mathcal{H}^2)\cosh(2hN),\quad A_2 = 8\beta N\mathcal{H}^2\sinh(2hN),\quad A_3 = 2N^2\mathcal{H}^2(\beta^2N^2+1)\cosh(2hN),$
$A_4 = 2N^2(\beta^2N^2\mathcal{H}^2 + N^2 - 2\mathcal{H}^2),\quad A_5 = 4\beta\mathcal{H}^2N^3\sinh(2hN),\quad A_6 = \mathcal{H}^2 + N^2,$
$A_7 = 16\mathcal{H}^2(\beta n\sinh(hn) + \cosh(hN)),\quad A_8 = 16\beta N\mathcal{H}^2\sinh(hN),\quad A_9 = 8\mathcal{H}^2,$
$F(h,y) = (A_3 - A_4 + A_5)(h^2 - y^2) - A_6\cosh(2Ny) + A_7\cosh(Ny),$
$G(h,y) = F(h,y) + (A_8 - A_7)\cosh(Ny) + A_9(\cosh(N(h - y)) + \cosh(N(h+y))).$

IV. RESULTS AND DISCUSSION

During this work we study the effects of slip velocity with Hall Currents on peristaltic motion of Maxwell fluid through a porous medium in a symmetric channel with heat and mass Transfer. The ohmic and viscous dissipations are taken into consideration. The system of nonlinear partial differential equations which describe the velocity, temperature and concentration of the fluid are simplified under the approximations of long wavelength and low Reynolds number, then solved analytically subjected to suitable boundary conditions. The distributions of stream function, velocity, pressure rise, temperature and concentration are obtained as functions of physical parameters of the problem, then the effects of these parameters on those solutions are discussed numerically and illustrated graphically through some figures. Figs. (2 - 5) illustrate the relation between the pressure rise ΔP and the mean volume flow rate $\overline{Q}$ for different values of the physical parameters of the problem. It is observed in these figures that the pressure rise ΔP decreases when the mean volume flow rate $\overline{Q}$ increases. Also, it is clear that ΔP increases with the slip parameter β, as shown in Fig. (2). It is clear from Fig. (3) that ΔP decreases with the increasing of the Hartmann number M. In Fig. (4) we see that ΔP increases with the increasing of the Hall parameter m. It is clear from Fig. (5) that ΔP decreases with the increasing of permeability parameter K.

The relation between the velocity component u and the distance y is cleared in Figs. (6-9) for different values of the parameters. It is clear from Fig. (6) that the velocity decreases with the increasing of β, in the region $-1 < y < 0$ the velocity icreases with y and then in the region $0 < y < 1$ the contrary effect occur. The same behavior occurs with M as shown in Fig.(7). Figs.(8) and (9) showed that u increases with the increasing of both m and K. In the region $-1 < y < 0$ we observe u increases with y and then in the region $0 < y < 1$ the contrary effect occur. Figs.(10 -14) illustrate the effects of Brinkman number B_r and β, m, K and M on the temperature field Θ, it is found that Θ increases with B_r, K and M, while it decreases with the increasing of the parameters β and m. Figs. (15-21) study the effects of B_r, Soret number S_r, Schmidt number S_c β, M, $m$and K, on the concentration field Φ It is found that Φ increases with both of β and m, while it decreases with the increasing of B_r, S_r, S_c, M and K.

V. CONCLUSION

In this study, the problem of two dimensional peristaltic flow with heat and mass transfer of a non-Newtonian Maxwell fluid through a porous medium in a symmetric channel has been investigated. The equations governing the fluid flow in the presence of ohmic and viscous dissipations with the effect of hall currents, subjected to a set of appropriate boundary conditions, have been solved analytically under low Reynolds number and long wavelength approximations. The solutions of these equations are obtained as functions of the physical parameters of the problem. The effects of these parameters on these solutions are discussed numerically and cleared graphically. It is found that, the main results are

- The pressure rise ΔP has the maximum value at the minimum value of the mean volume flow rate $\overline{Q}$ and vise versa.
- The temperature distribution Θ decreases with increasing both of β and m.
- The concentration distribution Φ increases with β and m.

FIGURES

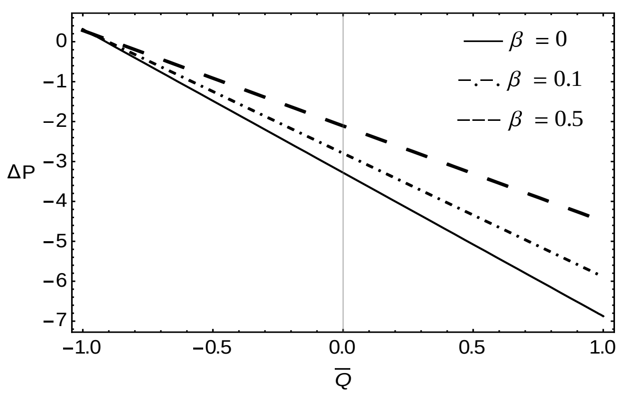

Fig.(2) For different values of slip parameter β, the pressure rise ΔP is plotted against the mean volume flow rate $\overline{Q}$ when $\phi = 0.1,\ K = 4,\ m = 0.5,\ M = 2.$

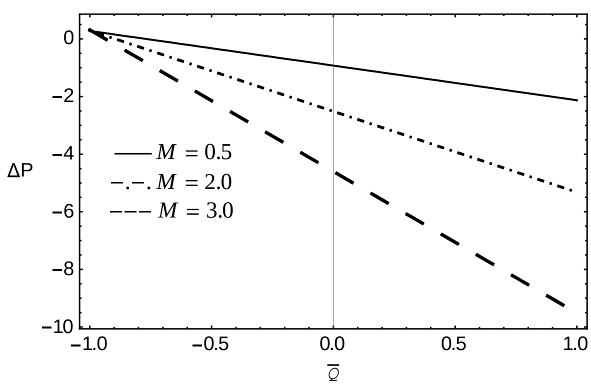

Fig.(3) For different values of Hartman number M, the pressure rise ΔP is plotted against the mean volume flow rate $\overline{Q}$ when $\phi = 0.1,\ \beta = 0.2,\ K = 4,\ m = 0.5.$

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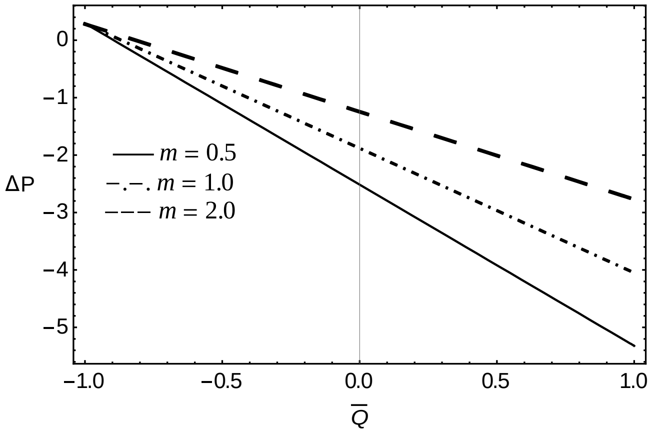

Fig.(4) For different values of Hall parameter m, the pressure rise ΔP is plotted against the mean volume flow rate $\overline{Q}$ when $\phi = 0.1,\ \beta = 0.2,\ K = 4,\ M = 2.$

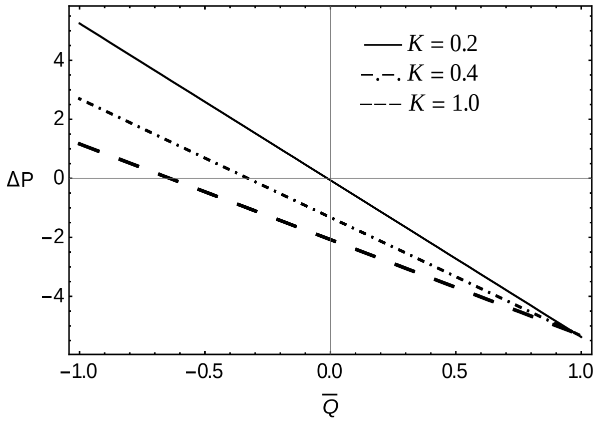

Fig.(5) For different values of permeability parameter K, the pressure rise ΔP is plotted against the mean volume flow rate $\overline{Q}$ when $\phi = 0.2,\ \beta = 0.5,\ m = 0.1,\ M = 2.$

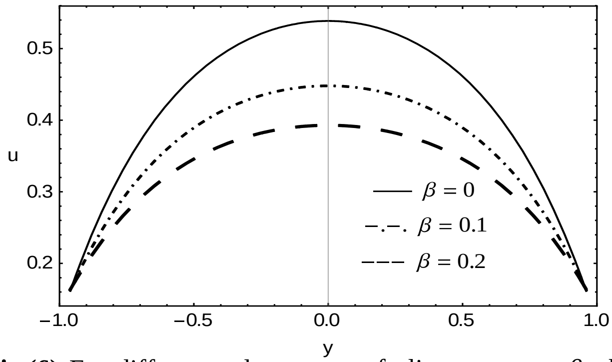

Fig.(6) For different values of slip parameter β, the longitudinal velocity profile u is plotted against y when $\phi = 0.5,\ x = 1,\ q = 0.5,\ K = 0.2,\ M = 0.5,\ m = 0.2.$

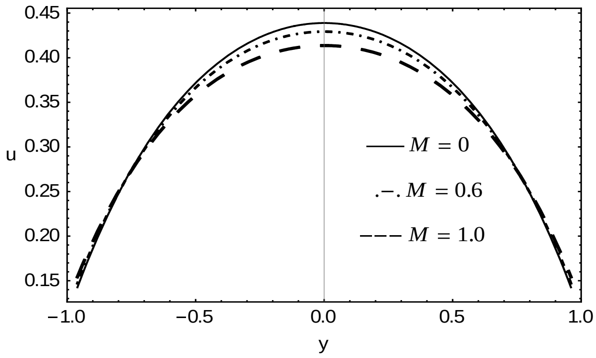

Fig.(7) For different values of Hartmann number M, the longitudinal velocity profile u is plotted against y when $\phi = 0.5,\ x = 1,\ q = 0.5,\ \beta = 0.2,$ $m = 0.2,\ K = 0.3.$

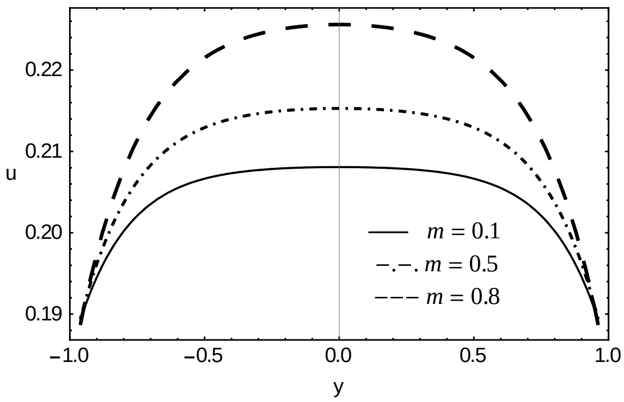

Fig.(8) For different values of Hall parameter m, the longitudinal velocity profile u is plotted against y when $\phi = 0.5,\ x = 1,\ q = 0.5,\ \beta = 0.5,\ M = 5,\ K = 0.3.$

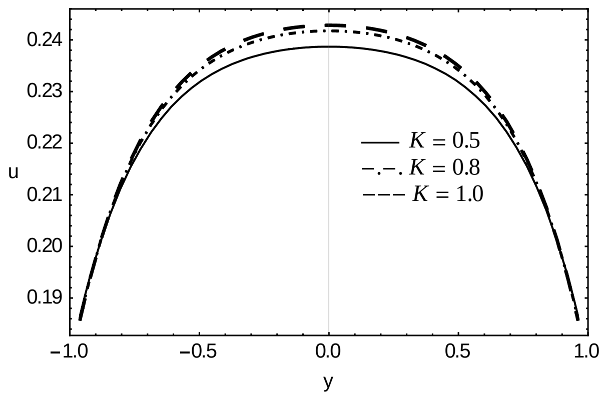

Fig.(9) For different values permeability parameter K, The longitudinal velocity profile u is plotted against y when $\phi = 0.5,\ x = 1,\ q = 0.5,\ \beta = 0.5,\ M = 5,$ $m = 1.$

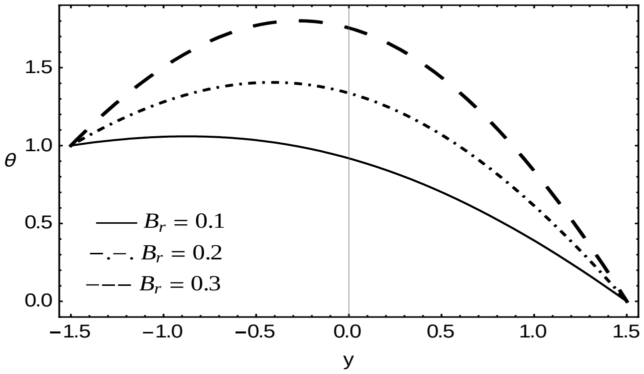

Fig.(10) For different values of the Brinkman number B_r, the temperature distribution Θ is plotted against y when $\phi = 0.5$, $x = 1$, $q = 0.5$, $\beta = 0.5$, $M = 1.5$, $m = 0.1$, $K = 1.2$.

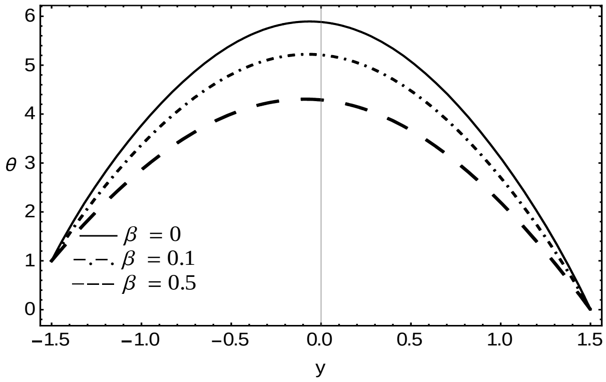

Fig.(11) For different values of the slip parameter β, the temperature distribution Θ is plotted against y when $\phi = 0.5$, $x = 1$, $q = 0.5$, $B_r = 1$, $M = 2$, $m = 1$, $K = 1.2$.

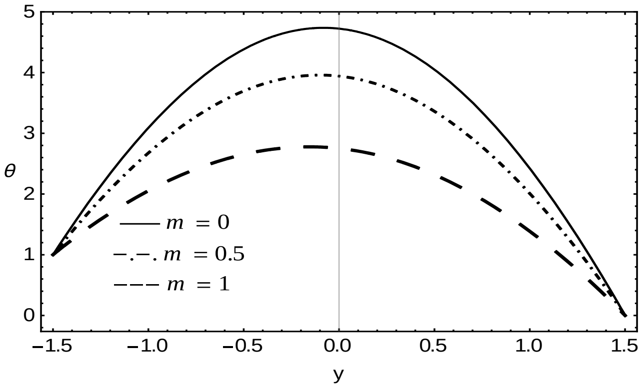

Fig.(12) For different values of the Hall parameter m, the temperature distribution Θ is plotted against y when $\phi = 0.5$, $x = 1$, $q = 0.5$, $\beta = 0.5$, $B_r = 1$, $M = 1.5$, $K = 1.2$.

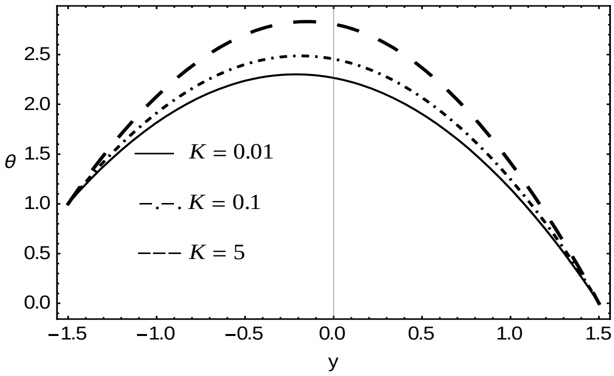

Fig.(13) For different values of the permeability parameter K, the temperature distribution Θ is plotted against y when $\phi = 0.5$, $x = 1$, $q = 0.5$, $\beta = 0.5$, $B_r = 1$, $M = 1.5$, $m = 1$.

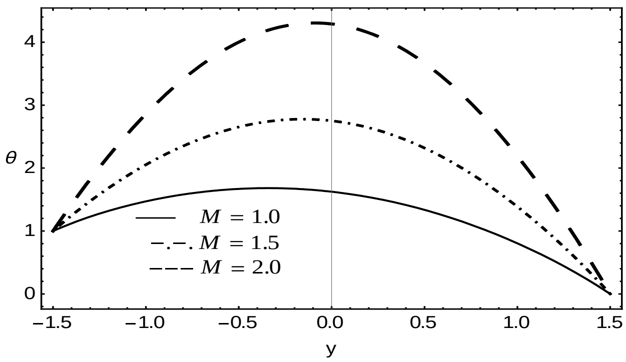

Fig.(14) For different values of the Hartman number M, the temperature distribution Θ is plotted against y when $\phi = 0.5$, $x = 1$, $q = 0.5$, $\beta = 0.5$, $B_r = 1$, $K = 1.2$, $m = 1$.

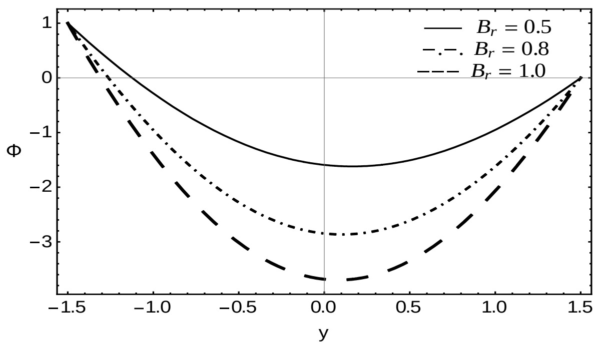

Fig.(15) For different values of the Brinkman number B_r, the concentration distribution Φ is plotted against y when $\phi = 0.5$, $x = 1$, $q = 0.5$, $\beta = 0.5$, $M = 1.5$, $m = 0.1$, $K = 1.2$, $S_c = 1$, $S_r = 1$.

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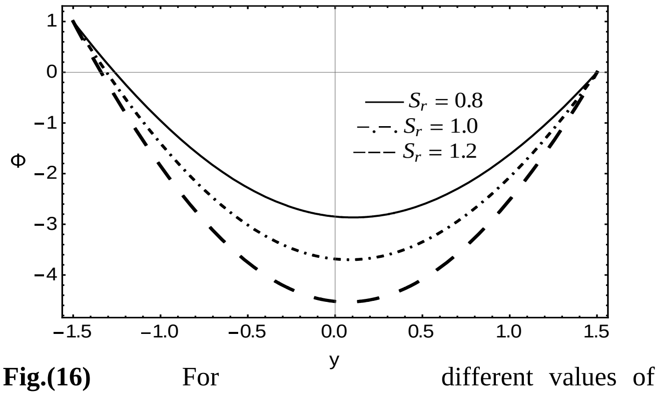

Fig.(16) For different values of the Soret number S_r, the concentration distribution Φ is plotted against y when $\phi = 0.5$, $x = 1$, $q = 0.5$, $\beta = 0.5$, $B_r = 1$, $M = 1.5$, $m = 0.1$, $K = 1.2$, $S_c = 1$.

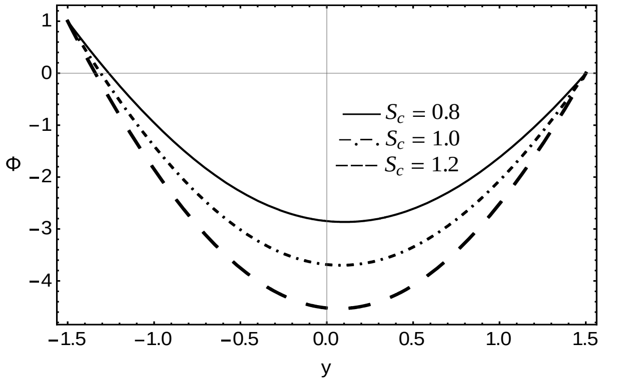

Fig.(17) For different values of the Schmidt number S_c, the concentration distribution Φ is plotted against y when $\phi = 0.5$, $x = 1$, $q = 0.5$, $\beta = 0.5$, $B_r = 1$, $M = 1.5$, $m = 0.1$, $K = 1.2$, $S_r = 1$.

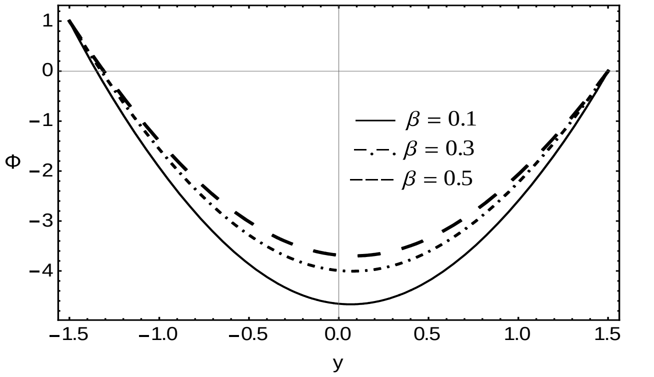

Fig.(18) For different values of the slip parameter β, the concentration distribution Φ is plotted against y when $\phi = 0.5$, $x = 1$, $q = 0.5$, $B_r = 1$, $M = 1.5$, $m = 0.1$, $K = 1.2$, $S_c = 1$, $S_r = 1$.

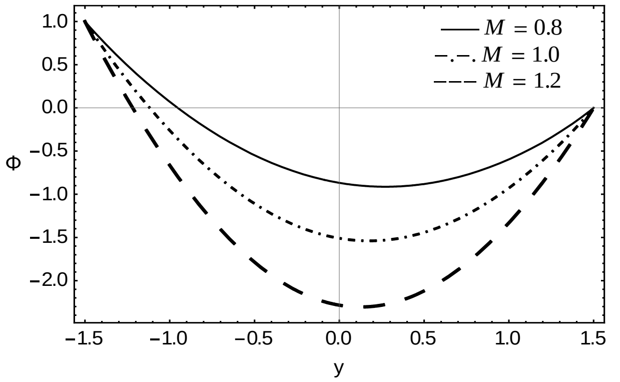

Fig.(19) For different values of the Hartman number M, the concentration distribution Φ is plotted against y when $\phi = 0.5$, $x = 1$, $q = 0.5$, $\beta = 0.5$, $B_r = 1$, $K = 1.2$, $m = 0.1$, $S_c = 1$, $S_r = 1$.

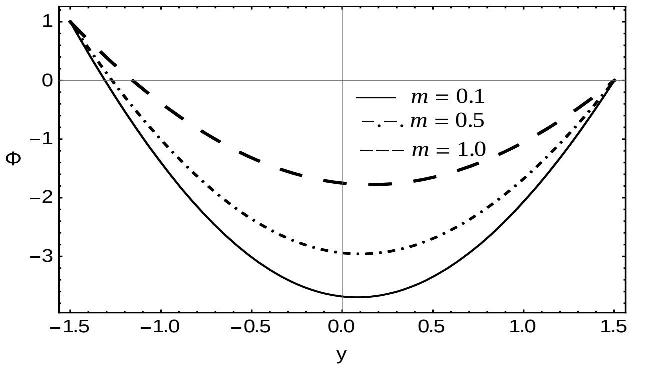

Fig.(20) For different values of the Hall number m, The concentration distribution Φ is plotted against y when $\phi = 0.5$, $x = 1$, $q = 0.5$, $\beta = 0.5$, $B_r = 1$, $K = 1.2$, $M = 1.5$, $S_c = 1$, $S_r = 1$.

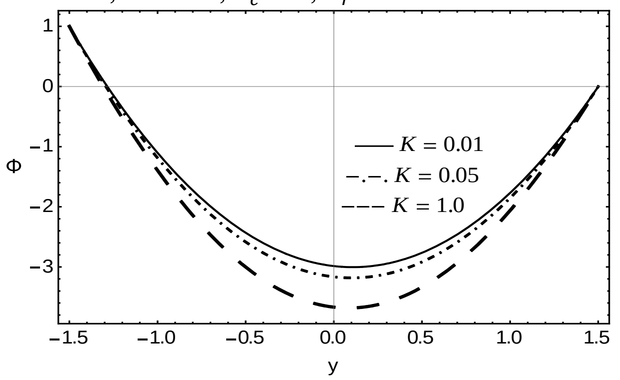

Fig.(21) For different values of the permeability parameter K, the concentration distribution Φ is plotted against y when $\phi = 0.5$, $x = 1$, $q = 0.5$, $\beta = 0.5$, $B_r = 1$, $M = 1.5$, $m = 0.1$, $S_c = 1$, $S_r = 1$.

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AUTHOR PROFILE

Nabil T. M. Eldabe, received the B. S. in 1970 at the Department of Mathematics in Faculty of Education, Ain Shams University, Cairo, Egypt. Also, he received another B.S. in 1972 at the Department of Mathematics in Faculty of Science, Ain Shams University. Furthermore, he received M.S. degree from Girls College, Ain Shams University, in 1975. In1980, he received the PH.D. From Faculty of Science, Assiut University, Assiut, Egypt. From 1970 to 1974, he had been employed as demonstrator, from 1975 to 1979, he had worked as an assistant lecturer, from 1980 to 1984, he had worked as a lecturer, from 1985 to 1989 he had worked as an assistant professor and from 1990 to 2009, he had worked as a professor at Department of Mathematics, Faculty of Education, Ain Shams University. Also, from 1999 to 2005 and 2006 to 2009, he had been employed as head of the Department of Mathematics, Faculty of Education, Ain Shams Un. From 2007 to 2008, he had been employed as director for the Center of Development and Teaching Science at Ain Shams Un. From 2009 until now, he is emeritus professor in Faculty of Education, Ain Shams Un. Further, he had been employed as a head of the Department of Mathematics, Jouner College and Teachers College, Saudi Arabia from (1983 to 1988) and from (1994 to 1999). Also, he received Amin Loutfy prize, one of the Egyptian encouragements prizes in scientific fields (1989). He was teaching most of the mathematics curriculum in Saudi Arabia, the United Arab Emirates and participating in the conferences of the Hashemite Kingdom of Jordan.

Amira S. A. Asar, received the B. S. in 2003 in Mathematics from Mathematics Department, Faculty of Science (Girls), Al-Azhar University, Cairo, Egypt. Furthermore, she received M. Sc degree in Applied Mathematics form Mathematics Department, Faculty of Science (Girls), Al-Azhar University in 2011. In 2015 she got the Ph.D. in Applied Mathematics form Mathematics Department, Faculty of Science (Girls), Al-Azhar University. From 2004 to 2011, she had been employed as demonstrator, from 2012 to 2015, she had been worked as an assistant lecturer at the Department of Mathematics in the Faculty of Science (Girls), Al-Azhar University. She has been worked as a lecturer at Mathematics Department, Faculty of Science (Girls), Al-Azhar University from 2015 until now. Her research field interested in the branches of applied mathematics especially Fluid mechanics

Shaimaa F. Ramadan, received the B. S. in 2006 at the Department of Mathematics in the Faculty of Science (Girls), Al-Azhar University, Cairo, Egypt. Furthermore, she received M.S. degree from the Faculty of Science (Girls) Al-Azhar University in 2011. In 2014 she got the Ph.D. degree from the Faculty of Science (Girls) Al-Azhar University. From 2009 to 2011, she had been employed as demonstrator, from 2012 to 2014, she had been worked as an assistant lecturer at the Department of Mathematics in the Faculty of Science (Girls), Al-Azhar University. She has been worked as a lecturer at Mathematics Department, Faculty of Science (Girls), Al-Azhar University from 2015 until now. Her research field interests are in the branches of applied mathematics especially Fluid mechanics.

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