

Effect of Carbon Nanotubes Volume Fraction on the Deflections and Stresses of Laminated Hybrid Composite Plates using First-Order Shear Deformation Theory



D. Dhanunjaya Raju, V. V. Subba Rao

Abstract: Carbon Nanotubes reinforced composite materials are widely used in aerospace engineering due to high strength-to-weight ratios, which can be tailored per requirements. The behaviour of the composite structures depends on various factors like, Lamination scheme, Ply orientation and loading conditions. In the present work, Static deflection and stresses analysis of a Carbon Nano Tube reinforced laminated hybrid composite plate extends the First Order Shear Deformation Theory with variable ply angles and volume fractions of CNT. A micromechanics model based on the Mori-Tanaka method is used to calculate the properties of CNT reinforced laminate. In this analysis, hybrid composite plates mainly consist of Graphite/epoxy, Kevlar/epoxy and CNT/ polystyrene sub-laminates. The lamination scheme and CNT volume fraction play a vital role in the non-dimensional deflections and stresses. The main intention of this work is to enhance the suitability of CNT reinforced Hybrid composite plates under static loading for structural applications.

Keywords: Carbon Nanotubes, CNT Reinforced Laminates, Hybrid Composites, Static Deflection, Stress Analysis.

I. INTRODUCTION

Composite materials are mostly replacing traditional metals in many engineering applications due to their lightweight and high strength. These properties attract researchers in structural engineering applications. Carbon nanotubes (CNT), with extraordinary mechanical strength, low density, and highest thermal conductivity, improves the stability of traditional composite materials. Such composites are of paramount interest in the aerospace and automobile industries. There are numerous theoretical and experimental results have shown that both single-walled carbon nanotubes (SWCNT) and multi-walled carbon nanotubes (MWCNTs)

have [1] TPa Young's moduli in the axial direction, depending on the diameter and chirality [2]. The Elastic properties of crystals of single-walled carbon nanotubes are evaluated by V.N. Popov[3]. Numerous plate theories are proposed to predict composite plates' transverse deflections and stress distribution. In the classical laminate plate theory (CLPT), transverse shear stresses are neglected, however, the idea gives inaccurate results for thick and moderately thick laminated plates as there are considerable shear deformations. So, it is evident that transverse shear deformations must be considered in the analysis. In FSDT, a shear correction factor compensates for the assumed uniform transverse shear strain variations over the entire plate thickness. Static and dynamic of carbon nanotube-reinforced functionally graded cylindrical panels [4]. Non-dimensional deflections and stresses in laminated plates subjected to sinusoidal load using FSDT are given in [5]. Deflection and stress Analysis of laminated hybrid composite plates using First-order shear deformation theory by D. D. Raju [6]. F.Heidari, K. Taheri, M. Sheybani, M. Janghorban, and A.Tounsi[8] presented the mechanics of nano-composites reinforced by wavydefectedaggregated nanotubes. D. D. Raju and V. V. S. Rao [9-10] presented the deformation of CNT reinforced hybrid laminated composite plates induced by piezoelectric actuators and the effect of thickness ratios. H. S. Shen [11] studied the nonlinear bending behaviour of simply supported, functionally graded CNT-reinforced composite panels under a uniform or sinusoidal shear load with temperature change using a higher-order shear strain plate theory. R. Zerrouki[12] analysed the bending responses of carbon nanotube-reinforced composite beams using the higher-order shear deformation beam theory to determine strain-displacement relationships. This article considers the static analysis of simply-supported laminated composite plates with CNT reinforced laminas subjected to sinusoidally distributed load.

II. MICROMECHANICS MODEL OF CNT REINFORCED LAMINA

In the micromechanics model, Single-wall nanotubes are considered solid fibres with anisotropic material properties.

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The composite is considered transversely isotropic, and the nanotube-polymer interface is perfectly bonded. The stress-strain relation of an elementary cell of the composite material can be expressed as follows,

$$\begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \tau_{23} \\ \tau_{13} \\ \tau_{12} \end{bmatrix} = \begin{bmatrix} n & l & l & 0 & 0 & 0 \\ l & k+m & k-m & 0 & 0 & 0 \\ l & k-m & k+m & 0 & 0 & 0 \\ 0 & 0 & 0 & 2m & 0 & 0 \\ 0 & 0 & 0 & 0 & 2p & 0 \\ 0 & 0 & 0 & 0 & 0 & 2p \end{bmatrix} \begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ \varepsilon_{23} \\ \varepsilon_{13} \\ \varepsilon_{12} \end{bmatrix} \quad (1)$$

where k, l, m, n and p are Hill's elastic moduli
 n = uni-axial tension modulus in the fibre direction
 k = plane strain bulk modulus normal to the fibre direction
 l = cross modulus
 m = shear moduli in planes normal to the fibre direction
 p = shear moduli in planes parallel to the fibre direction

Hill's elastic moduli are calculated by Mori- Tanaka method.

$$= \frac{E_m \{ E_m c_m + 2k_r (1 + v_m) [1 + c_r (1 - 2v_m)] \}}{2(1 + v_m) [E_m (1 + c_r - 2v_m) + 2c_m k_r (1 - v_m - 2v_m^2)]} \quad (2)$$

$$l = \frac{E_m \{ v_m c_m [E_m + 2k_r (1m + v_m)] + 2c_r l_r (1 - v_m^2) \}}{(1 + v_m) [E_m (1 + c_r - 2v_m) + 2c_m k_r (1 - v_m - 2v_m^2)]} \quad (3)$$

$$n = \frac{E_m^2 c_m (1 + c_r - c_m v_m) + 2c_m c_r (k_r n_r - l_r^2) (1 + v_m)^2 (1 - 2v_m)}{(1 + v_m) [E_m (1 + c_r - 2v_m) + 2c_m k_r (1 - v_m - 2v_m^2)]} + \frac{E_m [2c_m^2 k_r (1 - v_m) + c_r n_r (1 - 2v_m + c_r) + 4c_m c_r l_r v_m]}{[E_m (1 + c_r - 2v_m) + 2c_m k_r (1 - v_m - 2v_m^2)]} \quad (4)$$

$$p = \frac{E_m [E_m c_m + 2(1 + c_r) p_r (1 + v_m)]}{2(1 + v_m) [E_m (1 + c_r) + 2c_m p_r (1 + v_m)]} \quad (5)$$

$$m = \frac{E_m [E_m c_m + 2m_r (1 + v_m) (3 + c_r - 4v_m)]}{2(1 + v_m) \{ E_m [c_m + 4c_r (1 - v_m)] + 2c_r m_r (3 - v_m - 4v_m^2) \}} \quad (6)$$

Where k_r, l_r, m_r, n_r and p_r are Hill's elastic moduli for the reinforcing phase.

C_r, E_m and v_m reinforce phase volume fraction, matrix Young's modulus and matrix Poisson's ratio.

The equations of the CNRP as functions of the stiffness constants are determined for a unidirectional composite as follows,

$$E_{11} = n - \frac{l^2}{k} \quad (7)$$

$$E_{22} = \frac{4m(kn - l^2)}{kn - l^2 + mn} \quad (8)$$

$$G_{12} = 2p ;$$

$$G_{23} = \frac{E_{22}}{2(1 + v_{23})}; \quad (9)$$

$$v_{12} = \frac{l}{2k}$$

III. FIRST ORDER SHEAR DEFORMATION THEORY

The First Order Shear Deformation Laminated Plate Theory (FSDT) is an extension of the Classical Lamination Plate theory (CLPT) based on Kirchhoff's assumptions, where transverse normal and shear stresses are not considered. The transverse shear stresses are derived by using 3D elasticity equilibrium equations. These are inaccurate, but the equilibrium-derived transverse stress derived from equilibrium is sufficiently precise for homogeneous and thin plates. These are inaccurate if the plates are relatively thick, i.e. ($a/h < 20$). In FSDT, transverse shear stresses are calculated, and normal transverse stress is neglected. Classical lamination plate theory is suitable for only thin plates, but the First order shear deformation plate theory is ideal for relatively thick plates ($a/h < 20$).

IV. SIMPLY SUPPORTED CROSS-PLY LAMINATED PLATE

In this study, simply-supported laminated composite plates with CNT reinforced laminas are subjected to sinusoidally distributed load considered with various combinations of lamination schemes and fibre orientations.

The mechanical loads are developed in double Fourier sine series as,

$$q(x, y) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} Q_{mn} \sin \alpha x \sin \beta y \quad (10)$$

It is a one-term solution ($Q_{mn} = q_0$ and $m = n = 1$) for a sinusoidally distributed load, the solution is a closed-form solution. For other loads, the Navier solution is a series solution, that can be evaluated for a sufficient number of terms in the series.

$$Q_{mn} = \frac{4}{ab} \int_0^b \int_0^a q(x, y) \sin \alpha x \sin \beta y dx dy \quad (11)$$

Where

$$\alpha = \frac{m\pi}{a}$$

and

$$\beta = \frac{n\pi}{b}$$

For specially orthotropic symmetric laminated plates from the equation of motion



$$\begin{bmatrix} \hat{S}_{11} & \hat{S}_{12} & 0 & 0 & 0 \\ \hat{S}_{12} & \hat{S}_{22} & 0 & 0 & 0 \\ 0 & 0 & \hat{S}_{33} & \hat{S}_{34} & \hat{S}_{35} \\ 0 & 0 & \hat{S}_{34} & \hat{S}_{44} & \hat{S}_{45} \\ 0 & 0 & \hat{S}_{35} & \hat{S}_{45} & \hat{S}_{55} \end{bmatrix} \begin{Bmatrix} U_{mn} \\ V_{mn} \\ W_{mn} \\ X_{mn} \\ Y_{mn} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ Q_{mn} \\ 0 \\ 0 \end{Bmatrix} \quad (12)$$

Where,

$$\hat{S}_{11} = (A_{11}\alpha^2 + A_{66}\beta^2) \quad (13)$$

$$\hat{S}_{12} = (A_{12} + A_{66})\alpha\beta \quad (14)$$

$$\hat{S}_{22} = (A_{66}\alpha^2 + A_{22}\beta^2) \quad (15)$$

$$\hat{S}_{34} = K \cdot A_{55}\alpha \quad (16)$$

$$\hat{S}_{33} = K (A_{55}\alpha^2 + A_{44}\beta^2) \quad (17)$$

$$\hat{S}_{44} = (D_{11}\alpha^2 + D_{66}\beta^2 + KA_{55}) \quad (18)$$

$$\hat{S}_{45} = (D_{12} + D_{66})\alpha\beta \quad (19)$$

$$\hat{S}_{35} = K \cdot A_{44}\beta \quad (20)$$

$$\hat{S}_{55} = (D_{66}\alpha^2 + D_{22}\beta^2 + KA_{44}) \quad (21)$$

Where A_{ij} =Extensional stiffness matrix, D_{ij} =Bending Stiffness Matrix. [5]

The transverse deflections are expressed as

$$w_0(x, y) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} W_{mn} \sin \alpha x \sin \beta y \quad (22)$$

Where

$$W_{mn} = \frac{Q_{mn}}{b_{mn}} \quad (23)$$

$$X_{mn} = \frac{b_1 Q_{mn}}{b_1 b_{mn}}$$

$$Y_{mn} = \frac{b_2 Q_{mn}}{b_0 b_{mn}}$$

$$b_{mn} = \hat{S}_{33} + \hat{S}_{34} \frac{b_1}{b_0} + \hat{S}_{35} \frac{b_2}{b_0}$$

$$b_0 = \hat{S}_{44}\hat{S}_{55} - \hat{S}_{45}\hat{S}_{45} \quad (24)$$

$$b_1 = \hat{S}_{45}\hat{S}_{35} - \hat{S}_{34}\hat{S}_{55}$$

$$b_2 = \hat{S}_{34}\hat{S}_{45} - \hat{S}_{44}\hat{S}_{35}$$

The in-plane normal stresses Eq.(25-26), shear stress Eq. (27), and transverse shear stresses Eq (28-29) are expressed as follows,

$$\sigma_{xx}^{(k)} = -z \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} (\bar{Q}_{11} \alpha X_{mn} + \bar{Q}_{12} \beta Y_{mn}) \sin \alpha x \sin \beta y \quad (25)$$

$$\sigma_{yy}^{(k)} = -z \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} (\bar{Q}_{12} \alpha X_{mn} + \bar{Q}_{22} \beta Y_{mn}) \sin \alpha x \sin \beta y \quad (26)$$

$$\sigma_{xy}^{(k)} = -z \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} (-\bar{Q}_{66} \beta X_{mn} + \bar{Q}_{66} \alpha Y_{mn}) \cos \alpha x \cos \beta y \quad (27)$$

$$\sigma_{yz}^{(k)} = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} (\bar{Q}_{44} Y_{mn} + \bar{Q}_{44}\beta W_{mn}) \sin \alpha x \cos \beta y \quad (28)$$

$$\sigma_{xz}^{(k)} = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} (\bar{Q}_{55} X_{mn} + \bar{Q}_{55} \alpha W_{mn}) \cos \alpha x \sin \beta y \quad (29)$$

Non-dimensional maximum transverse deflection Eq. (30), normal stresses Eq. (31-32), shear stress Eq. (33) and Transverse shear stresses Eq. (34-35) are expressed as follows,

$$\bar{w} = w_0 \left(\frac{a}{2}, \frac{b}{2}, \frac{h}{2} \right) \left(\frac{E_2 h^3}{a^4 q_0} \right) \times 10^2 \quad (30)$$

$$\bar{\sigma}_{xx} = \sigma_{xx} \left(\frac{a}{2}, \frac{b}{2}, \frac{h}{2} \right) \left(\frac{h^2}{a^2 q_0} \right) \quad (31)$$

$$\bar{\sigma}_{yy} = \sigma_{yy} \left(\frac{a}{2}, \frac{b}{2}, \frac{h}{4} \right) \left(\frac{h^2}{a^2 q_0} \right) \quad (32)$$

$$\bar{\sigma}_{xy} = \sigma_{xy} \left(a, b, -\frac{h}{2} \right) \left(\frac{h^2}{a^2 q_0} \right) \quad (33)$$

$$\bar{\sigma}_{xz} = \sigma_{xz} \left(a, b, -\frac{h}{2} \right) \left(\frac{h}{b q_0} \right) \quad (34)$$

$$\bar{\sigma}_{yz} = \sigma_{yz} \left(a, b, -\frac{h}{2} \right) \left(\frac{h}{b q_0} \right) \quad (35)$$

V. RESULTS AND DISCUSSIONS

Consider a simply supported square, cross-ply [0/90/90/0] composite plate subjected to sinusoidally distributed load. Non-dimensional transverse maximum central deflections, Normal stresses, Shear stress, and Transverse Shear stress are calculated and validated with the published results [5]. The material properties are $E_1=25$, $E_2=1$, $G_{12}=G_{13}=0.5E_2$, $G_{23}=0.2E_2$ and $\nu_{12}=0.25$, $K=5/6$.



Table I. Effect of Aspect ratio on the Non-dimensional deflections and Normal stresses

a/h	Reference \bar{w}	Present \bar{w}	Reference $\bar{\sigma}_{xx}$	Present $\bar{\sigma}_{xx}$	Reference $\bar{\sigma}_{yy}$	Present $\bar{\sigma}_{yy}$
10	0.6627	0.6627	0.4989	0.4989	0.3614	0.3614
15	0.5367	0.5367	0.5192	0.5192	0.3144	0.3144
20	0.4912	0.4912	0.5273	0.5273	0.2957	0.2957
50	0.4409	0.4409	0.5368	0.5368	0.2737	0.2737
100	0.4337	0.4337	0.5382	0.5382	0.2705	0.2705

Table II. Effect of Aspect ratio on the Non-dimensional shear and Transverse shear stresses

a/h	Reference $\bar{\sigma}_{xy}$	Present $\bar{\sigma}_{xy}$	Reference $\bar{\sigma}_{xz}$	Present $\bar{\sigma}_{xz}$	Reference $\bar{\sigma}_{yz}$	Present $\bar{\sigma}_{yz}$
10	0.0241	0.0241	0.4165	0.4165	0.1292	0.1292
15	0.0227	0.0227	0.4311	0.4311	0.1145	0.1145
20	0.0221	0.0221	0.4370	0.4370	0.1087	0.1087
50	0.0214	0.0214	0.4438	0.4438	0.1019	0.1019
100	0.0213	0.0213	0.4448	0.4448	0.1008	0.1008

The numerical results were validated against the published results by developed code and agreed well with the published results. Then the MATLAB code is further extended to find out the deflections and stresses of Simply supported square laminated plates Subjected to SSL. The properties of the lamina, SWCNT, and Polystyrene matrix material are as given in the Table. 3

Table III. Properties of the Composite lamina

Engineering constant [GPa]	Graphite/epoxy (G) [5]	Kevlar/epoxy(K) [7]	SWCNT (C) [9]	polystyrene
E_{11}	137.9	87	–	1.9
E_{22}	8.96	5.5	–	–
G_{12}	7.20	2.2	–	–
G_{13}	7.20	2.2	–	–
G_{23}	6.21	2.2	–	–
V_{12}	0.62	0.3	–	0.3
n_r	–	–	450	–
k_r	–	–	30	–
m_r	–	–	1	–
p_r	–	–	1	–
l_r	–	–	10	–

The Non-dimensional maximum transverse deflections, normal stresses, shear stresses and transverse shear stresses are evaluated for various Hybrid composite plates with a/h=10 subjected to SSL, with different fibre orientations as given in Table IV.

Table IV. Effect of fibre orientation and lamination Schemes on the Non-dimensional deflections and stresses.

Fibre Orientation	Lamination Scheme	\bar{w}	$\bar{\sigma}_{xx}$	$\bar{\sigma}_{yy}$	$\bar{\sigma}_{xy}$	$\bar{\sigma}_{xz}$	$\bar{\sigma}_{yz}$
0/90/90/0	CNT/CNT/CNT/CNT	0.2671	0.5049	0.5034	0.0141	0.4479	0.1479
	G/G/G/G	0.7097	0.459	0.26	0.0498	0.3079	0.1023
	K/K/K/K	0.8387	0.4908	0.3062	0.0271	0.2903	0.0917
	CNT/G/G/CNT	0.1791	0.5696	0.2492	0.0127	0.1629	0.0296
	G/CNT/CNT/G	0.7501	0.4336	0.4123	0.0486	0.494	0.2001
	CNT/K/K/CNT	0.2399	0.5867	0.2006	0.0149	0.3233	0.0716
	K/CNT/CNT/K	0.7987	0.4042	0.6787	0.0233	0.3762	0.1485
0/90/0/90	CNT/CNT/CNT/CNT	0.2294	0.0083	0.289	0.0108	0.2979	0.084
	G/G/G/G	0.6879	0.0392	0.2363	0.0481	0.2051	0.1769
	K/K/K/K	0.7927	0.0427	0.2573	0.0253	0.191	0.191
	CNT/G/G/CNT	0.1614	0.0086	0.187	0.0111	0.0983	0.0277
	G/CNT/CNT/G	0.7217	0.0378	0.3609	0.0465	0.3425	0.2954
	CNT/K/K/CNT	0.2119	0.0089	0.1238	0.0116	0.2239	0.0632
	K/CNT/CNT/K	0.7465	0.0371	0.5566	0.022	0.2384	0.2384
0/45/45/0	CNT/CNT/CNT/CNT	0.2365	0.4665	0.1156	0.0125	0.385	0.1431
	G/G/G/G	0.6669	0.4258	0.0829	0.0466	0.3075	0.0948
	K/K/K/K	0.7912	0.4462	0.0908	0.0252	0.3009	0.0811
	CNT/G/G/CNT	0.1705	0.5374	0.0788	0.0121	0.158	0.0264
	G/CNT/CNT/G	0.6764	0.38	0.1005	0.0432	0.4816	0.1851
	CNT/K/K/CNT	0.2364	0.5587	0.0613	0.0146	0.33	0.0613
	K/CNT/CNT/K	0.6861	0.3358	0.1547	0.0196	0.3411	0.1358

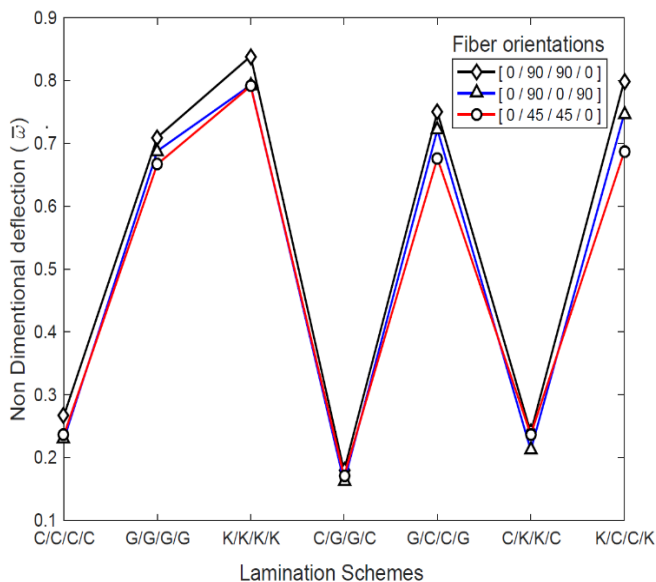


Fig. 1. Effect of hybrid lamination schemes on non-dimensional deflections

From Fig. 1, it can be seen that [0/45/45/0] fibre orientation laminated plates are stiffer than the others. The composite laminate with outer CNT reinforced lamina can carry heavier loads with minimal deflection.

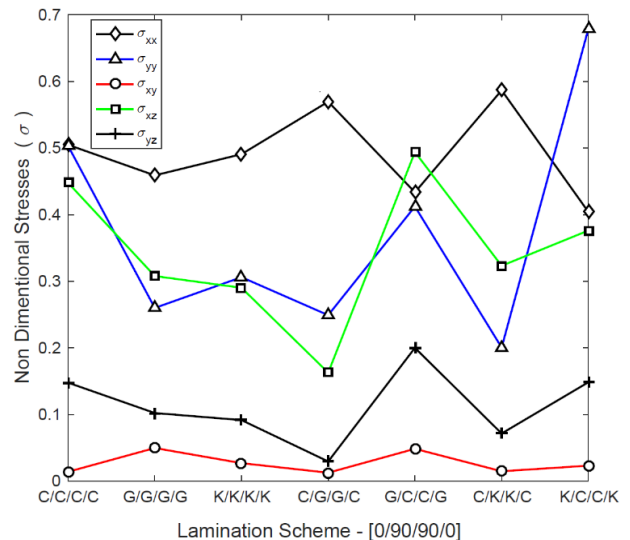


Fig. 2. Effect of hybrid lamination schemes on Non-dimensional stresses with [0/90/90/0] fibre orientation

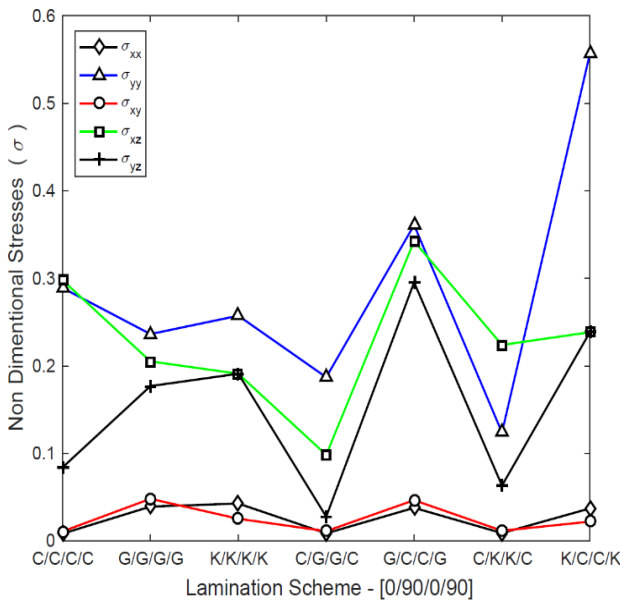


Fig. 3. Effect of hybrid lamination schemes on Non-dimensional stresses with [0/90/0/90] fibre orientation

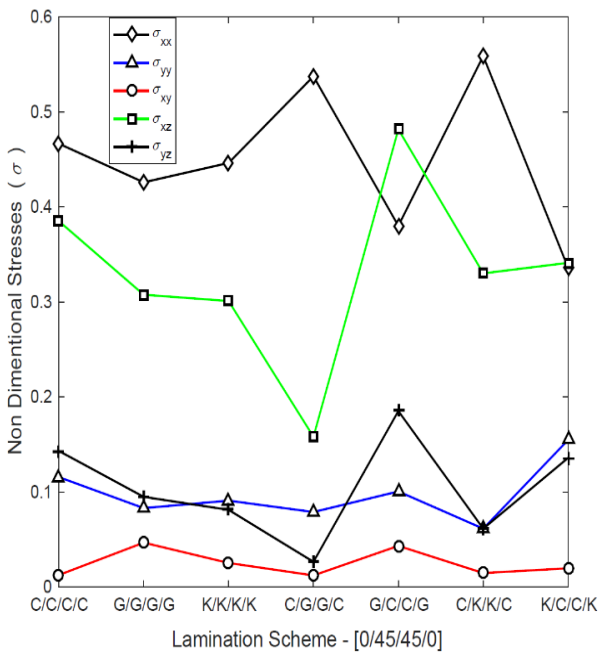


Fig. 4. Effect of hybrid lamination schemes on Non-dimensional stresses with [0/45/45/0] fibre orientation

From Fig. 2-4, symmetrically laminated plates can withstand the highest in-plane stresses, but these are weak in terms of in-plane shear stresses. Ply orientation, and a hybrid lamination scheme are vital in dimensionless stresses.

A. CNT Reinforced Laminate

Further, it investigated the strength of the CNT-reinforced Laminated plates with the various fibre orientations and lamination schemes. The volume fraction of the CNT Reinforcement plays a vital role in the non-dimensional deflections. From Fig. 5, as the volume fraction (fibre to matrix material) increases, the in-plane strength of the composite increases, but after a specific limit, the debonding of fibre to matrix material will occur.

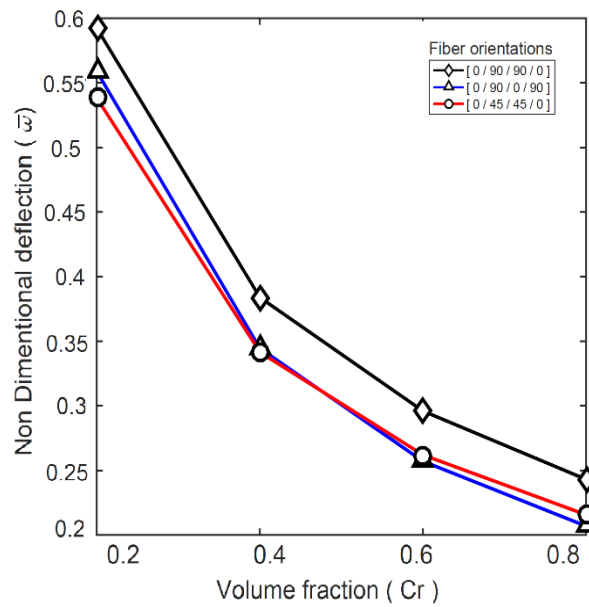


Fig. 5. Effect of fibre volume fraction orientation on the Non-dimensional deflections

VI. CONCLUSION

In this article, the static responses of simply-supported laminated composite plates with CNT reinforced laminae are subjected to sinusoidally distributed load are carried. The non-dimensional transverse central deflections, Normal stresses, Shear stress, and Transverse Shear stresses were evaluated using FSDT. This analytical procedure is extended to the hybrid composite plates with CNT reinforced laminates. Symmetric laminated plates can withstand the highest in-plane stresses, but these are weak in in-plane shear stresses. Ply orientation and a Hybrid lamination scheme plays a vital role in the Non-dimensional stresses. As the CNT volume fraction (fibre to matrix material) increases, the in-plane strength of the composite increases, but after a certain limit, the debonding of fibre to matrix material occurs.

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