Systemic Multi-Objective Optimization of Induction Motor-Driven Electromechanical Systems

Noureddine Ferchichi, Houssem Ben Aribia

Abstract: The optimization of electromechanical systems, such as those involving induction motors and gearboxes, is crucial for improving energy efficiency, system performance, and reliability in industrial applications. This paper presents an advanced methodology for optimizing the energy efficiency of electromechanical systems, integrating both mechanical and electrical subsystems to minimize the system's overall weight, energy losses, and transient response time. The optimisation problem is approached holistically, considering the interdependence of various system parameters and employing multi-objective optimisation techniques to address conflicting objectives. The analysis focuses on optimizing the gearbox speed ratio to minimize the relative weight of the motor-gearbox system while maintaining operational efficiency. A systematic approach, utilising convex surrogate modelling and multi-stage gearboxes, is proposed to enhance the scalability of the solution. The results demonstrate the existence of optimal gearbox speed ratios for various motor sizes and configurations, providing insights into the best design choices for minimising system weight and optimising performance. These findings apply to a range of real-world systems, including electric vehicles and industrial machinery, where minimising weight and optimising energy efficiency are crucial for enhancing overall system performance and reducing operational costs.

Keywords: Energy Efficiency, Multi-Objective Optimization, Minimum System Weight, Systemic Approach.

I. INTRODUCTION

In various industrial sectors, the increasing need for real-time management, improved quality, higher productivity, reduced production costs, and enhanced operational safety has led to the development of advanced methodologies. A notable approach involves optimizing production system topologies using genetic algorithms (GAs) combined with neural networks [1]. This method employs similarity-based mutation and recombination to generate new system designs, while discrete-event simulation is used to evaluate their performance. To reduce computational costs, neural networks serve as surrogate models, approximating the outcomes of simulations. Studies have demonstrated that both unassisted and neural network-assisted GAs effectively identify optimal solutions in industrial settings, with the latter offering superior scalability as the number of potential solutions increases [2].

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Optimization is a continually evolving field, driven by technological advancements and industrial transformations, as companies strive to maximize the efficiency of their production systems. Theoretically, the goal of optimization is to identify optimal solutions that enable electromechanical systems to operate at their maximum capacity. However, in practice, unforeseen failures often caused by incorrect system sizing can result in catastrophic consequences for businesses. Traditional selection methods for the fundamental components of an electromechanical system, which typically involve selecting each element independently based on empirical criteria, often result in issues with adaptation and sizing.

Intelligent design methods, such as machine learning algorithms, have been explored to optimise the technological design of microelectromechanical systems (MEMS). These methods analyze efficiency and implement generative design techniques to solve complex optimization problems, aiming to enhance system performance and reliability [3].

By integrating intelligent design methods, engineers can address the limitations of traditional selection approaches, leading to better-adapted and appropriately sized electromechanical systems. This integration enhances system performance and reduces the risk of unforeseen failures, contributing to more efficient and reliable production systems.

The parameters of an electromechanical system are interdependent and interact with one another as presented in Figure 1. Designing electromechanical systems requires a holistic methodology that considers the interdependence of parameters and the integration of subsystems, including electricity, electronics, mechanics, hydraulics, and chemistry. Traditional optimisation techniques often address parameters and shapes separately, which can result in suboptimal designs. The integrated subsystems, which may encompass electricity, electronics, mechanics, hydraulics, or chemistry, must be optimized collectively as part of the overall system. When the objective is to design a heterogeneous system characterised as "complex with interacting parameters," the problem must be addressed with global consistency under a set of criteria that are optimised simultaneously. This approach ensures high quality, enhanced reliability, and reduced costs [4].
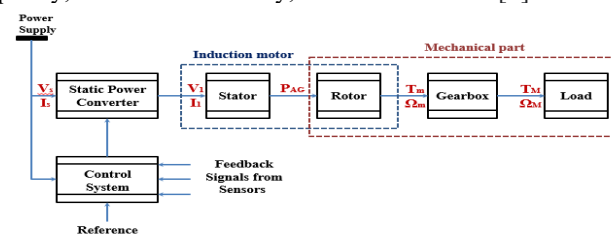
[Fig.1: Typical Diagram of an Electromechanical System]

The systemic approach Contrasts sharply with the early 20th-century mechanistic perspective, which decomposed systems into elementary subsystems, each addressed independently by

specialists. This reductionist method often led to locally optimized components that failed to coalesce into a globally optimal system, primarily due to a lack of interdisciplinary collaboration. In contrast, the systemic approach positions the system within its broader environment and examines it holistically, integrating its intended purpose. Multidisciplinary research, viewed as a complex system, emphasizes the importance of integrating diverse disciplinary perspectives to address intricate problems effectively. By fostering collaboration among various fields, this approach enhances the quality and reliability of outcomes, ensuring that the assembled system aligns with original specifications and operates optimally within its intended context [5].

Optimization in problem-solving extends beyond the mere application of mathematical techniques; it is a systematic process involving multiple phases, each subject to revision if the solution proves unsuccessful. In the context of multi-objective optimization for metal-cutting processes, evolutionary algorithms like NSGA-II have been employed to balance conflicting objectives such as production quality and time [6]. When production requirements change, process parameters must be re-optimized, often necessitating costly simulations. To mitigate this, solution adaptation strategies have been developed, enabling algorithms to transfer solutions from previous optimisation tasks, thereby reducing the number of evaluations required for re-optimisation. This approach underscores that optimization tools and methods are not universally fixed; they must be tailored to the specific problem context to enhance performance and reliability.

Induction motors are integral to industrial applications, as they are the most widely used machines in production processes. To ensure these motors deliver the expected quality of service, it's crucial to optimize and monitor their operating conditions within the overall electromechanical system. A promising approach involves using convex surrogate modelling techniques to create scalable models that predict motor losses based on operating points and design parameters [7]. This method enables the efficient optimisation of the entire system, including the induction motor, gearbox, and mechanical application, by formulating the design problem as a second-order cone program that can be solved with optimality guarantees. By addressing factors such as optimal electrical and mechanical performance, economic parameters, system weight, energy losses, cost efficiency, and reliability, this approach facilitates the achievement of both operational and financial excellence.

The formulation of a comprehensive optimization problem begins with a detailed analysis of the modulation strategies for each system component. In electromechanical systems, the development of models is intricately tied to the selection of optimization criteria and the interdisciplinary nature of the design approach. When studying an integrated system like an electric vehicle's active suspension with an In-Wheel-Motor, challenges arise from the complex interaction between mechanical and electrical subsystems [8]. To mitigate these challenges, techniques such as linearising specific nonlinear characteristics and accounting for the inertial effects of all moving parts are employed.

Similarly, in the context of an electromechanical actuator system, the rotor of the induction motor is treated as an electromechanical converter and integrated into the mechanical subsystem [9]. This mechanical segment,

comprising the motor's rotor and gearbox, facilitates the effective transmission of power to the mechanical load [10]. The multi-objective optimization of such systems considers criteria like efficiency, reliability, and system dynamics to address the adverse effects of electromechanical coupling while achieving robust system performance [11].

This study aims to develop an advanced tool for optimizing energy efficiency in electromechanical systems by adopting a systemic approach. Specifically, it investigates the influence of the gearbox speed ratio on the overall system parameters, including weight reduction and transient response time. The methodology is structured into two primary phases. The first phase involves the rigorous formulation of the optimization problem, encompassing the definition of constraints and performance metrics tailored to the system's requirements. The second phase is dedicated to implementing and simulating the proposed optimization strategy using a systemic framework. The results are subsequently analyzed to evaluate the trade-offs and synergies between the gearbox design, energy efficiency, and dynamic performance.

II. PROPOSED METHODOLOGY

An electromechanical system is considered adequate when it minimizes overall dimensions, weight, energy losses, transient response time, and operating costs, as outlined in [12]. The analytical expression for the gearbox weight can be represented as follows:

$$W_g = K.K_g.T_n.F_n(r) \quad \dots \quad (1)$$

Where: W_g - weight of the gearbox, T_n - rated torque of the induction motor, K - ratio of the starting torques to the rated torque, K_g - proportionality coefficient that accounts for the physical dimensions and design characteristics of the gearbox unit. The following equation mathematically defines it:

$$K_g = \frac{1}{\sqrt[3]{V}} \quad \dots \quad (2)$$

$$V = l.w.h \quad \dots \quad (3)$$

Where, V - gearbox volume.

$$Fnr = 1 + r12r - 1r1n - 1 \quad \dots \quad (4)$$

.

Where n is the number of gearbox stages, and r is the gearbox speed ratio.

Calculating the weight of an induction motor involves considering various factors, including the motor's dimensions, materials, and design specifications. While a specific formula like $W_m = K_m \cdot T_n$ (where Wm is the motor weight, Km is a constant representing the motor construction, which is not standard in the literature, the weight can be estimated by analysing the motor's components and their respective densities).

$$Wm = Km.Tn \quad \dots \quad (5)$$

The power losses in a gearbox typically range from 1–2% for a single-stage reduction gearbox and 3–5% for a complex gearbox with two or three reduction stages. In this study, the gearbox efficiency is assumed to be

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100%, allowing the rated torque to be determined using the following relation:

$$T_n = \frac{P_n}{r.\Omega_n} \quad ... \quad (6)$$

Where, Ω_n - output angular speed of the gearbox, P_n - rated power of the induction motor.

Thus, using equation 6, the weight equations for the induction motor and the gearbox are expressed as follows:

$$W_g = \frac{K.K_g.P_n.F_n(r)}{r.\Omega_n} \quad ... \quad (7)$$

$$W_m = \frac{K_m.P_n}{r.\Omega_n} \quad ... \quad (8)$$

The total weight of the motor-gearbox system is calculated using the following relation:

$$W_s = W_g + W_m \quad ... \quad (9)$$

Let's put $g_s = \frac{W_s.\Omega_n}{P_n}$ relative weight of the system motor-gearbox   ...   (10)

Thereby:

$$g_s = \frac{K.K_g.F_n(r) + K_m}{r} \quad ... \quad (11)$$

The expressions for the optimal gearbox ratios (r_{op}), which minimize the relative weight of the system, can be determined as follows:

$$K.K_g.\frac{\partial F_n(n,r)}{\partial r} - K.K_g.F_n(n,r) - K_m = 0 \quad ... \quad (12)$$

For each value of the number of gearbox stages (n), a corresponding expression is obtained, as presented in equations (13), (14), (15), and (16):

For $n = 1$:

$$K.K_g.r^2 - K.K_g - K_m = 0 \quad ... \quad (13)$$

For $n = 2$:

$$\frac{1}{2}K.K_g.r^{\frac{5}{2}} - K.K_g r^2 - K_m.r + \left(\frac{1}{3}.K.K_g + 2.K_m\right).r^{\frac{1}{2}} - (K.K_g + K_m) = 0 \quad ... \quad (14)$$

For $n = 3$:

$$\frac{1}{3}.K.K_g.r^2 - \frac{2}{3}.K.K_g.r^{\frac{83}{50}} - \frac{1}{3}.K.K_g.r^{\frac{133}{100}} - \frac{2}{3}.K.K_g.r$$
$$- \left(\frac{1}{3}.K.K_g + K_m\right).r^{\frac{2}{3}}$$
$$+ \left(\frac{133}{100}.K.K_g + 2.K_m\right).r^{\frac{1}{3}}$$
$$- (K.K_g + K_m) = 0 \quad ... \quad (15)$$

For $n = 4$:

$$\frac{1}{4}.K.K_g.r^{\frac{7}{4}} - \frac{1}{2}.K.K_g.r^{\frac{2}{3}} - \frac{1}{2}.K.K_g.r^{\frac{5}{4}} + \frac{3}{4}.K.K_g.r^{\frac{3}{4}}$$
$$- \left(\frac{1}{2}.K.K_g + K_m\right).r^{\frac{1}{2}}$$
$$+ \left(\frac{5}{4}.K.K_g + 2.K_m\right).r^{\frac{1}{2}} - (K.K_g + K_m)$$
$$= 0 \quad ... \quad (16)$$

The range of parameters for the induction motor used in the simulations, including their minimum and maximum values, is summarized in Table 1:

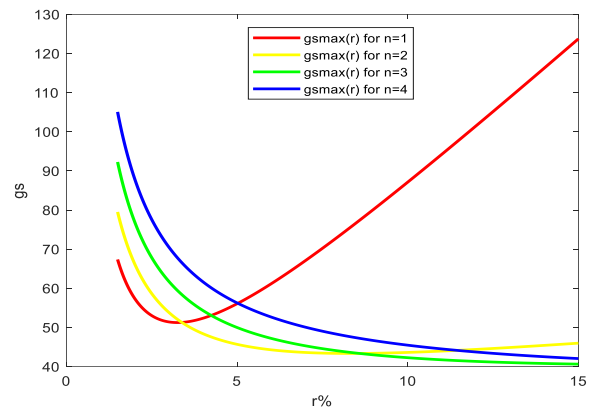**Table 1: The Minimum and Maximum Values of the Induction Motor Parameters**

Parameters	Minimum Value	Maximum Value
Power P_n	0.25 kW	30 kW
Speed Ω_n	900 r/min	1800 r/min
Coefficient K_m	7.66 m^{-1}	75.46 m^{-1}
Starting torque ratio K	1.6	3.6
Construction coefficient Kg	1.16 m^{-1}	2.19 m^{-1}
$K.Kg$	1.85 m^{-1}	7.88 m^{-1}

III. RESULTS AND DISCUSSION

Based on the boundary values of the coefficients K_m and $K.K_g$ provided in Table 1, the minimum and maximum relative weights of the 'induction motor-gearbox' system can be calculated. The relative weight of the system is expressed as a function of the number of gearbox stages (n) and the speed ratio (r).
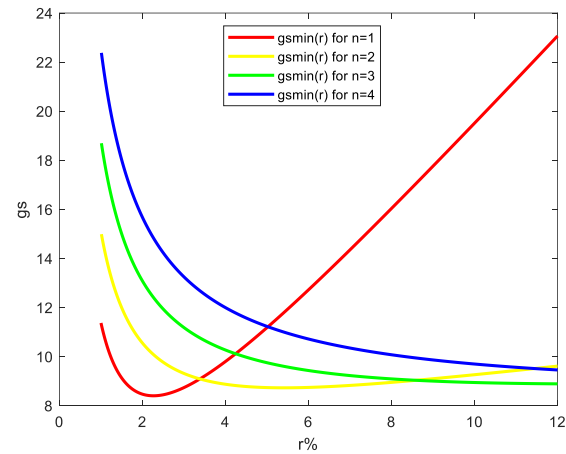
[Fig.3: Variations of g_s as a Function of (n) and (r) for the Minimum Values of $K.K_g$ and K_m]
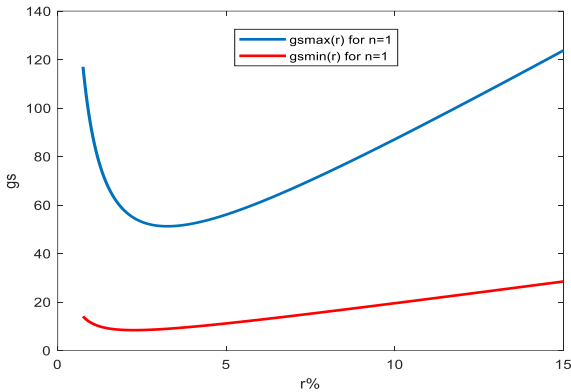

[Fig.4: Variations of g_s as a Function of (n) and (r) for the Maximum Values of $K.K_g$ and K_m]

Figures 3 and 4 illustrate the variation in the system's relative weight across these parameters.

For both the minimum and maximum values of $K.K_g$ and K_m, the relative weight of the system tends to

increase rapidly as the speed ratio decreases relative to the optimal value. Conversely, an increase in the speed ratio compared to the optimal value remains critical for systems with a single-stage gearbox, whereas this increase has a negligible effect on systems with two or more stages.

For the analysed range of induction motors, spanning from 0.25 kW to 30 kW, the coefficients K.Kgand Km exhibit variations between their minimum and maximum values. Consequently, the relative weight of g_s responds to these variations as illustrated in Figure 5.

[Fig.5: Variations of g_s as a Function of (r) for $n = 1$]

Here, we have only presented the variations of g_s as a function of r for $n = 1$. For other numbers of stages, the curves exhibit a similar shape. However, beyond the optimal point, the increase in g_s becomes negligible. The analysis of Figure 5 indicates that, for any number of gearbox stages (n), an optimal speed ratio (r_{op}) exists that minimizes the relative weight of the "motor-gearbox" system. The findings can be detailed as follows:

When $r < r_{op}$, the relative weight of the system increases rapidly, regardless of the number of gearbox stages (n).

When $r > r_{op}$, the relative weight of the system increases more gradually compared to the previous case, with the rate of increase varying based on the number of gearbox stages. For $n = 1$, the relative weight increases significantly. For $n = 2$, the growth is moderate. For $n \geq 3$, the relative weight exhibits minimal variation.

Within the range defined by the minimum and maximum values of $K.K_g$ and K_m, there exist infinitely many curves representing the optimal gearbox speed ratios that minimize the relative system weight. These curves account for all possible combinations of motors and gearboxes. The optimal gearbox speed ratios that result in the minimum relative system weight are detailed in Table 2.

Table 2: Optimal Gear Ratios Values

	$K_m(m^{-1})$	KK_g (m^{-1})	Number of Gearbox Stages			
			$n = 1$	$n = 2$	$n = 3$	$n = 4$
			r_{op}			
Minimum Values	7.66	1.85	2.26	4.95	11.98	30.40
Maximum Values	75.46	7.88	3.25	8.43	25.44	47.84

In general, the extreme values of the optimal gearbox speed ratios, corresponding to (n) varying from 1 to 4, identify the regions where the relative weight of the "induction motor-

gearbox" system is minimized. These regions represent the optimal design zones for achieving the minimum possible system weight.

The equations describing the optimal relative weight as a function of the optimal gearbox speed ratios, with (n) held constant, are provided below:

$$\text{For } n = 1, \quad g_{s1op} = r_{op1}^{2,845} \quad ... \quad (17)$$
$$\text{For } n = 2, \quad g_{s2op} = r_{op2}^{2} \quad ... \quad (18)$$
$$\text{For } n = 3, \quad g_{s3op} = 0,0613.r_{op3}^{2} \quad ... \quad (19)$$
$$\text{For } n = 4, \quad g_{s4op} = 0,133.r_{op4} - \frac{107,05}{r_{op4}} \quad ... \quad (20)$$

The equations representing the speed ratios for any type of gearbox are given below:

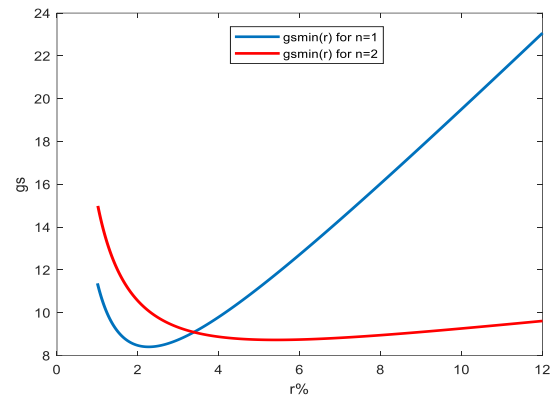$$\text{For } n = 1, \quad r^2 - 2.r^{\frac{3}{2}} + 1 = 0 \quad ... \quad (21)$$
$$\text{For } n = 2, \quad r - r^{0,83} - r^{0,66} + r^{0,33} - r^{0,16} + 1 = 0 \quad ... \quad (22)$$
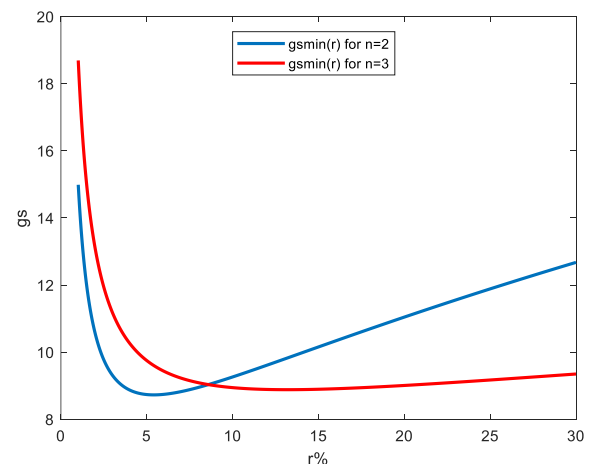$$\text{For } n = 3, \quad r^{0,66} - r^{0,58} - r^{0,41} + r^{0,25} - r^{0,083} + 1 = 0 \quad ... \quad (23)$$
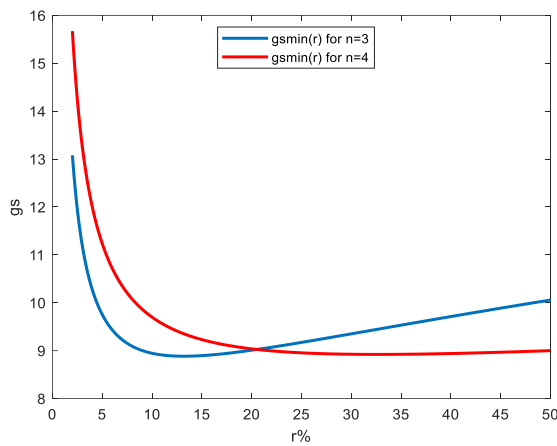
The intersection of two characteristics, corresponding to two different gearbox stages, as shown in Figures 6, 7, and 8, enables the use of both types of gearboxes with equal feasibility while maintaining the minimum relative weight.

[Fig.6: Variations of g_s as a Function of (r) for $n = 1$ and $n = 2$]

[Fig.7: Variations of g_s as a Function of (r) for $n = 2$ and $n = 3$]

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[Fig.8: Variations of g_s as a Function of (r) for $n = 3$ and $n = 4$]

From Figures 6, 7 and 8, the solution to the equations describing the speed ratios can be determined graphically, providing insight into the most suitable type of gearbox to use. The intersections between the characteristic curve of the gearbox with (n) stages and that of the gearbox with $n+1$ stages are presented as follows.

$$r_{1-2} = 3.38; \quad r_{2-3} = 8.52; \quad r_{3-4} = 20.93$$

Based on these results, it can be concluded that the value of $r_{n-(n+1)}$ It is independent of K_m and $K.K_g$. Therefore, if (r) is less than 3.38, a single-stage gearbox provides the minimum system weight. If $3.38 \leq r < 8.52$, a double-stage gearbox is recommended. If $8.52 \leq r \leq 20.93$, a three-stage gearbox is the most efficient choice. Consequently, if (r) exceeds 20.93, a four-stage gearbox is the most recommended option.

For real-world applications, such as electric vehicles or industrial machinery, the results indicate that selecting the optimal gearbox design (in terms of stages and speed ratio) is crucial for improving energy efficiency and reducing unnecessary weight, which directly impacts performance and cost.

IV. CONCLUSION

This study examines the optimisation of the motor-gearbox system's weight through a comprehensive analysis of the relationship between gearbox stages, speed ratios, and system performance. By reviewing key factors such as gearbox power losses, motor torque, and system efficiency, the study develops a set of mathematical models to determine the optimal gearbox ratios that minimise the system's relative weight.

To achieve a positive energy performance in terms of transient time, it is essential to comply with operational, exploitation, and control constraints, including the optimal coupling with the gearbox. Selecting an incorrect gear ratio increases energy losses, resulting in motor overheating and a reduced system lifespan.

The analysis clearly shows that there is an optimal gearbox speed ratio (*ROP*) *that minimises* the relative weight of the "motor-gearbox" system for each number of gearbox stages (1 to 4). This optimal ratio varies with the number of stages and motor/gearbox characteristics.

The study finds that when the gearbox speed ratio (r) is lower than the optimal ratio (r_{op}), the system weight increases

significantly. On the other hand, when (r) exceeds (r_{op}), the rate of increase in weight slows down, with minimal variation occurring for systems with three or more stages.

The results of the simulations using the range of induction motor parameters, from *0.25 kW* to *30 kW*, demonstrate that the relative weight of the system is highly sensitive to both the motor's specifications and the gearbox's design parameters (such as the construction coefficient and starting torque ratio). For each set of motor and gearbox characteristics, the optimal gearbox speed ratios were calculated and graphically illustrated, revealing clear trends based on the number of gearbox stages.

The study also provides a guide for selecting the optimal and suitable number of gearbox stages based on the system's power rating and required efficiency, making it a valuable reference for engineers designing powertrains or other electromechanical systems.

The study's focus on the weight optimization of motor-gearbox systems could be expanded to include additional system-level considerations, such as cost, reliability, and thermal management, which play significant roles in practical applications.

DECLARATION STATEMENT

After aggregating input from all authors, I must verify the accuracy of the following information as the article's author.

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- **Funding Support:** This article has not been funded by any organizations or agencies. This independence ensures that the research is conducted with objectivity and without any external influence.
- **Ethical Approval and Consent to Participate:** The content of this article does not necessitate ethical approval or consent to participate with supporting documentation.
- **Data Access Statement and Material Availability:** The adequate resources of this article are publicly accessible.
- **Author's Contributions:** The authorship of this article is contributed equally to all participating individuals.

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