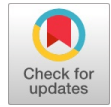


RNGA based Centralized PI Controller for Multivariable Non Square Systems using Direct Synthesis Method



C.V. Nageswara Rao, Putta Vihari

Abstract: The design of centralised PI controllers for multivariable non-square systems is proposed in the present work. The centralised controller is designed using the direct synthesis method. The method includes approximating the inverse of the process transfer matrix with the effective transfer function matrix. The practical transfer function for each element in the process transfer function matrix is derived by using the relative normalized gain array (RNGA), and relative average residence time array (RARTA) concepts proposed by Cai et al [1]. The transfer function models used in the present work include first-order processes with time delay (FOPDT). The Maclaurin series is applied to reduce the resulting controllers into standard PI forms. The design method requires a single tuning parameter (filter time constant) to adjust the performance of the controller. A simulation study is conducted for various case studies, and the results demonstrate the advantages of the proposed method over the literature-reported methods. The control algorithms are comparatively analysed using a standard robust stability measure. The designed controllers give a good performance with lesser interaction compared to the literature methods, the Davison Method [2] and Tanntu and Lieslehto's method [3].

Keywords: First Order Plus Dead Time, Multivariable, Centralized, Maclaurin Series, Pi Controllers, Relative Normalized Gain Array, Relative Average Residence Time Array and Effective Transfer Function.

I. INTRODUCTION

Multivariable systems with an unequal number of input and output variables are referred to as non-square systems. Systems with more outputs than inputs are generally undesirable, as it is impossible to maintain all outputs at the set point due to system limitations. Systems with more inputs than outputs are frequently encountered in process industries. The occurrence of such systems is more common in the chemical process industries. Literature reveals that most commonly dealt with non-square systems are the Shell control problem [4],[5] with three manipulated variables and two controlled variables, and the mixing tank problem with three manipulated variables and two controlled variables [6].

One of the techniques to control such systems is to break down the system and design the decentralized multivariable controllers [7]. Literature study [2] shows that it results in poor performance because of the information neglected by its structure.

Cai et al [1] proposed a new loop pairing criterion based on a new method for interaction measurement. In their work, both the steady-state and transient information of the process transfer function are investigated, and the RNGA is introduced for loop interaction measurements. They have demonstrated the effectiveness of the method applied in designing decentralised controllers for square systems, for which the RGA-based loop pairing criterion yields an inaccurate interaction assessment.

Loh and Chiu [8] have deduced that non square systems should be controlled in their original state instead of squaring down by adding or deleting the variables. Sharma and Chidambaram [9] have proposed a method to control non-square systems using Davison's method [2] to design centralized controllers. They have extended Davison's method [2] to control non square systems. Vijay et al [10] proposed centralized multivariable PI controllers for MIMO processes. Two centralized controllers (one using RGA and the other using the practical transfer function (ETF) derived from an RNGA-RARTA) are designed based on the direct synthesis method. In the present work, the direct synthesis method [11], [12] is used to design the centralized controller for non-square systems. In this method, the inverse of the process transfer function matrix required for this method is estimated by using the Relative Average Residence Time Array (RARTA) and the Relative Normalized Gain Array (RNGA) concept proposed by Cai et al [1].

Moore-Penrose Pseudo-inverse is used to find the inverse of Non-Square multivariable systems (Non-square Matrices). The overall design method includes three steps.

1. Using the concepts of normalised gain, find (i) the relative normalised gain array (RNGA) and (ii) the relative average residence time array (RARTA) of a given transfer function matrix using the inverse of the matrix.
2. Use the information obtained in the first step to get an effective transfer function matrix for the closed-loop system.
3. Design the centralized controller by approximating the inverse of the process transfer function matrix with the transpose of the effective transfer function matrix in the direct synthesis method.

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The resultant controller is not in the standard PI form. To obtain the controller in standard PI form, the Maclaurin series expansion is applied to the controller matrix.

II. CONTROL SYSTEM DESIGN

Consider an M-input and N-output open-loop stable multivariable system. $G(s)$ and $G_c(s)$ are the process transfer function matrix and full-dimensional controller matrix with compatible dimensions, expressed by

$$G(s) = \begin{bmatrix} g_{11}(s) & g_{12}(s) & \cdots & g_{1n}(s) \\ g_{21}(s) & g_{22}(s) & \cdots & g_{2n}(s) \\ \vdots & \vdots & \ddots & \vdots \\ g_{m1}(s) & g_{m2}(s) & \cdots & g_{mn}(s) \end{bmatrix} \quad (1)$$

$$G_c(s) = \begin{bmatrix} g_{c,11}(s) & g_{c,12}(s) & \cdots & g_{c,1m}(s) \\ g_{c,21}(s) & g_{c,22}(s) & \cdots & g_{c,2m}(s) \\ \vdots & \vdots & \ddots & \vdots \\ g_{c,n1}(s) & g_{c,n2}(s) & \cdots & g_{c,nm}(s) \end{bmatrix} \quad (2)$$

Let each element of the process transfer function matrix be represented by a first-order plus dead time (FOPDT) model, i.e.

$$g_{ij}(s) = \tilde{g}_{ij}(s)e^{-\theta_{ij}s}, \quad i, j = 1, 2, \dots, n \quad (3)$$

We know that Controller design using the direct synthesis method by

$$G_c(S) = \frac{G_{cl}(S)}{G(S)(I + G_{cl}(S))} \quad (4)$$

Where $G_{cl}(S)$ Is the desired closed-loop transfer function. According to IMC theory (Morari and Zafiriou [13], Nageswara Rao and Padmasree [14]), the desired closed-loop transfer function $G_{cl}(S)$ The i^{th} loop is chosen as

$$G_{cl}(S) = \frac{e^{-\theta_{ij}s}}{(\lambda_{ij}s+1)^{r_{ij}}} \prod_{k=1}^{q_{ij}} \frac{z_{k-1}}{z_{k-1}+s} \quad (5)$$

After substituting Equation (5) in Equation (4) and using Penrose Moore pseudo-inverse whenever the inverse of the process transfer function matrix is required, we get

$$g_{c,ji}(s) = \{[G^+(s)]^T\} \left(\frac{e^{-\theta_{ij}s} \prod_{k=1}^{q_{ij}} \frac{z_{k-1}}{z_{k-1}+s}}{(\lambda_{ij}s+1)^{r_{ij}} e^{-\theta_{ij}s} \prod_{k=1}^{q_{ij}} \frac{z_{k-1}}{z_{k-1}+s}} \right) \quad (6)$$

$G^+(s)$ is estimated by using the concepts RNGA, RARTA and ETF proposed by Cai et al [1]. 2009. Finally, the Controller for FOPDT is obtained as

$$g_{c,ji}(s) = \frac{(\hat{\tau}_{ij}s+1)}{\hat{k}_{ij}} \left(\frac{e^{(\hat{\theta}_{ij}-\theta_{ij})s}}{(\lambda_{ij}+\theta_{ij})s} \right) \quad (7)$$

The Controller Obtained now is converted into a standard PI controller form using the Maclaurin series Expansion.

$$g_{c,ji}(s) = \frac{1}{s} [P_{ji}(0) + sP'_{ji}(0) + \cdots] \quad (8)$$

Compared to the standard PI controller form, the resulting controller parameters are

$$k_{I,ji} = P_{ji}(0) = \frac{1}{\hat{k}_{ij}(\lambda_{ij}+\theta_{ij})} \quad (9)$$

$$k_{c,ji} = P'_{ji}(0) = \frac{[(\hat{\theta}_{ij}-\theta_{ij})+\hat{\tau}_{ij}]}{\hat{k}_{ij}(\lambda_{ij}+\theta_{ij})} \quad (10)$$

Where $\hat{\theta}_{ij}, \hat{\tau}_{ij}$ and \hat{k}_{ij} They are closed-loop Time delay, Time constant and gains.

The controller is expected to provide better performance because, in designing the controller, we utilised dynamic interaction effects (RNGA-RARTA) in conjunction with steady-state effects (RGA). The proposed controller response is compared with that of the literature methods proposed by Davison [2] and Tanttu & Lieslehto [3]. Performance analysis is also carried out in terms of TV, IAE, ISE, and ITAE, and compared with the literature-reported methods.

A. Case Study – 1: Shell control problem

The process considered is a Shell control problem [4], [5] with two controlled variables and three manipulated variables, in which the two controlled variables are the composition of the top product and that of the side stream. The manipulated variables are top draw, side draw and the bottom reflux. As the delay compensator is significant for delay-dominant processes, here the time delays are considered to be five times the time constants of each transfer function in the process.

The resulting process is given by

$$G(s) = \begin{bmatrix} \frac{4.05e^{-250s}}{50s+1} & \frac{1.77e^{-300s}}{60s+1} & \frac{5.88e^{-250s}}{50s+1} \\ \frac{5.39e^{-250s}}{50s+1} & \frac{5.72e^{-300s}}{60s+1} & \frac{6.9e^{-200s}}{40s+1} \end{bmatrix}$$

Normalized gain matrix (K_N), RGA (Λ), RNGA (ϕ), and RARTA (Γ) can be calculated as

$$K_N = \begin{bmatrix} 0.0135 & 0.0049 & 0.0196 \\ 0.0180 & 0.0159 & 0.0288 \end{bmatrix};$$

$$\Lambda = \begin{bmatrix} 0.3203 & -0.5946 & 1.2744 \\ -0.0170 & 1.5733 & -0.5563 \end{bmatrix};$$

$$\phi = \begin{bmatrix} 0.8893 & -0.7626 & 0.8734 \\ -0.5162 & 1.7346 & -0.2184 \end{bmatrix};$$

$$\Gamma = \begin{bmatrix} 2.7767 & 1.2825 & 0.6853 \\ 30.3495 & 1.1025 & 0.3926 \end{bmatrix};$$

Centralized controller matrix based on RNGA-RARTA, using Equations (9) and (10), is given as

$$G_{cl}(s) = \begin{bmatrix} 0.1222 + \frac{1.0e^{-3} \times 0.1647}{s} & -0.0098 + \frac{1.0e^{-3} \times -0.0053}{s} \\ -0.1252 + \frac{1.0e^{-3} \times -0.6999}{s} & 0.0360 + \frac{1.0e^{-3} \times 0.4596}{s} \\ -0.0163 + \frac{1.0e^{-3} \times 0.4515}{s} & 0.0150 + \frac{1.0e^{-3} \times -0.1347}{s} \end{bmatrix}$$

The controller is tuned by adjusting filter time constants as $\lambda_1 = 75$ and $\lambda_2 = 112$.

Controller settings using Davison's method [2] is calculated as in the Equation below.



$$G_{C2}(s) = \begin{bmatrix} 1.0e^{-3} \times 0.0791 + \frac{1.0e^{-3} \times 0.0949}{s} & 1.0e^{-3} \times -0.0032 + \frac{1.0e^{-3} \times -0.0038}{s} \\ 1.0e^{-3} \times -0.3360 + \frac{1.0e^{-3} \times -0.4031}{s} & 1.0e^{-3} \times 0.2751 + \frac{1.0e^{-3} \times 0.3301}{s} \\ 1.0e^{-3} \times 0.2167 + \frac{1.0e^{-3} \times 0.2601}{s} & 1.0e^{-3} \times -0.0806 + \frac{1.0e^{-3} \times -0.0967}{s} \end{bmatrix}$$

And by Tantt and Lieslehto's method [3], Controller is given as

$$G_{C3}(s) = \begin{bmatrix} 0.0809 + \frac{1.0e^{-3} \times 0.1665}{s} & -0.0353 + \frac{1.0e^{-3} \times -0.0066}{s} \\ -0.1905 + \frac{1.0e^{-3} \times -0.7073}{s} & 0.1341 + \frac{1.0e^{-3} \times 0.5791}{s} \\ 0.0547 + \frac{1.0e^{-3} \times 0.4563}{s} & -0.0093 + \frac{1.0e^{-3} \times -0.1697}{s} \end{bmatrix}$$

Fig. 1 shows the closed-loop response of the Shell control problem subject to sequential unit step changes at $t = 0$ and $t = 3000$ s, respectively. It can be seen that the interactions are lowered in the RNGA-based centralized PI controller compared to the controllers proposed by Davison [2] and Tantt & Lieslehto [3]. This is because RNGA-RARTA includes dynamic interaction effects rather than only steady-state interactions. Hence, the interaction effects due to changes in other outputs are comparatively less than those of any other method. The controller performance is also checked by comparing the values of Total variation (TV), IAE, ISE, ITAE values of the controller obtained from

the proposed work with those of Davison's and Tantt and Lieslehto's methods [2], [3]. Table 1 presents a comparison of the values of TV, IAE, ISE, and ITAE for the three methods under consideration. Fig. 2 is a graphical representation of the frequency plot of the spectral radius, which represents the stability bounds for the shell control problem. The controller has been designed to exhibit robust stability, as it satisfies the minor gain theorem in terms of spectral radius. The proposed controller demonstrated intermediate robust stability compared to the controllers proposed by Davison and Lieslehto. The controller based on RNGA is found to be more stable than the Lieslehto controller. All three controllers show more stability at higher frequencies.

To demonstrate the robust performance of the proposed method, a simulation study was conducted by introducing perturbation uncertainty into all parameters simultaneously in the actual process. Nominal process controller settings are used for perturbation models. The resulting performance index for the model mismatches (perturbation in all parameters) is tabulated in Table I. A perturbation of $\pm 10\%$ and $\pm 30\%$ in the process gain, time delay and time constant is considered, and the corresponding responses are shown in Figs. 3, 4, 5 and 6. The proposed method yields a significant performance improvement compared to the methods reported in the literature. This shows that a centralised controller provides comparable performance to literature method-based controllers, even under uncertain conditions.

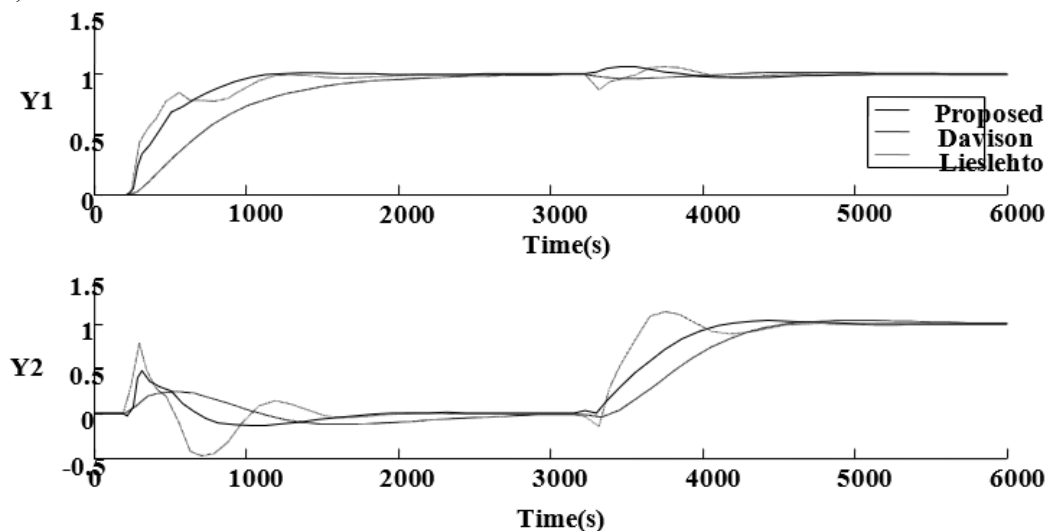


Fig. 1. Closed-loop responses to the sequential step changes in the set point for Shell Control Problem (First half of the bottom plot represents interaction when step change in Y1, second half represents response when step change in Y2)

The robust stability analysis of the perturbed model with $\pm 10\%$ and $\pm 30\%$ uncertainty also reveals that the proposed method is strong even in the presence of model uncertainties. Refer to Table 1.

B. Case Study 2: Mixing tank problem

The process considered is a mixing tank problem with two controlled variables and three manipulated variables, where the two controlled variables are the height of the liquid in the tank and the exit concentration. The manipulated variables are the flow rates of the three input streams. Time delays are intentionally added to demonstrate the improvement of the delay compensator.

The resulting process is given as

$$G(s) = \begin{bmatrix} \frac{4e^{-100s}}{20s+1} & \frac{4e^{-100s}}{20s+1} & \frac{4e^{-100s}}{20s+1} \\ \frac{3e^{-50s}}{10s+1} & \frac{-3e^{-100s}}{10s+1} & \frac{5e^{-50s}}{10s+1} \end{bmatrix}$$

Normalized gain matrix (K_N), RGA (Λ), RNGA (ϕ), and RARTA (Γ) could be calculated as

$$K_N = \begin{bmatrix} 0.0333 & 0.0333 & 0.0333 \\ 0.0500 & -0.0273 & -0.0833 \end{bmatrix};$$

$$\Lambda = \begin{bmatrix} 0.2692 & 0.55577 & 0.1731 \\ 0.1154 & 0.4038 & 0.4808 \end{bmatrix};$$

$$\phi = \begin{bmatrix} 0.2529 & 0.6772 & 0.0699 \\ 0.1137 & 0.2653 & 0.6210 \end{bmatrix};$$

$$\Gamma = \begin{bmatrix} 0.9394 & 1.2143 & 0.4038 \\ 0.9857 & 0.6569 & 1.2917 \end{bmatrix};$$

Centralized controller matrix based on RNGA-RARTA using Equations (9) and (10) as

$$G_{C1}(s) = \begin{bmatrix} 0.0422 + \frac{1.0e^{-3} \times 0.4417}{s} & -0.0008 + \frac{1.0e^{-3} \times 0.2657}{s} \\ 0.0535 + \frac{1.0e^{-3} \times 0.9150}{s} & 0.0009 + \frac{1.0e^{-3} \times -0.9299}{s} \\ -0.0183 + \frac{1.0e^{-3} \times 0.2685}{s} & -0.0004 + \frac{1.0e^{-3} \times 0.6642}{s} \end{bmatrix}$$

The controller is tuned by adjusting filter time constants as $\lambda_1 = 15$ and $\lambda_2 = 13$. Controller for this case using Davison's method [2] is calculated as

$$G_{C2}(s) = \begin{bmatrix} 0.0289 + \frac{0.0007}{s} & 0.0165 + \frac{0.0004}{s} \\ 0.0600 + \frac{0.0014}{s} & -0.0579 + \frac{-0.0013}{s} \\ 0.0186 + \frac{0.0004}{s} & 0.0413 + \frac{0.0010}{s} \end{bmatrix}$$

TABLE I - Performance analysis comparison for all uncertainties for Case Study – 1

Model (nominal/uncertain)	Method	TV	IAE	ISE	ITAE
No Uncertainties	RNGA (Proposed)	1.168	1.46801×10^3	0.966×10^3	2.8237×10^5
	Davison Method	1.032	2.1612×10^3	1.443×10^3	4.3557×10^5
	Tanttu and Lieslehto method	1.322	1.4770×10^3	0.961×10^3	2.5425×10^5
+30% uncertainty	RNGA (Proposed)	0.94	1.8349×10^3	1.096×10^3	4.0246×10^5
	Davison Method	0.98	2.5247×10^3	1.686×10^3	5.5090×10^5
	Tanttu and Lieslehto method	1.162	1.6149×10^3	0.976×10^3	2.9539×10^5
-30% uncertainty	RNGA (Proposed)	2.77	2.5625×10^3	1.5158×10^3	5.8586×10^5
	Davison Method	1.36	2.0641×10^3	1.3162×10^3	4.3439×10^5
	Tanttu and Lieslehto method	2.203	1.9998×10^3	1.2264×10^3	4.0096×10^5
+10% uncertainty	RNGA (Proposed)	1.013	1.5467×10^3	992.3421	3.0803×10^5
	Davison Method	1.004	2.2701×10^3	1.5209×10^3	4.6366×10^5
	Tanttu and Lieslehto method	1.224	1.4909×10^3	958.7869	2.5746×10^5
-10% uncertainty	RNGA (Proposed)	1.45	1.5636×10^3	988.9643	3.1412×10^5
	Davison Method	1.087	2.0738×10^3	1.3736×10^3	4.1598×10^5
	Tanttu and Lieslehto method	1.46	1.5225×10^3	986.2283	2.7030×10^5

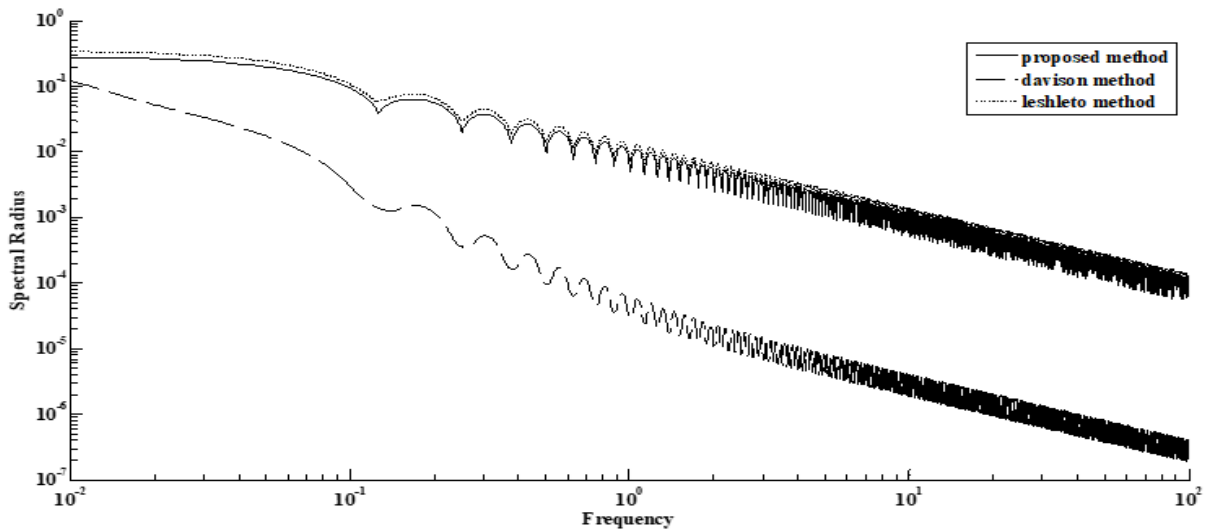


Fig. 2. Frequency plot of spectral radius with no perturbations included.

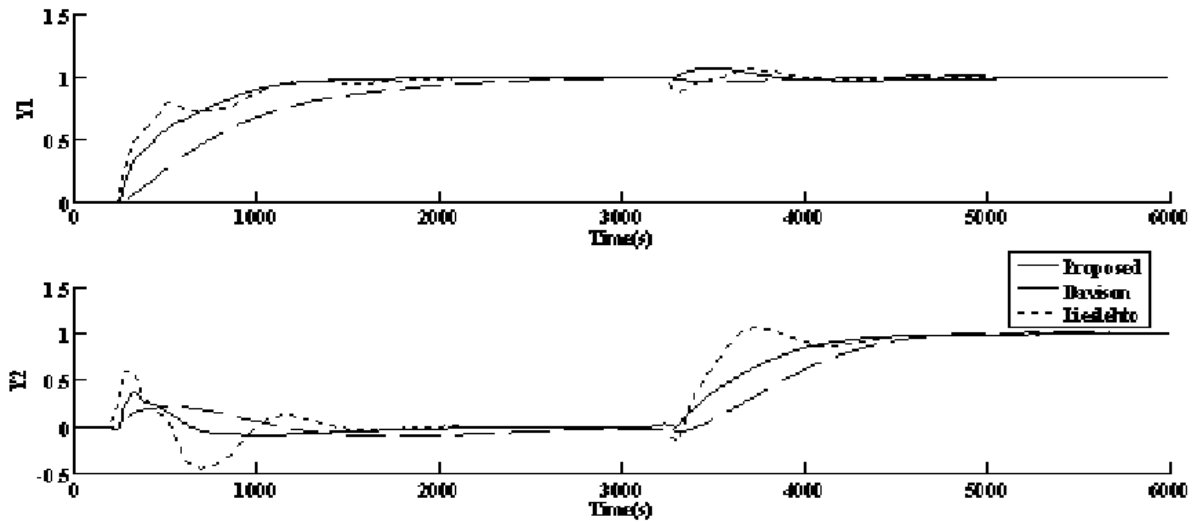


Fig.3. Response curve with +10% uncertainty in all parameters.

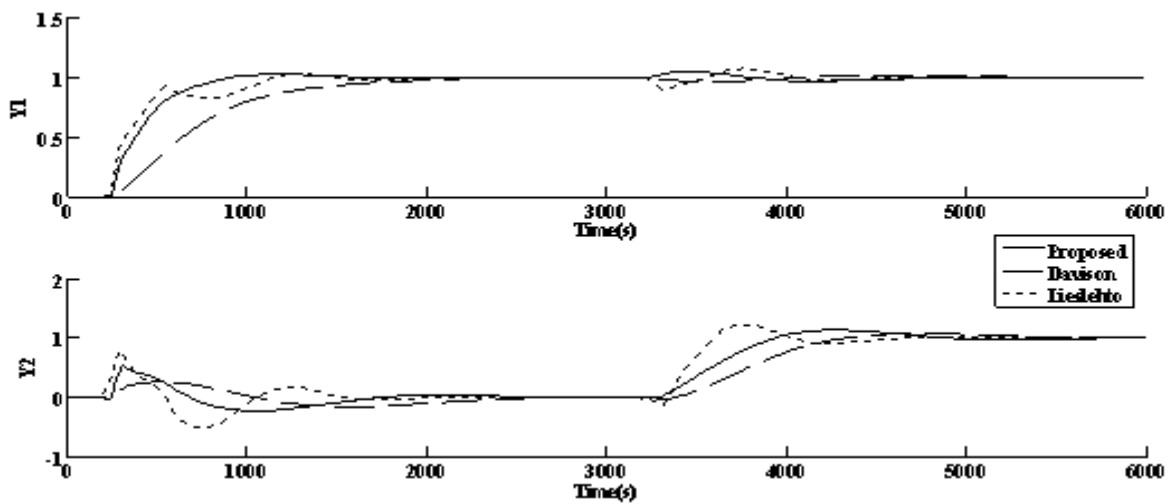


Fig.4. Response curve with -10% uncertainty in all parameters.

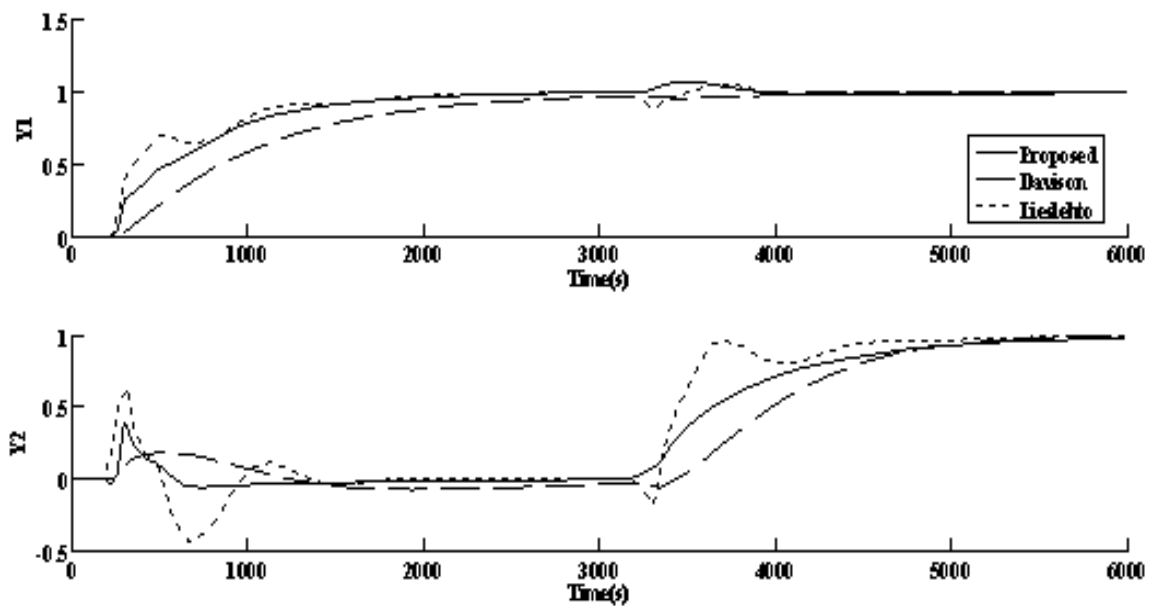


Fig.5. Response curve with +30% uncertainty in all parameters.

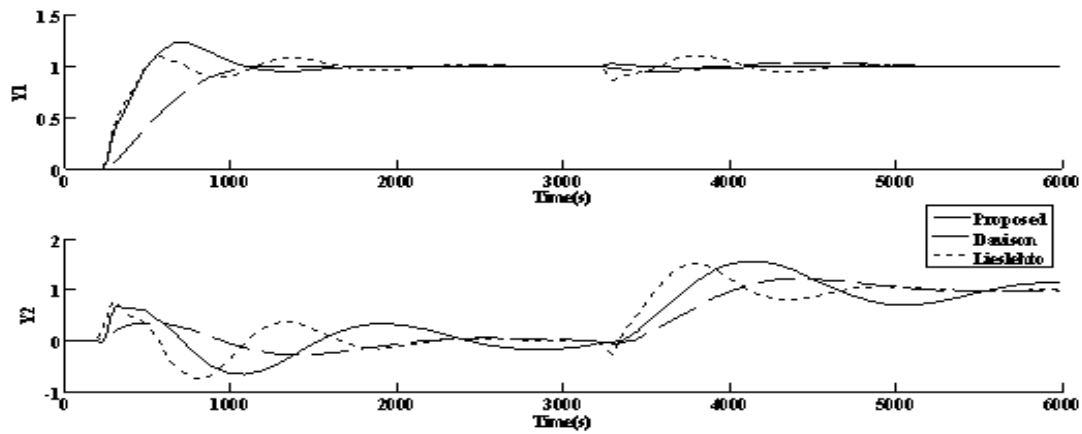


Fig.6. Response curve with -30% uncertainty in all parameters.

Controller by Tanttu and Lieslehto's method [3] as

$$G_{C3}(s) = \begin{bmatrix} 0.0306 + \frac{1.0e^{-3} \times 0.4642}{s} & 0.0092 + \frac{1.0e^{-3} \times 0.2653}{s} \\ 0.0803 + \frac{1.0e^{-3} \times 0.9615}{s} & -0.0385 + \frac{1.0e^{-3} \times -0.9284}{s} \\ 0.0097 + \frac{1.0e^{-3} \times 0.2984}{s} & 0.0293 + \frac{1.0e^{-3} \times 0.6631}{s} \end{bmatrix}$$

Fig. 7 shows the closed-loop response of the mixing tank problem subject to sequential unit step changes at $t = 0$ and $t = 900$, respectively. It can be seen that the interactions are lower in RNGA-based centralised controllers compared to the controllers proposed by Davison, Tanttu, and Lieslehto. In this case, the interaction effects due to changes in other outputs are comparatively less than those of any other method.

The controller performance is also checked by comparing the values of TV, IAE, ISE and ITAE values of the controller obtained from the proposed work with those of Davison's and Tanttu and Lieslehto's methods [2], [3]. Table 2 shows the comparison of values for TV, IAE, ISE, and ITAE for the three methods under consideration. This indicates that the proposed controller exhibits fewer interaction effects while maintaining good performance. Fig. 8 is a graphical representation of the frequency plot of the spectral radius, which represents the stability bounds for the shell control problem. The controller has

been designed to exhibit robust stability, as it satisfies the minor gain theorem in terms of spectral radius. The proposed controller demonstrated intermediate robust stability compared to the controllers proposed by Davison and Lieslehto. The controller based on RNGA is found to be more stable than the Lieslehto controller. All three controllers show more stability at higher frequencies. To demonstrate the robust performance of the proposed method, a simulation study was conducted by introducing perturbation uncertainty into all parameters simultaneously in the actual process. Nominal process controller settings are used for perturbation models.

A perturbation of $\pm 10\%$ and $\pm 30\%$ in the process gain, time delay and time constant is considered, and the corresponding closed-loop responses are shown in Figures Fig. 9, Fig. 10, Fig. 11 and Fig. 12.

The resulting performance index for the model mismatches (perturbation in all parameters) is tabulated in Table II. The proposed method yields a significant performance improvement compared to literature method-based controllers. IAE values are less affected in centralized controller scheme. This shows that a centralized controller provides comparable performance to literature method-based controllers, even under uncertain conditions. The response curves indicate that the method proposed by Davison renders the controller unstable at a 30% uncertainty.

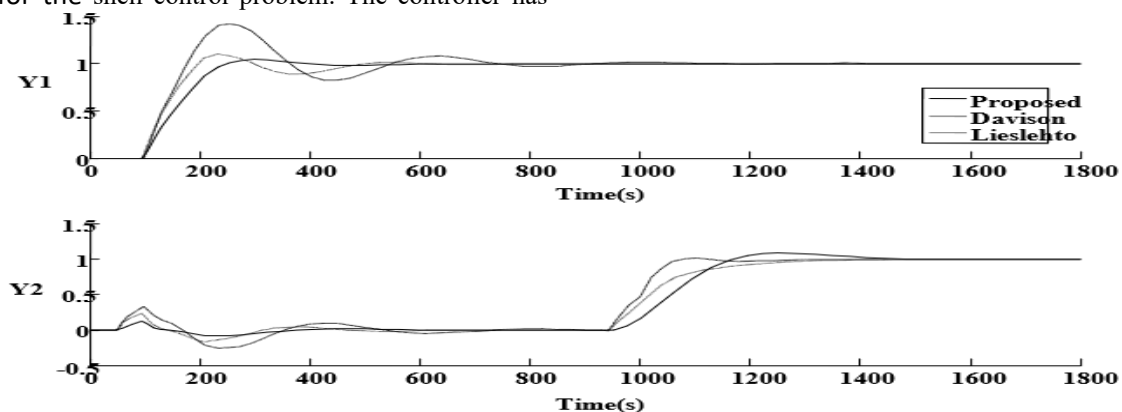


Fig.7: Closed-loop responses to the sequential step changes in the set point for the Mixing Tank. (First half of the bottom plot represents interaction when step change in Y1, second half represents response when step change in Y2)

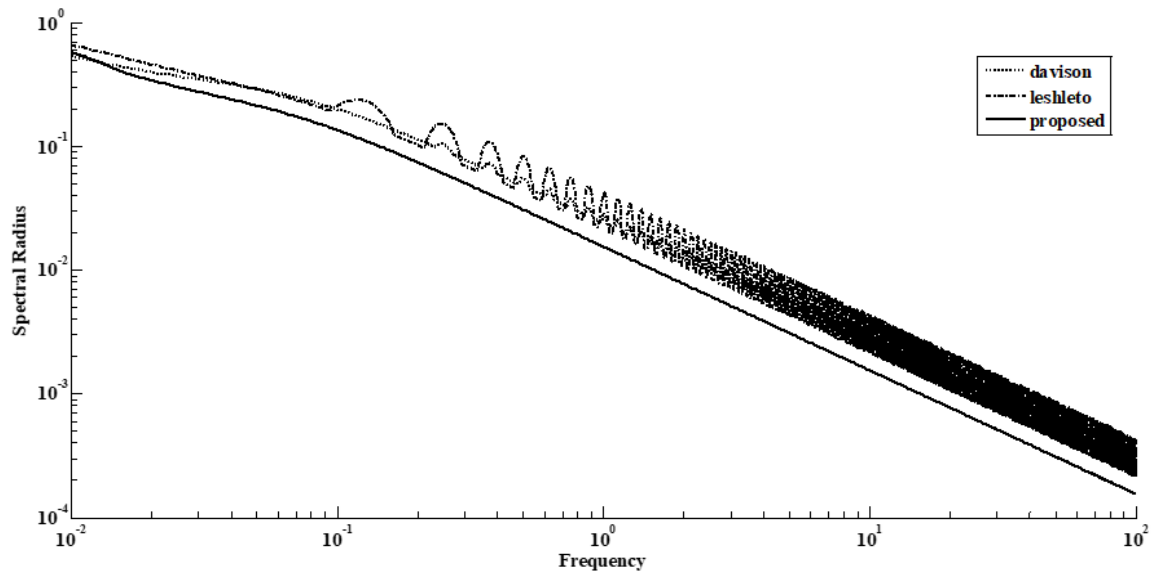


Fig. 8: Frequency plot of spectral radius with no perturbations included.

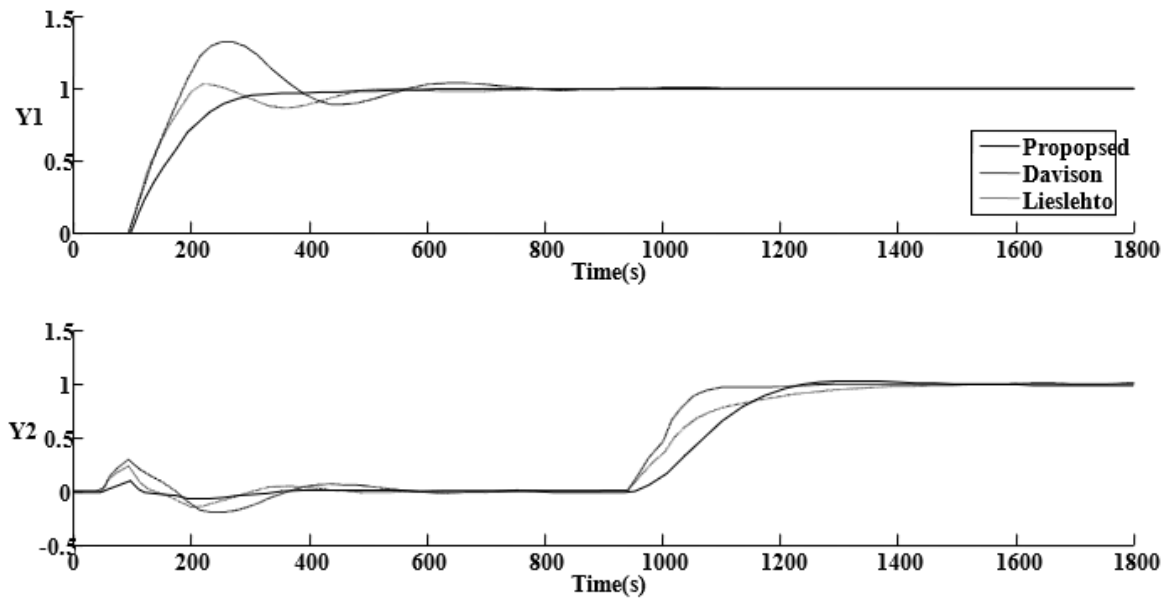


Fig.9: Response curve with +10% uncertainty in all parameters.

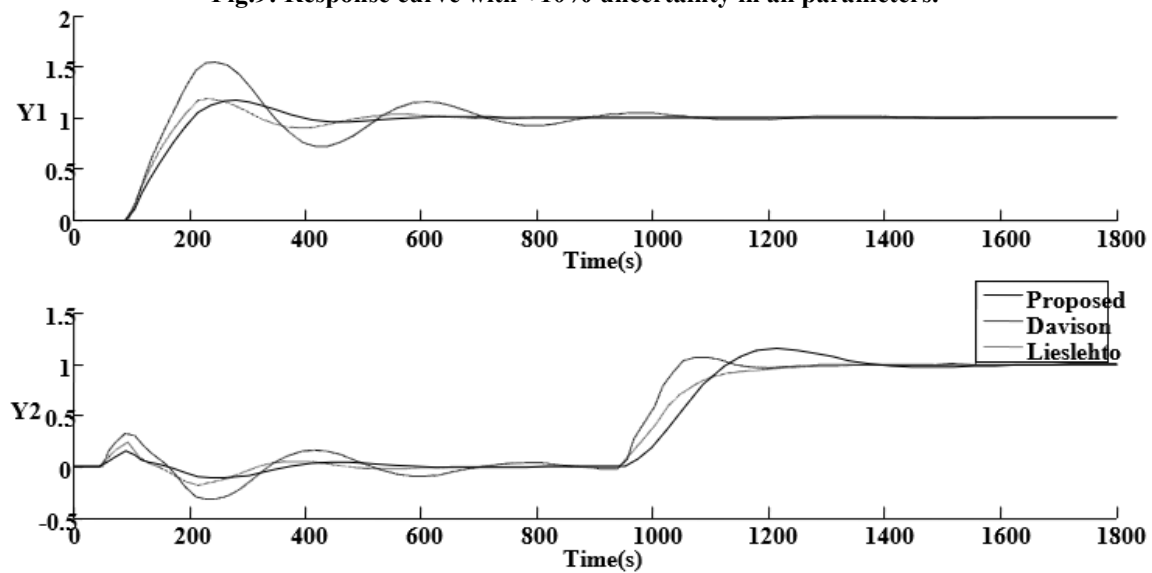


Fig.10: Response curve with -10% uncertainty in all parameters.

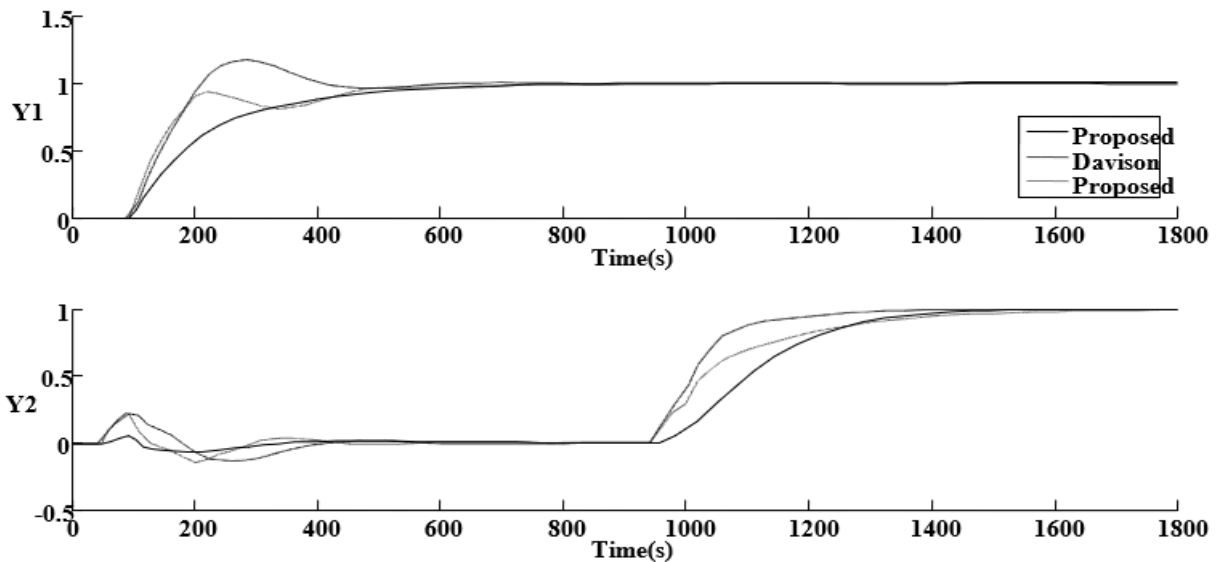


Fig. 11: Response curve with +30% uncertainty in all parameters.

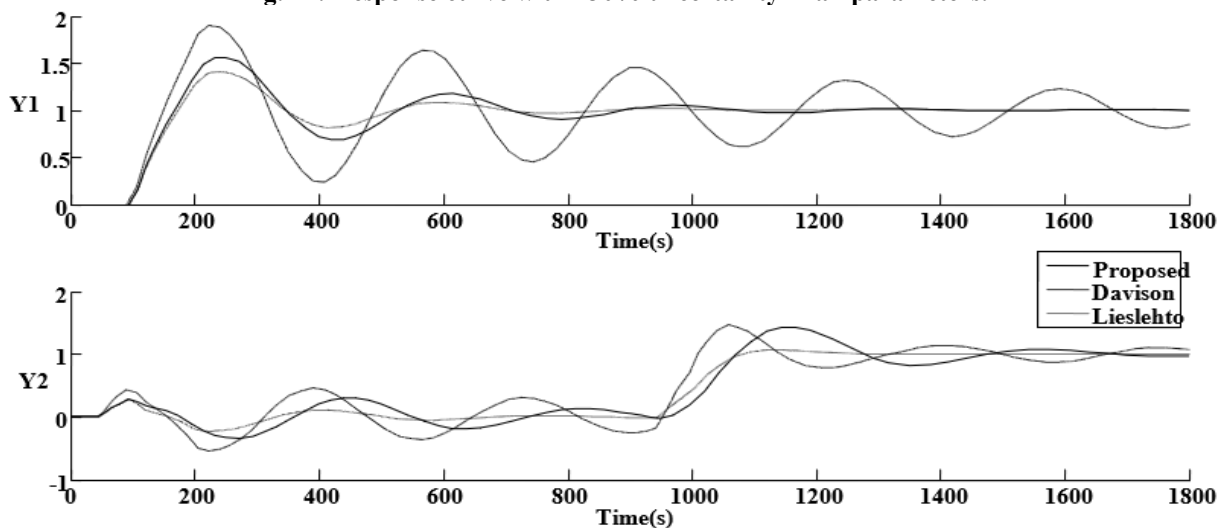


Fig.12: Response curve with -30% uncertainty in all parameters.

TABLE-II. Performance analysis comparison for nominal and model uncertainties for Case Study – 2

Model (nominal/uncertain)	Method	TV	IAE	ISE	ITAE
No Uncertainties	RNGA (Proposed)	0.5877	380.5549	285.5328	1.9351×10^4
	Davison Method	0.8889	412.0307	251.2634	1.7110×10^4
	Tanttu and Lieslehto method	0.5778	360.5407	245.3196	1.8357×10^4
+30% Uncertainty	RNGA (Proposed)	0.4648	505.4479	348.9271	2.9642×10^4
	Davison Method	0.5703	364.5779	245.6360	1.6862×10^4
	Tanttu and Lieslehto method	0.5242	424.5589	263.1792	2.4575×10^4
-30% Uncertainty	RNGA (Proposed)	1.7781	648.0409	341.7058	3.8376×10^4
	Davison Method	3.4023	1.0523×10^3	553.3447	7.1014×10^4
	Tanttu and Lieslehto method	0.8949	407.2970	251.0726	1.8098×10^4
+10% Uncertainty	RNGA (Proposed)	0.4998	403.9143	302.9529	2.2468×10^4
	Davison Method	0.7210	385.1315	244.9739	1.6300×10^4
	Tanttu and Lieslehto method	0.5521	371.4965	248.3244	1.9934×10^4
-10% Uncertainty	RNGA (Proposed)	0.7416	398.1495	275.2281	2.1947×10^4
	Davison Method	1.1612	475.9689	268.4038	2.0533×10^4
	Tanttu and Lieslehto method	0.6213	357.6549	242.3320	1.6994×10^4

The robust stability analysis of the perturbed model with 30% uncertainty is shown in Figs. 13 and 14, respectively. From these graphs, it can be concluded that the proposed method shows comparable robust stability even in the presence of model uncertainties.

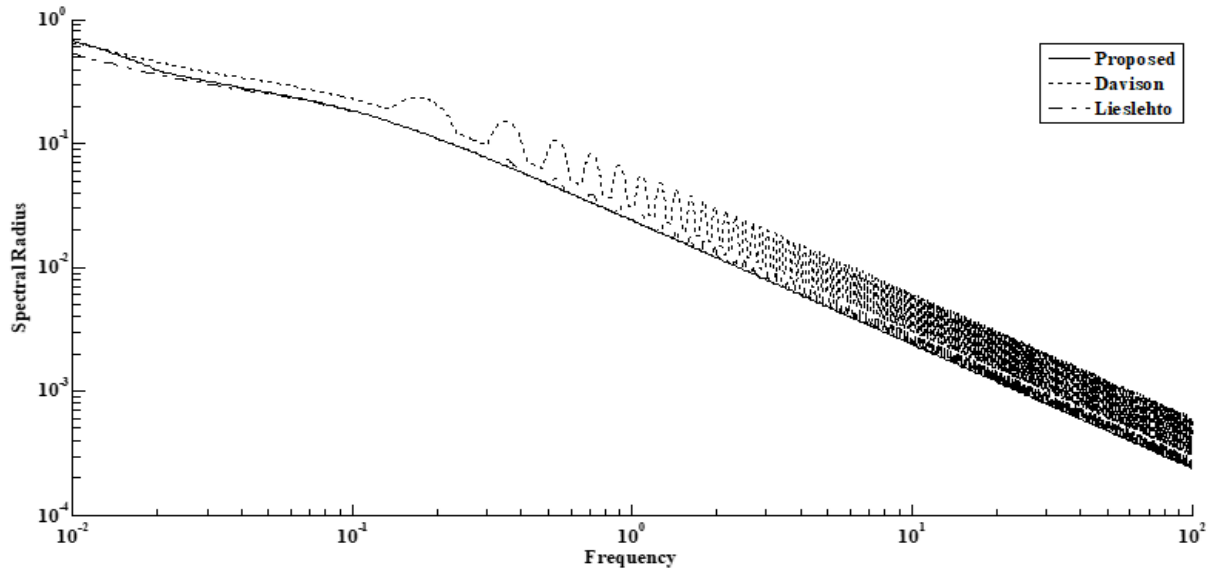


Fig.13. Frequency plot of spectral radius with -30% uncertainty in all parameters.

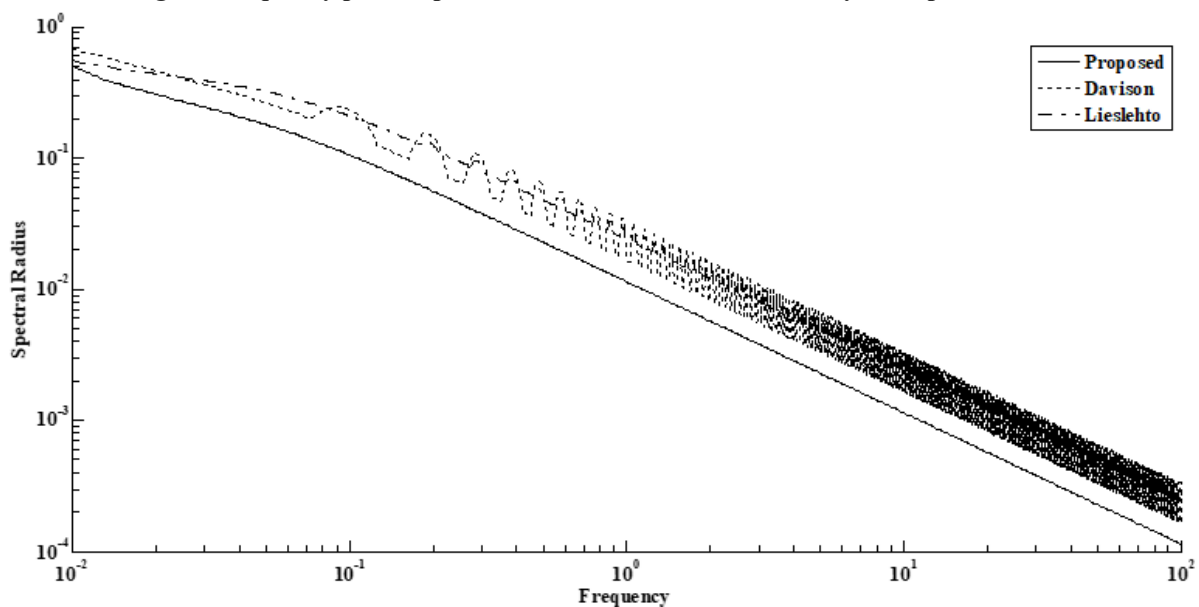


Fig.14. Frequency plot of spectral radius with +30% uncertainty in all parameters.

III. CONCLUSION

In this work, centralized multivariable PI controllers are designed for non-square multivariable systems. The proposed controller is designed based on the direct synthesis method. RNGA & RARTA are used to estimate the inverse of the process transfer function matrix in the direct synthesis method. Since the inverse does not exist for non-square multivariable systems, the Moore-Penrose Pseudo-inverse is used in place of the inverse of a matrix when it is required for calculation. The resulting transfer function model of the controller is converted into standard PI controller forms using a Maclaurin series expansion. The performance analysis is carried out by comparing the performance of the proposed controller with that of controllers obtained by literature methods in terms of ISE, ITAE and TV. Since RNGA includes both steady-state and transient information, the proposed method has a less pronounced effect of interactions than the methods proposed by Davison & Tantt and Lieslehto when a step change is given in one of the set points. The performance parameters, such as ISE, ITAE, and Total Variation (TV), are appreciable when compared to

conventional methods. The proposed method also demonstrated comparable robust stability compared to existing literature methods.

DECLARATION

Funding/ Financial Support	Grants/ Grants/ Financial Support	No, I did not receive.
Conflicts of Interest/ Competing Interests	Conflicts of Interest/ Competing Interests	No conflicts of interest to the best of our knowledge.
Ethical Approval and Consent to Participate	Ethical Approval and Consent to Participate	No, the article does not require ethical approval or consent to participate, as it presents evidence.

Availability of Data and Material/ Data Access Statement	Not relevant.
Authors Contributions	Mr. Putta Vihari has worked to obtain the data for the case study considered. Accordingly, the controller design methodology is decided upon. The second author, Dr. C. V. Nageswara Rao, conducted a simulation study using a Simulation tool while training the first author. A method of RNGA-based controller design is utilised, based on a brainstorming discussion among the authors. As there are few research works that address non-square systems, this methodology is adopted. Generally, a PI controller is sufficient for controlling a maximum of variables, and thus, the design of a PI controller.

AUTHORS PROFILE



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Mr. Vihari Putta is an operations manager at the coal gasification plant in JSP-Angul. He has pursued his master's in chemical engineering from Gayatri Vidya Parishad College of Engineering (Autonomous). He has 17 years of experience in the industrial sectors of refining, petrochemicals, and gasification. His research areas of interest include Process Control, Gasification, Gas Sweetening and Process Optimization. He has participated in and presented papers at International Conferences and national conferences organised by premier institutes. He has attended the National Seminar on State Environment: Environment Protection, Management and Governance at Andhra University. Additionally, he has attended workshops on Advances in Separation Processes, Linear and Nonlinear Finite Element Methods, and MATLAB Applications in Chemical Engineering.

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