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Stabilization and Performance Analysis of an Inverted Pendulum using Classical and Intelligent Control Techniques

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Abstract: ***The inverted pendulum system is a classical benchmark in control theory, known for its nonlinear and unstable dynamics. This paper presents a comparative study of three advanced control strategies: Linear Quadratic Regulator (LQR), Model Predictive Control (MPC), and Fuzzy Logic Control (FLC) for stabilizing an inverted pendulum system. Each controller is designed to stabilise the pendulum upright while minimising cart displacement. Performance is evaluated based on settling time, rise time, steady-state error, and control effort under nominal and perturbed conditions, including varying pendulum mass and initial angle. Results indicate that while LQR offers a fast response, it demands high control energy. MPC ensures precise tracking but is computationally intensive. FLC provides a robust and energy-efficient balance, making it ideal for uncertain environments.***

Keywords: ***Control Effort, Fuzzy Logic Control, Inverted Pendulum, LQR, MPC, Robustness.***

Abbreviations:
RK: Runge-Kutta
LQR: Linear Quadratic Regulator
MPC: Model Predictive Control
MFs: Membership Functions

I. INTRODUCTION

The inverted pendulum system is a fundamental problem in dynamics and control theory, often used as a benchmark for testing control strategies due to its inherently unstable nature. It consists of a pendulum mounted on a moving cart, with the pendulum's centre of mass positioned above its pivot point. Unlike a traditional pendulum, which naturally stabilizes at its lowest potential energy state, the inverted pendulum requires active control to maintain its upright position. This makes it an essential case study in nonlinear control systems, stability analysis, and modern control techniques [1].

One of the inverted pendulum's most important characteristics is that it is an underactuated system, meaning it has fewer control inputs than degrees of freedom. The system has two degrees of freedom: the cart's position and the pendulum's angle, but typically, only the force applied to the cart is available as a control input. This makes the control problem significantly more challenging since not all states can be directly controlled. Instead, controllers must leverage the system's natural dynamics to achieve stabilization [2].

Studying inverted pendulum systems extends beyond theoretical interest, offering valuable insights for practical engineering applications. They are simplified models for complex systems such as bipedal robots, Segway transporters, missile guidance systems, and rocket stabilisation. In each case, maintaining balance or trajectory in the presence of instability is critical, and the control strategies developed for the inverted pendulum provide foundational techniques for these real-world implementations.

II. SYSTEM MODELING

A. System Description

The inverted pendulum on a cart system consists of a rigid pendulum attached to a moving cart that can travel along a horizontal track. The goal is to apply a controlled force to the cart so that the pendulum remains upright. The system can be described using the following parameters:

- M: Mass of the cart (kg)
- m: Mass of the pendulum (kg)
- L: Length to pendulum centre of mass (m)
- g: Acceleration due to gravity (m/s^2)
- b: Damping coefficient (Ns/m)
- θ: Angle of the pendulum from the vertical (rad)
- x: Position of the cart along the horizontal axis (m)
- F: Control force applied to the cart (N)

The nonlinear equations of motion for the inverted pendulum on a cart are derived using Newtonian mechanics. The state-space representation includes the cart position x, the pendulum angle θ, and their derivatives. The system is simulated using the 4th-order Runge-Kutta (RK4) method for numerical integration.

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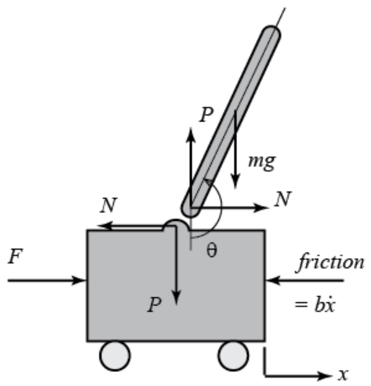

[Fig.1: Free-Body Diagram of the Inverted Pendulum System]

B. Derivation of Equations of Motion using Newtonian Approach

Summing forces on the cart in the horizontal direction:

$$M\ddot{x} + b\dot{x} + N = F \quad ... \quad (1)$$

Forces on the pendulum (horizontal):

$$N = m\ddot{x} + ml\ddot{\theta}\cos\theta - ml\dot{\theta}^2\sin\theta \quad ... \quad (2)$$

Substituting into the cart equation:

$$(M + m)\ddot{x} + b\dot{x} + ml\ddot{\theta}cos\theta - ml\dot{\theta}^2 sin\theta = F \quad ... \quad (3)$$

Perpendicular forces on the pendulum:

$$Psin\theta + Ncos\theta - mgsin\theta = ml\ddot{\theta} + m\ddot{x}cos\theta \quad ... \quad (4)$$

Moments about the pendulum centre:

$$-Plsin\theta - Nlcos\theta = I\ddot{\theta} \quad ... \quad (5)$$

Combining leads to:

$$(I + ml^2)\ddot{\theta} + mglsin\theta = -ml\ddot{x}cos\theta \quad ... \quad (6)$$

Here, I is the Moment of inertia of the pendulum about the pivot point (kg · m²).m²).

C. Linearization of the System

Linearising about $\theta = \pi$(upright position), with $\theta = \pi + \phi$ and small ϕ:

$$\cos(\theta) \approx -1$$
$$\sin(\theta) \approx -\phi$$
$$\dot{\theta}^2 = \dot{\phi}^2 \approx 0$$

Linearized equations of motion (with $u = F$):

$$(I + ml^2)\ddot{\phi} - mgl\phi = ml\ddot{x} \quad ... \quad (7)$$

$$(M + m)\ddot{x} + b\dot{x} - ml\ddot{\phi} = u \quad ... \quad (8)$$

III. SYSTEM REPRESENTATION

A. Transfer Function Representation

Taking the Laplace transform of the previous two equations, assuming zero initial conditions:

$$(I + mL^2)s^2\Phi(s) + mgL\Phi(s) = -mLs^2X(s) \quad ... \quad (9)$$

$$(M + m)s^2X(s) + bsX(s) + mLs^2\Phi(s) = U(s) \quad ... \quad (10)$$

Defining: $q = (M + m)(I + mL^2) - (mL)^2$,

Solving for $\Phi(s)$:

$$\frac{\Phi(s)}{U(s)} = \frac{\frac{mL}{q}s^2}{s^4 + \frac{b(I + mL^2)}{q}s^3 - \frac{(M + m)mgL}{q}s^2 - \frac{bmgL}{q}s} \quad ... \quad (11)$$

This represents the transfer function for the pendulum angle as the output in response to the control input. $U(s)$.

B. State-Space Representation

Rewriting the system equations into first-order differential equations, the state-space representation is:

$$\dot{x} = Ax + Bu \quad ... \quad (12)$$

$$y = Cx + Du \quad ... \quad (13)$$

Where the state, input, and output matrices are:

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & \frac{-(I+mL^2)b}{q} & \frac{m^2gL^2}{q} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \frac{-mLb}{q} & \frac{mgL(M+m)}{q} & 0 \end{bmatrix}, \quad ... \quad (14)$$

$$B = \begin{bmatrix} 0 \\ \frac{I + mL^2}{q} \\ 0 \\ \frac{mL}{q} \end{bmatrix}, C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \quad D = \begin{bmatrix} 0 \\ 0 \end{bmatrix}. \quad ... \quad (15)$$

This represents the state-space model of the system.

IV. STABILITY ANALYSIS

For analyzing the inverted pendulum system, we assume the following system parameters:

M = 5.0 kg (Mass of the cart)
m = 1.0 kg (Mass of the pendulum)
b = 0.1 N/m/s (Coefficient of friction for the cart)
L = 0.5 m (Length to pendulum centre of mass)
g = 9.81 m/s² (Acceleration due to gravity)
$I = \frac{1}{3}m(2L)^2 = \frac{1}{3} \cdot 1 \cdot (2 \cdot 0.5)^2 = \frac{1}{3}$ kgm²

Using the previously derived transfer function of the pendulum:

$$\frac{\Phi(s)}{U(s)} = \frac{\frac{ml}{q}s^2}{s^4 + \frac{b(I + ml^2)}{q}s^3 - \frac{(M + m)mgl}{q}s^2 - \frac{bmgl}{q}s}$$

Substituting the assumed values:

$$\frac{\Phi(s)}{U(s)} = \frac{0.1538s^2}{s^4 + 0.0179s^3 - 9.0554s^2 - 0.1509s} \quad ... \quad (16)$$

State-Space Representation Matrices

1. State Matrix (A):

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -0.0015 & 0.0621 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -0.0013 & 0.7451 & 0 \end{bmatrix}$$

2. Input Matrix (B):

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$$B = \begin{bmatrix} 0 \\ 0.0148 \\ 0 \\ 0.0127 \end{bmatrix}$$

3. Output Matrix (C):

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

4. Feedthrough Matrix (D):

$$D = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

The system's poles are the eigenvalues of the State Matrix A, which are -0.0167, -3.0099, and 3.0086. The presence of a positive pole confirms that the inverted pendulum system is inherently unstable in its upright position and requires external control effort to be stabilised.

V. CONTROL STRATEGIES FOR STABILIZATION

A. LQR Controller Design for Inverted Pendulum

i. Introduction

A linear quadratic regulator (lqr) is implemented to stabilize the inverted pendulum in the upright position. The LQR control strategy determines an optimal state-feedback control law to minimize a predefined cost function.
The control input is given by:

$$u = -Kx, \quad \dots \quad (17)$$

where K Is the optimal gain matrix and x Represents the system state vector.

ii. Quadratic Cost Function

The LQR controller minimizes the following quadratic cost function:

$$J = \int_0^\infty (x^T Q x + u^T R u)\, dt, \quad \dots \quad (18)$$

Where:

- Q It is a positive semi-definite matrix that penalises deviations from the desired state.
- R It is a positive definite matrix that penalises excessive control effort.

For this system, the weighting matrices are chosen as:

$$Q = \begin{bmatrix} 10 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 10 & 0 \\ 0 & 0 & 0 & 10 \end{bmatrix}, \quad R = 3. \quad \dots \quad (19)$$

The values of Q are set to place higher importance on controlling the pendulum angle (θ) and its velocity ($\dot{\theta}$), ensuring that the pendulum remains upright. The small value of R Allows more decisive corrective actions while keeping control effort within reasonable limits.

iii. Computing the LQR Gain Matrix

The optimal feedback gain matrix K It is computed by solving the Continuous-time algebraic Riccati equation (CARE):

$$A^T P + PA - PBR^{-1}B^T P + Q = 0, \quad \dots \quad (20)$$

Where P Is the solution to the Riccati equation.
To solve for PWe rewrite the equation as:

$$PA + A^T P - PBR^{-1}B^T P + Q = 0. \quad \dots \quad (21)$$

This is then solved numerically using MATLAB's care function:

[P, ~, ~] = care (A, B, Q, R);

Once P is determined, the optimal gain matrix K Is computed using:

$$K = R^{-1}B^T P. \quad \dots \quad (22)$$

In MATLAB, this is efficiently computed using:

K = lqr(A, B, Q, R);

In this work, solving the Riccati equation for the assumed set of system parameters yields:

$$P = \begin{bmatrix} 23.3766 & 26.8082 & 12.1519 & 1.1774 \\ 26.8082 & 56.5136 & 27.2297 & 3.5991 \\ 12.1519 & 27.2297 & 71.0687 & 2.3835 \\ 1.1774 & 3.5991 & 2.3835 & 7.0654 \end{bmatrix}. \quad \dots \quad (23)$$

Using this matrix, the calculated optimal gain matrix K Is:

$$K = [1.8257 \quad 4.1680 \quad 2.1937 \quad 2.2186]. \quad \dots \quad (24)$$

This matrix determines the feedback control law, ensuring system stability.

iv. Simulation and Control Implementation

During the simulation, the control input u It is applied iteratively at each time step. The system is then numerically integrated using the Runge-Kutta 4th-order (RK4) integration method with a specified sampling time. $T_s = 0.1$ The states are updated using the system equations, and the updated state is used for the following control computation.

B. Model Predictive Control

i. Introduction

Model Predictive Control (MPC) is an advanced optimal control technique that determines control actions by solving a constrained optimization problem over a finite prediction horizon at each sampling instant. It uses a model of the process to predict future system behaviour and optimises a cost function to determine the optimal control sequence. Only the first control input is implemented, and the procedure is repeated at the next time step in a receding horizon fashion [3].

MPC is particularly attractive for systems with constraints on inputs and states, making it suitable for the inverted pendulum, a classic benchmark in control theory, due to its instability and nonlinear dynamics. In this work, MPC is employed to stabilize an inverted pendulum mounted on a cart by minimizing a quadratic cost function over prediction and control horizons, denoted as N and N_u, respectively.

ii. System Modelling and Discretization

The nonlinear dynamics of the inverted pendulum are linearized around the upright equilibrium. The continuous-time state-space representation is:

$$\dot{x}(t) = Ax(t) + Bu(t), \quad y(t) = Cx(t) \quad \dots \quad (25)$$

With the state vector $x(t) = [x \quad \dot{x} \quad \theta \quad \dot{\theta}]^T$.

Using Zero-Order Hold with sampling time. $T_s = 0.1$ s, the discrete-time system becomes:

$$x_{k+1} = A_d x_k + B_d u_k, \quad y_k = C_d x_k \quad \dots \quad (26)$$

Where A_d, B_d, and C_d They are computed using MATLAB's c2d() function.

iii. Cost Function and Constraints

At each time step, MPC solves the optimization problem:

$$\min_{u_0,\dots,u_{N_u-1}} \sum_{k=0}^{N-1} (y_k^T Q y_k + u_k^T R u_k) \quad \dots \quad (27)$$

Subject to:

$$x_{k+1} = A_d x_k + B_d u_k \quad \dots \quad (28)$$

$$y_k = C_d x_k \quad \dots \quad (29)$$

$$u_{\min} \le u_k \le u_{\max} \quad \dots \quad (30)$$

Here, $Q \in \mathbb{R}^{4\times4}$, $R \in \mathbb{R}$ They are weight matrices that emphasize angle stabilization and reasonable control effort:

$$Q = \begin{bmatrix} 10 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 100 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad R = 0.1$$

Control force bounds:

$$u_{\min} = -10\text{N}, \quad u_{\max} = 10\text{N}$$

These constraints ensure feasibility and prevent actuator saturation [4].

iv. Simulation and Control Implementation

The total number of simulation steps is $N_{\text{sim}} = \frac{T_{\text{sim}}}{T_s}$. At each time step, the control input u_k is computed using MATLAB's mpcmove() function, which solves the optimization problem using the current state x_k as the initial condition.

The state evolution is governed by:

$$x_{k+1} = A_d x_k + B_d u_k. \quad \dots \quad (31)$$

C. Fuzzy Logic Control for Inverted Pendulum

i. Introduction

Fuzzy Logic Control (FLC) is a model-free, rule-based control strategy particularly well-suited to managing the nonlinear and underactuated nature of the inverted pendulum system. Unlike traditional model-based controllers, FLCs utilize heuristic rules and fuzzy membership functions to map linguistic variables to control actions [5]. This section presents the structure, design, and implementation of the fuzzy logic controller used to stabilize the pendulum in the upright position while regulating the cart's position.

ii. Structure of the Fuzzy Inference System

The fuzzy inference system (FIS) employed in this work is of the Mamdani type [6], which is favoured in control applications due to its intuitive rule-based framework and linguistic interpretability. The system is configured with three input variables and one output variable, corresponding to the key physical quantities of the system and the control force.

Each input undergoes a fuzzification process, converting crisp numerical inputs into fuzzy linguistic values using triangular membership functions (MFs). Triangular MFs are chosen for their simplicity, computational efficiency, and effectiveness in modeling gradual transitions in control behavior.

The universe of discourse for each variable is selected according to physical limitations and expected dynamics. Table 1 summarizes the controller's fuzzy variables, their ranges, and associated linguistic terms.

Table-I: FIS Variables, Ranges, and Linguistic Terms

Variable	Range	Terms
x_{error}	$[-1,\ 1]$	neg, zero, pos
θ_{error}	$[-\pi,\ \pi]$	neg, neg_small, zero, pos_small, pos
$\dot{\theta}$	$[-10,\ 10]$	neg, zero, pos
u	$[-50,\ 50]$	neg, zero, pos

The design of linguistic terms strikes a balance between resolution and computational efficiency. For example, the pendulum angle error utilises five terms to achieve finer control near the upright equilibrium, which is crucial for stabilisation.

iii. Triangular Membership Functions

Each fuzzy variable is represented using triangular membership functions (MFs), which are defined by three parameters: a, b, and c (with $a < b < c$). The membership function $\mu(x)$ is given by:

$$\mu(x; a, b, c) = \begin{cases} 0, & x \le a \\ \dfrac{x-a}{b-a}, & a < x \le b \\ \dfrac{c-x}{c-b}, & b < x < c \\ 0, & x \ge c \end{cases} \quad \dots . \quad (32)$$

Here, a and c Define the support of the fuzzy set. b is the peak, and $\mu(x)$ denotes the degree of membership. Each variable's MFs are designed to ensure smooth transitions between fuzzy sets. For instance, the cart position error (x_{error}) is divided into three overlapping fuzzy sets:

- Negative *(neg)*: $\mu_{\text{neg}}(x)$ with $(a = -1, b = -1, c = 0)$,
- Zero *(zero)*: $\mu_{\text{zero}}(x)$ with $(a = -1, b = 0, c = 1)$,
- Positive *(pos)*: $\mu_{\text{pos}}(x)$ with $(a = 0, b = 1, c = 1)$.

This overlap ensures that for a value like $x = 0.2$We obtain:

$$\mu_{\text{zero}}(0.2) = 0.8, \quad \mu_{\text{pos}}(0.2) = 0.2, \quad \mu_{\text{neg}}(0.2) = 0$$

Such graded memberships allow partial rule activation, promoting robust and nuanced control actions. This smoothness is essential in real-time applications, as it prevents abrupt changes in control force due to minor variations in input.

Figures 2 to 5 illustrate the membership functions for each variable used in the FIS.

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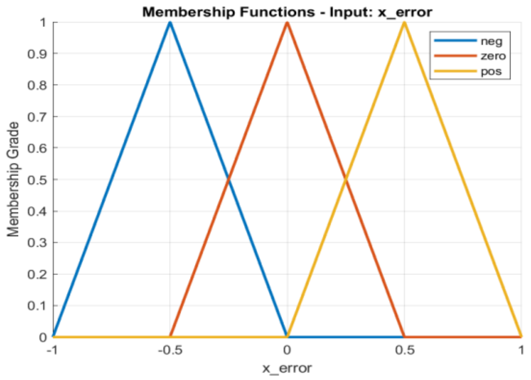
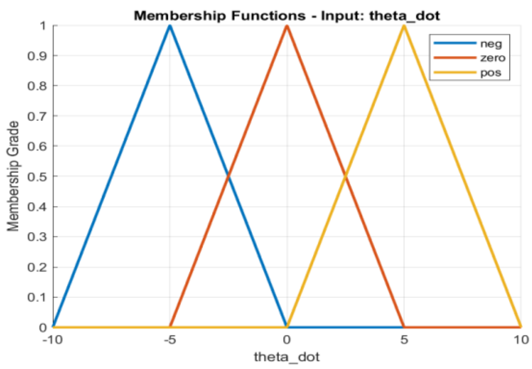
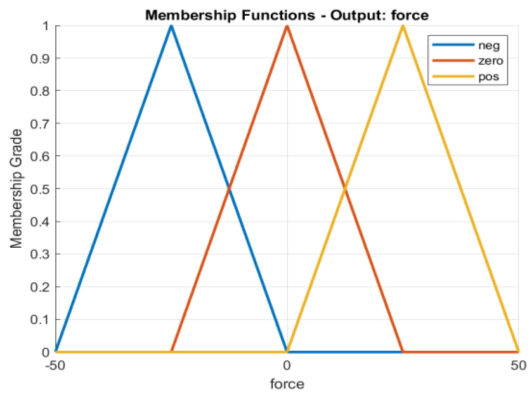


[Fig.2: Membership Functions for Cart Position Error]

[Fig.3: Membership Functions for Pendulum Angle Error]



[Fig.4: Membership Functions for Angular Velocity]



[Fig.5: Membership Functions for Control Force]

iv. Rule Base Design

The fuzzy rule base contains 9 Heuristic rules are based on expert knowledge of inverted pendulum dynamics. The rules are designed to drive the pendulum upright and centre the cart. The whole rule base can be represented in tabular form in Table 2:

Table-II: FIS Rule Base

x_{error}	θ_{error}	$\dot{\theta}$	Control Force
neg	neg	neg	neg
neg	zero	neg	neg
neg	pos	neg	zero
zero	neg	neg	zero
zero	zero	neg	zero
zero	pos	neg	zero
pos	neg	neg	pos
pos	zero	neg	pos
pos	pos	neg	pos

v. Control Signal Adjustment

To further improve performance, a proportional term is added based on the cart position error:

$$u = u_{FIS} + K_p \cdot x_{error} \quad \dots \quad (33)$$

Where u_{FIS} Is the control force output from the fuzzy inference system, and $K_p = 5$ It is a gain factor used to enhance the effect of cart position on the overall control signal.

vi. Simulation and Control Implementation

The controller is implemented in a discrete-time simulation with a sampling interval. $T_s = 0.1$ s. At each time step, the controller evaluates the inputs, applies fuzzy inference and defuzzification (centroid method) [7], and outputs a force command to the cart. This control force is passed to a fourth-order Runge-Kutta (RK4) integrator, which simulates the nonlinear dynamics of the inverted pendulum system.

VI. RESULTS AND DISCUSSION

This section comprehensively evaluates the three control strategies applied to the inverted pendulum on a cart. The controllers are assessed under nominal conditions and tested for robustness by varying the pendulum mass and initial angular displacement. Performance is measured through a set of quantitative metrics, and comparisons are drawn to highlight the strengths and limitations of each approach.

A. Performance Metrics

- **Settling Time (T_s)**: Time taken for the system output to remain within a specified range (e.g., $\pm 2°$).
- **Rise Time (T_r)**: Time taken for the pendulum angle to rise from 10% to 90% of its final absolute value at the upright position.
- **Steady-State Error (e_{ss})**: Difference between desired and final state values.
- **Control Effort ($\int |u(t)| dt$)**: Total magnitude of the control input over time.

B. Analysis Under Nominal Conditions

i. Simulation Setup

The simulation is conducted using the following nominal parameters:

- Pendulum Mass: $m = 1$ kg
- Cart Mass: $M = 5$ kg
- Pendulum Length: $L = 0.5$ m
- Initial Angle: $\theta_0 = 0°$ (measured from the downward vertical)

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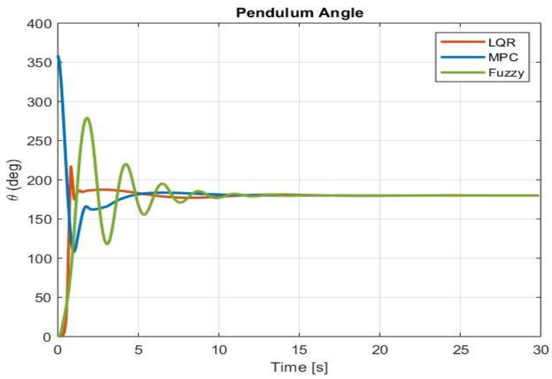
- Initial Angular Velocity: $\dot{\theta}_0 = 0$ rad/s
- Initial Cart Position and Velocity: $x_0 = 0$ m, $\dot{x}_0 = 0$ m/s
- Time Step: $\Delta t = 0.1$ s
- Total Simulation Time: $T = 30$ s
- Numerical Integration: 4th Order Runge-Kutta Method (RK4)

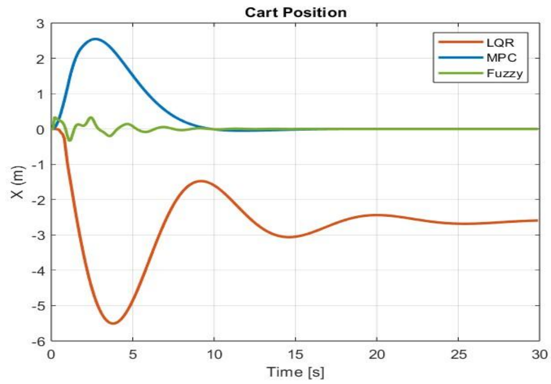
Each controller—LQR, MPC, and FLC—is applied under the same conditions. Their performance is compared using visual plots and summarized in a tabular format.

ii. System Response Comparisons

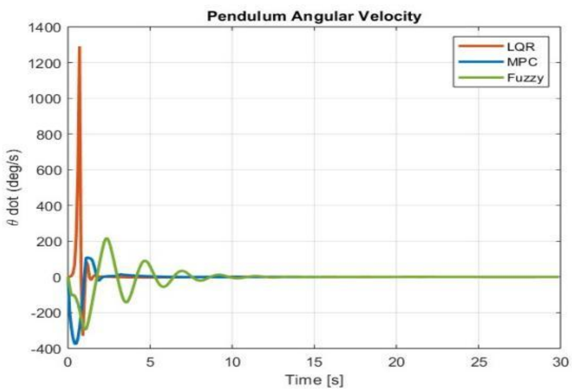
Fig. 6–10 illustrate the system responses of the inverted pendulum under nominal conditions for the LQR, MPC, and FLC controllers. These include the pendulum angle, cart position, angular velocity, cart velocity, and control effort over time.



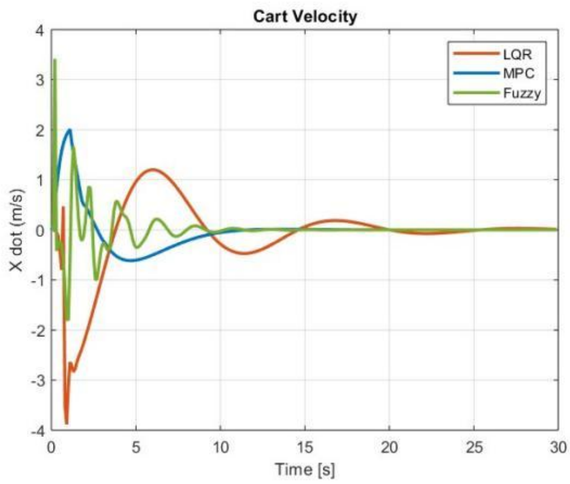
[Fig.6: Pendulum Angle vs. Time Under Nominal Conditions]



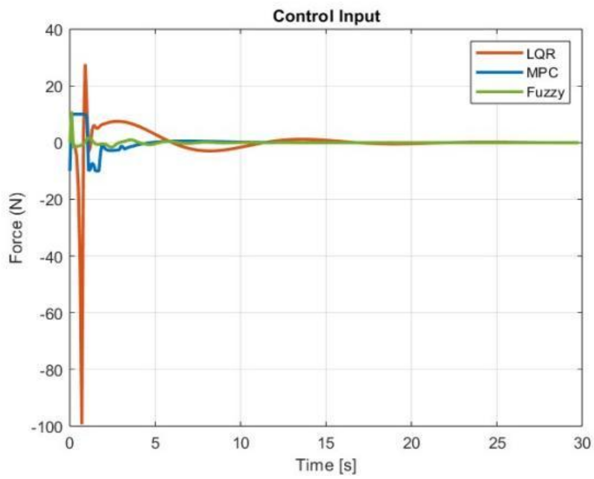
[Fig.7: Cart Position vs. Time Under Nominal Conditions]



[Fig.8: Pendulum Angular Velocity vs. Time Under Nominal Conditions]



[Fig.9: Cart Velocity vs. Time Under Nominal Conditions]



[Fig. 10: Control Effort vs. Time Under Nominal Conditions]

iii. Performance Summary

The key performance metrics for each controller under nominal conditions are summarized in Table 3, while the corresponding control effort metrics are reported in Table 4.

Table-III: Response Metrics under Nominal Conditions

Controller	T_s [s]	T_r [s]	e_{ss} (Angle) [deg]	e_{ss} (Position) [m]
LQR	1.20	0.30	0.06	2.59
MPC	4.30	8.90	0.01	0.00
FLC	2.50	0.80	0.0004	0.00

Table-IV: Control Effort Metrics under Nominal Conditions

Controller	Integral Effort	Mean Effort	Peak Effort
LQR	61.19	2.04	99.33
MPC	24.12	0.80	10.00
FLC	5.55	0.18	11.04

iv. Discussion

LQR offers the fastest response (rise time: 0.30 s, settling time: 1.20 s) but suffers from high steady-state error and the most significant control effort.

MPC ensures the highest accuracy with minimal steady-state errors but exhibits slow dynamics and moderate control effort.

FLC achieves a balanced trade-off, combining low errors, moderate speed, and minimal control effort to make it efficient and responsive.

C. Robustness Analysis

Robustness is assessed by varying the pendulum mass [8] and initial angle—key factors affecting system dynamics due to modelling errors or disturbances [9]. This analysis evaluates each controller's ability to maintain stability and performance under uncertainty, focusing on steady-state error, bounded response, and reliable stabilization. The comparison reveals which strategy—LQR, MPC, or FLC—is more resilient to parameter variation.

D. Variation in Pendulum Mass

i. Simulation Setup

- Masses tested: $m = \{0.5, 1.0, 1.5\}$ kg
- Fixed initial angle: $\theta_0 = 0.1^\circ$

The rest of the parameters are taken the same as before.

ii. Results and Discussion

The system is evaluated for three masses: 0.5 kg, 1.0 kg (nominal), and 1.5 kg. Key response metrics such as settling time (T_s), rise time (T_r), and steady-state errors for both angle and position are reported in Table 5. Control effort metrics, including the integral, mean, and peak values of the control signal, are summarised in Table 6.

Table-V: Controller Performance Under Varying Pendulum Mass

Mass (kg)	Controller	T_s [s]	T_r [s]	e_{ss} (Angle) [deg]	e_{ss} (Position) [m]
0.5	LQR	0.900	0.200	0.044	2.501
	MPC	0.700	12.90	0.041	0.001
	FLC	1.100	0.800	0.000	0.000
1.0	LQR	1.200	0.300	2.591	0.070
	MPC	4.301	8.910	0.002	0.001
	FLC	2.500	0.800	0.0004	0.000
1.5	LQR	1.500	0.300	0.0142	2.662
	MPC	1.700	7.700	0.000	0.001
	FLC	3.800	0.800	0.0020	0.000

Table-VI: Control Effort Metrics under Varying Pendulum Mass

Mass (kg)	Controller	Integral Effort	Mean Effort	Peak Effort
0.5	LQR	87.525	2.918	117.450
	MPC	14.490	0.480	10.001
	FLC	2.687	0.090	6.966
1.0	LQR	61.195	2.040	99.337
	MPC	24.090	0.800	10.002
	FLC	5.551	0.185	11.039
1.5	LQR	58.930	1.964	70.121
	MPC	29.811	0.990	10.112
	FLC	8.059	0.269	12.501

Settling and Rise Time: LQR maintains the fastest rise time (0.2–0.3 s) but suffers from increasing settling time at higher masses. MPC exhibits slow and variable dynamics (up to 12.9 s), while FLC provides stable and moderate response times ($T_r \approx 0.8$ s).

Steady-State Errors: FLC consistently achieves near-zero error, even as mass increases. MPC performs well, particularly in terms of position accuracy. In contrast, LQR exhibits significant angle and position errors, particularly at masses of 1.0 kg and 1.5 kg, indicating sensitivity to mass variation. Control Effort: LQR requires the highest effort (e.g., peak = 117.45 at 0.5 kg). MPC maintains moderate effort, while FLC is the most energy-efficient, with integral effort < 9 across all cases.

Summary: FLC is the most robust and efficient under mass variation. MPC offers good accuracy with longer rise times. LQR, though fast, struggles with accuracy and effort at higher masses.

FLC is the most reliable choice for applications with varying payloads, followed by MPC.

E. Variation in Initial Angle

i. Simulation Setup

- Initial angles tested: $\theta_0 = \{5^\circ, 10^\circ, 15^\circ\}$
- Fixed mass: $m = 1.0$ kg

The rest of the parameters are taken the same as before.

ii. Results and Discussion

To evaluate robustness under different initial conditions, the pendulum's initial angle was varied across 5°, 10°, and 15°, with other parameters held constant. The corresponding system response and control effort metrics are shown in Tables 7 and 8.

Table-VII: Controller Performance Under Varying Initial Angle

Angle (deg)	Controller	T_s [s]	T_r [s]	e_{ss} (Angle) [deg]	e_{ss} (Position) [m]
5°	LQR	0.950	0.200	0.045	2.599
	MPC	4.100	8.500	0.000	0.001
	FLC	4.000	0.900	0.002	0.000
eower of t power of 1(	LQR	0.900	0.200	0.058	2.593
	MPC	4.300	8.900	0.000	0.000
	FLC	1.300	0.800	0.0003	0.000
15°	LQR	0.800	0.300	0.0616	2.591
	MPC	4.500	9.200	0.002	0.005
	FLC	2.100	1.000	-0.005	0.000

Table-VIII: Control Effort Metrics under Varying Initial Angle

Angle(deg)	Controller	Integral Effort	Mean Effort	Peak Effort
5°	LQR	59.715	1.991	103.160
	MPC	23.910	0.803	10.020
	FLC	5.610	0.187	10.970
10°	LQR	54.661	1.822	72.126
	MPC	23.910	0.800	10.020
	FLC	5.610	0.187	10.970
15°	LQR	56.684	1.889	97.267
	MPC	23.960	0.833	10.010
	FLC	5.738	0.191	10.863

Settling and Rise Time: LQR delivers the fastest response ($T_r \approx 0.2$–0.3 s, $T_s < 1$ s) but at the cost of higher errors and effort. MPC exhibits the slowest dynamics ($T_r > 8$ s, $T_s > 4$ s), while FLC offers a good trade-off. At 10°, FLC

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achieves $T_r = 0.8$ s, $T_s = 1.3$ s.

Steady-State Errors: FLC and MPC maintain near-zero steady-state errors across all angles. FLC shows excellent stability (e.g., $-0.005°$ at $15°$). In contrast, LQR yields persistent angular errors (up to $0.0616°$) and significant cart position error (≈ 2.59 m).

Control Effort: LQR demands the highest control effort (peak > 100). MPC maintains a moderate effort (peak ≈ 10), while FLC is the most energy-efficient, with integral effort < 6 and mean effort < 0.2 across all angles.

Summary: FLC is the most robust and efficient under varying initial angles, balancing response speed, accuracy, and energy use. MPC offers high accuracy but a slow response. LQR is fast but lacks robustness and efficiency, making it less ideal for large initial deviations.

These findings highlight FLC as the preferred solution for scenarios with large initial deviations, offering both stability and low energy cost.

VII. COMPARATIVE ANALYSIS

A. Summary of Observations

- **LQR:** Fastest response but suffers from high steady-state position error and the highest control effort—limited robustness.
- **MPC:** Most accurate with minimal steady-state errors and moderate control effort. Slowest response time.
- **FLC:** Offers a balanced trade-off—accurate, energy-efficient, and moderately fast. Most robust overall.
- **Mass Variation:** LQR loses accuracy and demands more effort; MPC stays accurate with moderate effort; FLC remains stable and adaptive.
- **Initial Angle Variation:** LQR stays fast but inefficient; MPC remains precise but slow; FLC stays accurate and efficient, confirming its robustness.

B. Trade-offs and Limitations

Each controller presents distinct trade-offs:

- **Energy vs. Stability:** LQR offers a fast response but uses a lot of energy. FLC and MPC are more energy-efficient, with FLC maintaining good stability.
- **Computation vs. Robustness:** MPC is robust but computationally heavy. FLC achieves similar robustness with lower computational demand.
- **Tuning Effort:** LQR is easy to tune. MPC requires mathematical tuning. FLC needs expert-driven design, which can be time-consuming.

Choosing a controller depends on performance goals, hardware limits, and application needs.

C. Recommendations

Based on the analysis, the following recommendations are made:

- LQR is suitable for applications needing fast response and easy tuning, but it lacks precision and energy efficiency.
- MPC is ideal for robust and precise control in systems with constraints, at the cost of higher computational demand and slower response.
- FLC offers the best balance of accuracy, efficiency, and robustness, making it the most versatile option for practical applications.

VIII. CONCLUSION

FLC is recommended as the optimal control strategy for the inverted pendulum system in scenarios where energy efficiency, robustness, and stability are prioritized. For systems with real-time constraints and limited computation resources, LQR may serve as a pragmatic alternative. At the same time, MPC is preferable in environments where accuracy and constraint handling are more important than implementation complexity.

FUTURE WORK

Future research can explore advanced hybrid control strategies that combine the strengths of classical and intelligent methods. Adaptive fuzzy systems and neural network-based controllers may enhance robustness in highly nonlinear or time-varying environments. Additionally, implementing the controllers on real hardware will validate simulation results and assess real-time performance and computational feasibility.

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