A New Mathematical Dynamic Model for HVAC System Components Based on Matlab/Simulink

Ahmad Parvaresh, Seyed Mohammad Ali Mohammadi, Ali Parvaresh

Abstract— In this paper, a new and complete mathematical dynamic model of HVAC (Heating, Ventilating, and Air Conditioning) components such as heating/cooling coil, humidifier, mixing box, ducts and sensors is described. All of these components are proposed and simulated in Matlab/Simulink platform. The proposed model is presented in terms of energy mass balance equations for each HVAC component. We have considered two control loop for this model, namely, temperature control loop and humidity ratio control loop. The proposed model is a full dynamic model of HVAC system that includes least approximations and assumes.

Index Terms— HVAC system, HVAC components, Matlab/Simulink, HVAC model

I. INTRODUCTION

Initially, the most important issue of HVAC (Heating, ventilating and Air-Conditioning) systems factories was to maintain the zone conditions in predefined values related to occupants' thermal comfort. However, with start of energy crisis, the amount of energy consumption of these equipments became important. In order to, evaluate these two options, the designer have been started to modeling and design of new type of HVAC system components.

In this paper, some study has been focused on HVAC modeling. For instance, Wang et al. [1] have used heat transfer and energy balance principles to identify a linear model to represent a non-linear model of a cooling coil. Clarke [2] has presented a transient model for a HVAC system for some components such as humidifier and mixing box, but no specific model for cooling/heating coils was given. Stoecker [3] provided a model for cooling coil with empirical parameters under the assumptions of constant air flow and water flow. Braun and Rabehi [4,5] presented two cooling coil models that were too complex and iterative computations. Many researchers studied HVAC dynamic models using empirical or theoretical methods. Underwood and Crawford [6] developed a nonlinear model of a heat exchanger loop. Nassif et al. [7] presented a validation of the cooling coil, mixing box and a fan for a VAV (Variable Air Volume) system.

An issue related to HVAC systems modeling, is the use of an appropriate simulation tool. We can categorize these modeling tools on three sections.

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First, tools that used for pipe/duct sizing, as AFT Fathom, DOLPHIN, DUCTSIZE, EES (Energy Equation Solver) and PHYTON.

Second, tools that used for energy performance analysis, as Carrier HAP, TRANSYS, and HVACSIM+.

Third, tools that used for control and modeling analysis of these systems, as MATLAB and Energy Plus.

Advantage and disadvantage of these tools have mentioned in [8]. With these programs, the heating, cooling, lighting, ventilation and other energy related flows in a building can be simulated. Lebrun [9] presented a simulation of a HVAC system by using EES.

In these groups the reason for choosing Simulink, which is part of the Matlab package, was a larger degree of flexibility, modular structure, and transparency of the models and ease of use in the modeling process. The modular structure in Simulink makes it easier to maintain an overview of the models, and new models can just as easily be added to the pool of existing models [8]. Recently, some Matlab/Simulink-based simulations of HVAC systems have been developed. Some examples of these studies are Reiderer et al. [10], Mendes et al. [11], Dion et al. [12], Huang and Lam [13], Ghumari et al. [14], Oliveira et al. [15].

In this paper we have focused on the modeling of HVAC system components and simulation of the presented model. The HVAC system components that modeled are the zone, the cooling coil, the heating coil, the humidifier, the mixing box, the ducts, and the sensor models. This paper is organized as follows: Section 2 presents an introductory explanation of the HVAC system and its components models. Section 3 covers details of the new proposed model. In sections 4 and 5 we have simulated presented model in Matlab/Simulink and discussion about results, respectively. Finally, in section 6 we have mentioned conclusion.

II. MODEL OF HVAC SYSTEM COMPONENTS

In recent years, several numerical simulation models for HVAC systems performance were developed. Because of complexity of these systems a complete theoretical approach of formulating the model is too difficult. In order to, achieve full dynamic model of HVAC system we have to reach all of the important models of components. The major components considered in the system model can be divided in two groups, which are the zone model and Components of HVAC system. First we propose a model for the zone and then for all of components that are in HVAC system.

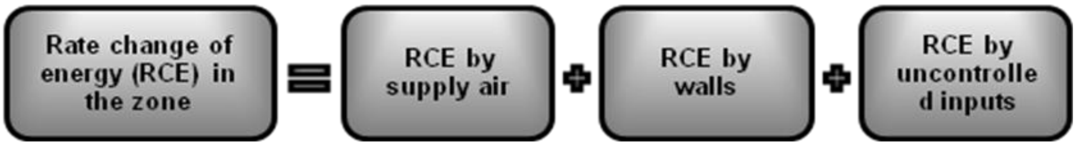

Figure 1. View of energy and mass balance equation that used for the zone modeling

A. The Zone Model

One of the most important sections of HVAC system model is the zone modeling. All of components in HVAC systems work to the zone achieve to optimal conditions. In previous works, because of simplify and approximation in presented models, they are far from their real performance.

In our proposed model we have considered effect of uncontrolled input as people, lights and so on, and effect of the north wall, south wall, east wall, west wall, floor and inner roof on the zone temperature. In this model we have considered eight state variables: the zone temperature (T_z), inner wall temperature (T_{ws},T_{wn},T_{we},T_{ww},T_f,T_r), and the zone humidity ratio (W_z). The pressure of air is assumed to be constant and air in the zone in fully mixed.

With these descriptions, energy and mass balance equations are (as shown in Fig. 1)

$$\begin{aligned} C_z \frac{dT_z}{dt} &= f_{sa}\rho_a C_{pa}(T_{sa} - T_z) \\ &+ H_{ws}A_{ws}(T_{ws} - T_z) + H_{wn}A_{wn}(T_{wn} - T_z) \\ &+ H_{we}A_{we}(T_{we} - T_z) + H_{ww}A_{ww}(T_{ww} - T_z) \\ &+ H_r A_r (T_r - T_z) + H_f A_f (T_f - T_z) + Q \end{aligned} \tag{1}$$

$$C_{ws}\frac{dT_{ws}}{dt} = H_{ws}A_{ws}(T_z - T_{ws}) + H_{ws}A_{ws}(T_o - T_{ws}) \tag{2}$$

$$C_{wn}\frac{dT_{wn}}{dt} = H_{wn}A_{wn}(T_z - T_{wn}) + H_{wn}A_{wn}(T_o - T_{wn}) \tag{3}$$

$$C_{we}\frac{dT_{we}}{dt} = H_{we}A_{we}(T_z - T_{we}) + H_{we}A_{we}(T_o - T_{we}) \tag{4}$$

$$C_{ww}\frac{dT_{ww}}{dt} = H_{ww}A_{ww}(T_z - T_{ww}) + H_{ww}A_{ww}(T_o - T_{ww}) \tag{5}$$

$$C_r\frac{dT_{wn}}{dt} = H_r A_r (T_z - T_r) + H_r A_r (T_o - T_r) \tag{6}$$

$$C_f\frac{dT_f}{dt} = H_f A_f (T_z - T_f) \tag{7}$$

$$V_z\frac{dW_z}{dt} = f_s (W_s - W_z) \tag{8}$$

With taking the Laplace transform of Eqs. (1)- (8) and rearrange we have

$$\begin{aligned} T_z(s) = G_z(s)[\beta T_s(s) + [\gamma^2 G_{ws}(s) \\ + \delta^2 G_{wn}(s) + \lambda^2 G_{we}(s) + \mu^2 G_{ww}(s) \\ + \sigma^2 G_r(s)](T_z(s) + T_o(s)) + \tau^2 G_f(s) T_f(s) + Q] \end{aligned} \tag{9}$$

$$W_z(s) = G_{wz}(s)W_s(s) + QQ \tag{10}$$

Where

$$\begin{aligned} \alpha = f_{sa}\rho_a C_{pa} + H_{ws}A_{ws} + H_{wn}A_{wn} \\ + H_{we}A_{we} + H_{ww}A_{ww} + H_r A_r + H_f A_f \end{aligned} \tag{11}$$

$$\begin{aligned} \beta = f_{sa}\rho_a C_{pa},\ \gamma = H_{ws}A_{ws},\ \delta = H_{wn}A_{wn}, \\ \lambda = H_{we}A_{we},\ \mu = H_{ww}A_{ww},\ \sigma = H_r A_r,\ \tau = H_f A_f. \end{aligned} \tag{12}$$

$$\begin{aligned} G_z(s) = \frac{1}{C_z s + 2\alpha},\ G_{ws}(s) = \frac{1}{C_{ws}s + 2\gamma}, \\ G_{wn}(s) = \frac{1}{C_{wn}s + 2\delta},\ G_{we}(s) = \frac{1}{C_{we}s + 2\lambda}. \end{aligned} \tag{13}$$

$$\begin{aligned} G_{ww}(s) = \frac{1}{C_{ww}s + 2\mu},\ G_r(s) = \frac{1}{C_r s + \sigma}, \\ G_f(s) = \frac{1}{C_f s + \tau},\ G_{wz}(s) = \frac{f_s}{V_z s + f_s}. \end{aligned} \tag{14}$$

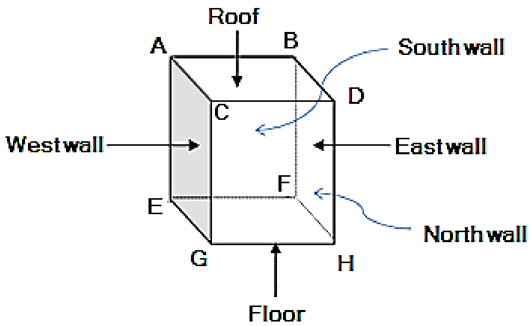

Figure 2. Graphical view of the zone

Fig. 2 shows a graphical view of the zone that modeled.

B. The Cooling Coil Model

The most important component of every HVAC system is cooling coil. The increased emphasis on variable operation of HVAC equipment warrants a greater understanding of the dynamic behavior of cooling coils. Due to importance of this component, many studies have done as [16], [17], and [18]. In Fig. 3 a view of cooling coil is shown.

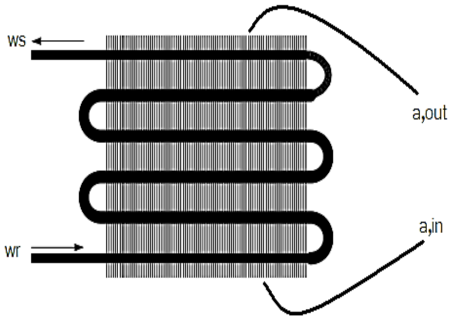

Figure 3. A view of cooling coil In air side

$$\frac{dT_{a,out}}{dt} = -\alpha(\bar{T}_a - \bar{T}_t) - \beta(T_{a,out} - T_{a,in}) \tag{15}$$

Where

$$\bar{T}_a = \frac{M_a c_{p,a} T_{a,in} + M_w c_w T_{a,out}}{M_a c_{p,a} + M_w c_w} \tag{16}$$

And

$$\alpha = \frac{h_t \eta_{s,ov} A_o}{\rho c_v A}, \ \beta = \frac{\gamma M_a}{\rho A L_c} \tag{17}$$

We can rewrite Eq. 16 as follow

$$\bar{T}_a(s) = \mu T_{a,in}(s) + \xi T_{a,out}(s) \tag{18}$$

That in which

$$\mu = \frac{M_a c_{p,a}}{M_a c_{p,a} + M_w c_w}, \ \xi = \frac{M_w c_w}{M_a c_{p,a} + M_w c_w}. \tag{19}$$

By taking Laplace transform Eq. 15

$$(s+\beta)T_a(s) = -\alpha(\bar{T}_a(s) - \bar{T}_t(s)) + \beta T_{a,in}(s) \tag{20}$$

We obtain $\bar{T}_t(s)$ by substituting Eq. 18 in Eq. 20

$$\bar{T}_t(s) = \frac{s+\beta+\alpha\xi}{\alpha} T_{a,out}(s) + \frac{\beta+\alpha\xi}{\alpha} T_{a,in}(s) \tag{21}$$

In water side

$$\frac{dT_{wr}}{dt} = \lambda(\bar{T}_t - \bar{T}_w) + \delta(T_{ws} - T_{wr}) \tag{22}$$

Where

$$\bar{T}_w = \frac{M_a c_{p,a} T_{wr} + M_w c_w T_{ws}}{M_a c_{p,a} + M_w c_w} \tag{23}$$

And

$$\lambda = \frac{h_{it} A_{it}}{m_w c_w}, \quad \delta = \frac{M_w}{m_w L_c} \tag{24}$$

With rewriting Eq. 23 we have

$$\bar{T}_w(s) = \mu T_{wr}(s) + \xi T_{ws}(s) \tag{25}$$

By taking Laplace transform Eq. 22

$$(s+\delta)T_{wr}(s) = \lambda \bar{T}_t(s) - \lambda \bar{T}_w(s) + \delta T_{ws}(s) \tag{26}$$

We obtain $\bar{T}_t(s)$ by substituting Eq. 25 in Eq. 26

$$\bar{T}_t(s) = \frac{s+\delta+\lambda\mu}{\lambda} T_{wr}(s) - \frac{\delta+\lambda\mu}{\lambda} T_{ws}(s) \tag{27}$$

By equating right-hand sides of Eq. 21 and Eq. 27

$$\begin{aligned} &\frac{s+\beta+\alpha\xi}{\alpha} T_a(s) + \frac{\beta+\alpha\xi}{\alpha} T_{a,in}(s) \\ &= \frac{s+\delta+\lambda\mu}{\lambda} T_{wr}(s) - \frac{\delta+\lambda\xi}{\lambda} T_{ws}(s) \end{aligned} \tag{28}$$

With simplify Eq. 28 we can obtain $T_a(s)$ in terms of $T_{wr}(s)$, $T_{ws}(s)$, and $T_{a,in}(s)$

$$\begin{aligned} T_a(s) = &\frac{\alpha(s+\delta+\lambda\mu)}{\lambda(s+\beta+\alpha\xi)} T_{wr}(s) - \frac{\alpha(\delta+\lambda\xi)}{\lambda(s+\beta+\alpha\xi)} T_{ws}(s) \\ &- \frac{\beta+\alpha\xi}{(s+\beta+\alpha\xi)} T_{a,in}(s) \end{aligned} \tag{29}$$

C. The Heating Coil Model

In HVAC systems heating coils are placed in the airstream to regulate the temperature of the air delivered to the space. During the heating mode, problems can occur if the hot water temperature in the heating coil has been set too low in an attempt to reduce energy consumption. If enough outdoor air to provide sufficient ventilation is brought in, that air may not be heated sufficiently to maintain thermal comfort or, in order to adequately condition the outdoor air, the amount of outdoor air may be reduced so that there is insufficient outdoor air to meet ventilation needs.

In proposed model for heating coil we considered effect of input/output water temperature through the coil, output temperature of the zone and input air from mixing box on output air temperature of coil.

$$C_{ah}\frac{dT_{co}}{dt} = f_{sw}\rho_w C_{pw}(T_{wi}-T_{wo}) + (UA)_a(T_o - T_{co}) + f_{sa}\rho_a C_{pa}(T_m - T_{co}) \quad (30)$$

$$[C_{ah}s + f_{sa}\rho_a C_{pa} + (UA)_a]\, T_{co}(s) = f_{sw}\rho_w C_{pw}[T_{wi}(s) - T_{wo}(s)] + (UA)_a T_o(s) + f_{sa}\rho_a C_{pa} T_m(s) \quad (31)$$

$$T_{co} = G_s(s)\{\nu[T_{wi}(s)-T_{wo}(s)] + \chi T_o(s) + \beta T_m(s)\} \quad (32)$$

Where

$$\xi = f_{sa}\rho_a C_{pa} + (UA)_a, \nu = f_{sw}\rho_w C_{pw}, \chi = (UA)_a, \quad G_s(s) = \frac{1}{C_{ah}s+\xi}. \quad (33)$$

$$(V_{ah} + f_{sa})W_{co}(s) = f_{sa}W_m(s) \quad (34)$$

$$W_s(s) = G_{ws}(s)W_{si}(s) \quad (35)$$

$$G_{ws}(s) = \frac{f_{sa}}{V_{ah}s + f_{sa}} \quad (36)$$

D. *The Humidifier Model*

The measurement and control of moisture in the air is an important phase of air conditioning [19]. In some references has been presented model for humidifier as [19] and [20].

$$C_h \frac{dT_h}{dt} = f_{sa}C_{pa}(T_{si}-T_h) + \alpha_h(T_o - T_h) \quad (37)$$

$$V_h\frac{dW_h}{dt} = f_{sa}(W_{si}-W_h) + \frac{h(t)}{\rho_a} \quad (38)$$

E. *The Mixing Box Model*

A mixing box is the section of an air handling unit used to mix the return air flow with the outside air flow. It consists of three sets of dampers whose operation is coordinated to control the fraction of the outside air in the supply air while maintaining the supply airflow rate approximately constant. In Fig. 4 structure of air flow in the mixing box unit is shown.

$$m_r C_{pa} T_r + m_o C_{pa} T_o = m_m C_{pa} T_m \quad (39)$$

$$m_r + m_o = m_m \quad (40)$$

$$T_m = \frac{m_r T_r + m_o T_o}{m_r + m_o} \quad (41)$$

$$W_m = \frac{m_r W_r + m_o W_o}{m_r + m_o} \quad (42)$$

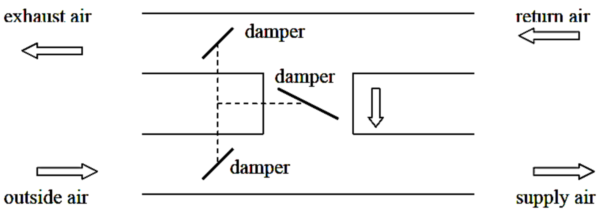

Figure 4. The mixing box structure

F. *The Duct Model*

The ducts are used in HVAC systems to deliver and remove air. Air ducts are one method of ensuring acceptable indoor air quality as well as thermal comfort.

$$\frac{dT_{out}}{dt} = \frac{(h_i + h_o)m_a C_p}{h_i M_c C_c}(T_{in} - T_{out}) \quad (43)$$

$$T_o(s) = G_{duct}(s)T_{in}(s) \quad (44)$$

$$G_{duct}(s) = \frac{\tau}{s+\tau}, \quad \tau = \frac{(h_i + h_o)m_a C_p}{h_i M_c C_c} \quad (45)$$

$$T_{s,temp}(s) = \frac{\tau_{s,temp}}{\tau_{s,temp}s+1}T_{m,temp}(s) \quad (46)$$

G. *The Sensors Model*

The sensors that are used in HVAC systems have different types as temperature sensor, Humidity sensor, pressure sensor, and flow sensor. In many studies different models have been presented to these sensors. For instance, Wang [17] by using of time constant manner considered a first order differential equation for temperature and humidity sensor.

Note that for achieve more exact model we have to consider different time constant. In proposed model, we have considered a first order differential equation with time constant 1 for temperature sensor and time constant 2 for humidity sensor.

Fig. 6 and Fig. 7 show changes in temperature and relative humidity over the period 1th-31th march 2012 in Kerman, respectively.

III. SIMULATION RESULTS OF PROPOSED MODEL

In order to simulate suggested system, the average temperature and relative humidity of Kerman are compared for mentioned period and are considered as two inputs for mixing box subsystem. In this regards, we consider two control loops so as to control temperature and relative humidity.

$$T_{s,hum}(s) = \frac{\tau_{s,hum}}{\tau_{s,hum}s+1}T_{m,hum}(s) \quad (47)$$

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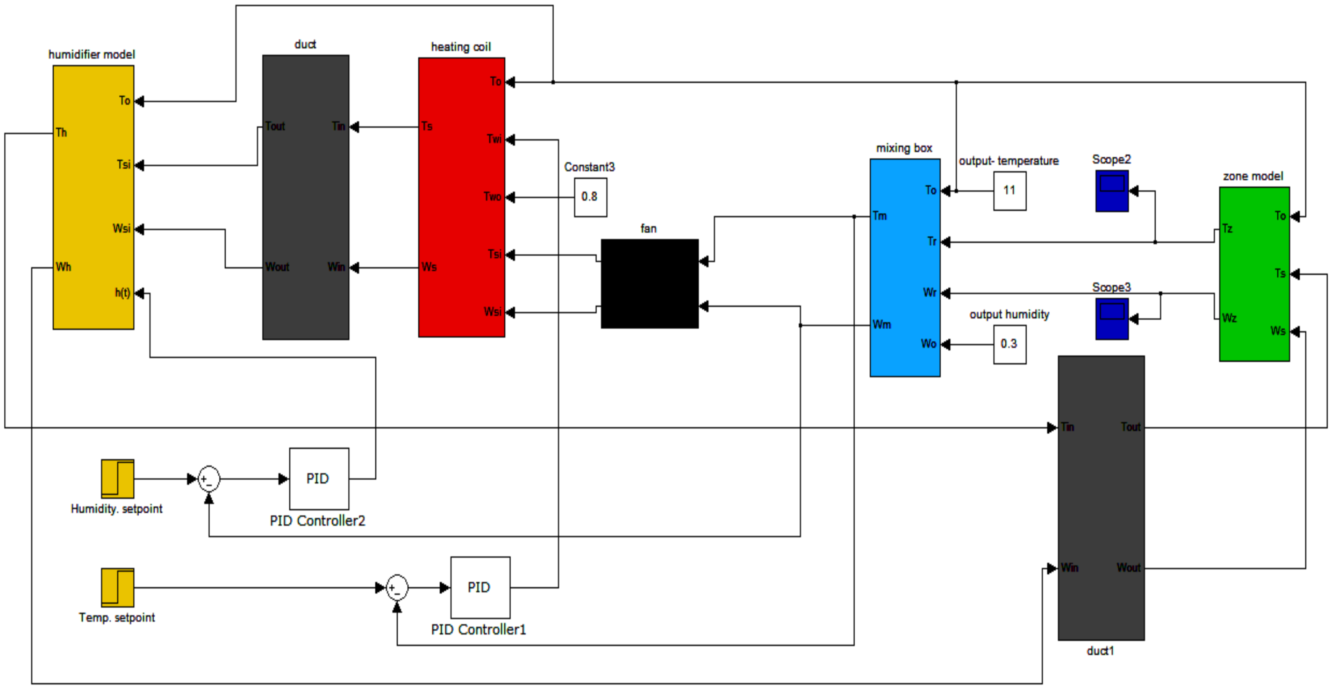

Figure 5. Proposed model for HVAC system

IV. PROPOSED MODEL

The proposed model for a HVAC system is presented in Fig. 5. This figure shows all components as subsystem in simulink platform. Moreover, the components arrangement is based on their real models.

We applied output temperature and relative humidity to model, then simulated it. We considered two PID controllers for control the loops.

Fig. 8 indicates the temperature-time curve of our suggested model. From this figure, the zone temperature tracks the set point very well, at time 40s arrive to desired value, and speed of convergence is high.

The relative humidity-time curve is presented in Fig. 9 for our suggested model. According to figures, it is revealed that suggested model track the set point with less time constant and high speed.

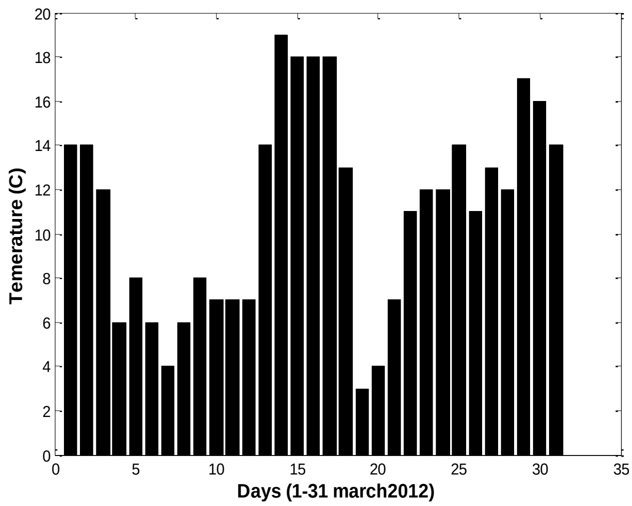

Figure 6. Changes of temperature over the period 1th-31th march 2012

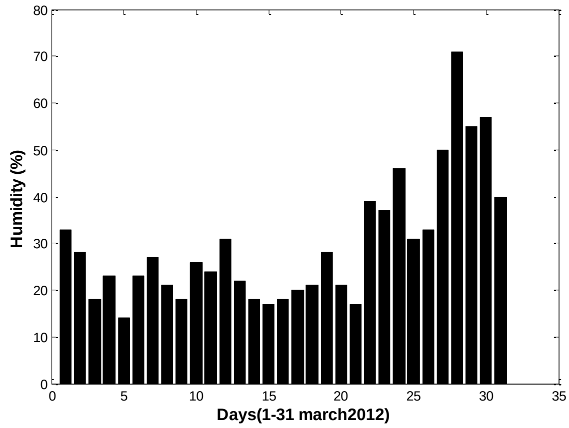

Figure 7. Changes of relative humidity over the period 1th-31th march 2012

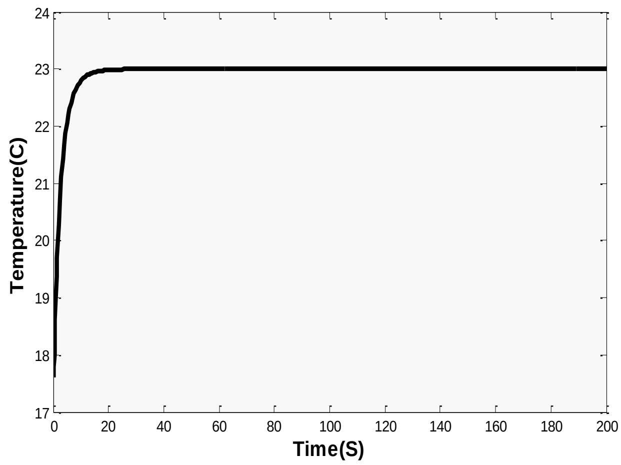

Figure 8. The zone temperature- time curve of proposed model

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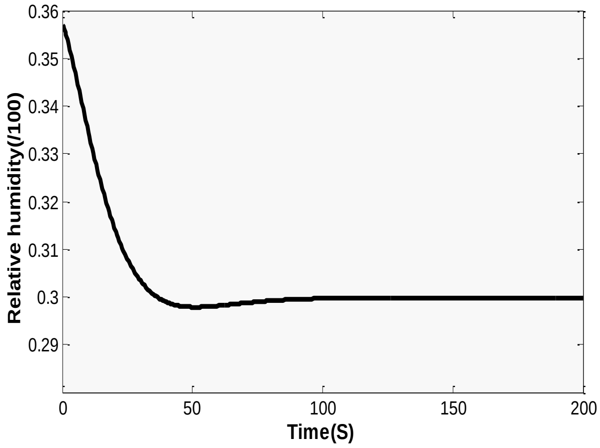

Figure 9. The humidity ratio- time curve of proposed model

V. CONCLUSION

In this paper we have presented a dynamic model for HVAC system components as the zone, cooling coil, heating coil, humidifier, mixing box, the ducts and the sensors. After derive mathematical models of components we analyzed simulation system of the complete HVAC system in Matlab/Simulink platform. In this models, which have been presented up to now, several simplifier assumptions such as effect of floor and roof on zone temperature, have been considered. But, we present a model, which is close to real model. Indeed, we present a complete mathematical dynamic model. The simulation results demonstrated that our suggested model for HVAC system components can be work very well.

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