A New Direct Search Method for Distributed Estimation in Wireless Sensor Networks

Zhenxing Luo

Abstract: Distributed estimation is a popular research topic in wireless sensor networks (WSNs). A maximum likelihood estimation (MLE) method is widely used in WSNs for distributed estimation. However, the MLE method is a computationally intensive method. To overcome this problem, in this paper, a new direct search method will be presented. This method has much lower computation complexity while can achieve estimation results similar to the results given by the MLE method.

Index Terms: Direct Search, Maximum Likelihood Estimation, Wireless Sensor Networks.

I. INTRODUCTION

Target estimation and parameter estimation have gained significant attention in wireless sensor networks (WSNs) [1]-[15]. A maximum likelihood estimation (MLE) method has been used widely because this method can provide satisfactory results [16]-[18]. However, the MLE method is computationally intensive. In this paper, we will present a new direct search method for distributed estimation, which has much lower computation cost. This method is different from the mean estimator in [19] because our method uses quantized data while the mean estimator in [19] uses analog data. Moreover, the sign estimator in [19] is also different from our method. The sign estimator uses the sign of signals and can be viewed as a particular example of methods using quantized data.

The rest of this paper is organized in the following way. Section II provides formulation of the MLE method, followed by the direct search method in Section III. Section IV provides simulation setup and Section V presents results and discussion. Finally, conclusion is presented in Section VI.

II. FORMULATION OF THE MLE METHOD

The MLE method has been widely used for distributed estimation. Because the direct search method will be compared with the MLE method, the formulation of the MLE method will be replicated here. The MLE has been presented in [16]-[18] and readers can refer to these references for more details.

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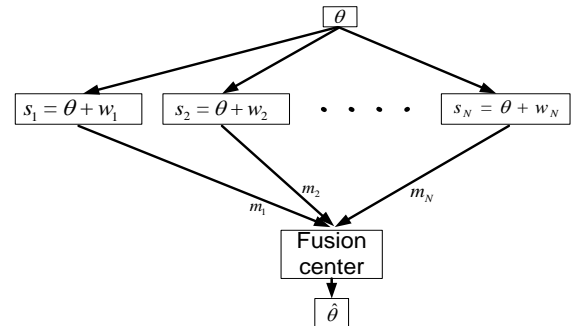
Fig. 1 Diagram of a WSN

In the distributed estimation problem, one or more parameters can be estimated. In this paper, for simplicity, we only estimate one parameter θ. The diagram of a WSN is shown in Fig. 1. The parameter θ is estimated using a total number of N identical sensors.

The signal received by the ith sensor can be expressed as

$$s_i = \theta + w_i \qquad (1)$$

where θ is the parameter to be estimated and w_i is an environmental noise. The noise w_i is a Gaussian noise and follows the distribution $N(0,1)$. The ith sensor, after acquiring the signal s_i, will quantize the signal s_i into a decision m_i according to threshold η_i. For a given θ, m_i takes value l with the probability

$$p_{il}(\eta_i, \theta) = Q(\eta_{il}) - Q(\eta_{i(l+1)}) = \int_{\eta_{il}}^{\eta_{i(l+1)}} f(s_i) ds_i \qquad (2)$$

where $R(x)$ is defined as

$$Q(x) = \int_x^\infty \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} dt . \qquad (3)$$

If the decision vector received by the fusion center is the

$$\mathbf{M} = [m_1, m_2, \cdots, m_{N-1} \; m_N] , \qquad (4)$$

fusion center can estimate θ by maximizing

$$\ln p(\mathbf{M}|\theta) = \sum_{i=1}^N \sum_{l=0}^{L-1} \delta(m_i - l) \ln \left[p_{il}(\eta_i, \theta) \right] \qquad (5)$$

where

$$\delta(x) = \begin{cases} 1, & x = 0 \\ 0, & x = 0 \end{cases} . \qquad (6)$$

Now, the MLE estimator can be expressed as

$$\hat{\theta} = \max_\theta \ln p(\mathbf{M}|\theta). \qquad (7)$$

If the estimate of θ is unbiased, the CRLB can be calculated by

$$E\{[\hat{\theta}(\mathbf{M}) - \theta][\hat{\theta}(\mathbf{M}) - \theta]^T\} \geq \mathbf{J}^{-1} \qquad (8)$$

$$\mathbf{J} = -E\left[\nabla_\theta \nabla_\theta^T \ln p\left(\mathbf{M}|\theta\right)\right]. \qquad (9)$$

Details about the method to calculate CRLB can be found in [16]-[18].

III. THE DIRECT SEARCH METHOD

The MLE method for distributed estimation requires iterative search in (7), which is computationally intensive. A new direct search method (DSM) can circumvent this problem. The direct search method can be expressed as

$$\int_{-\infty}^{\eta} \frac{1}{\sqrt{2\pi}} e^{-\frac{(t-\theta)^2}{2}} dt = \frac{\text{The number of } 0 \text{ decisions}}{N} . \qquad (10)$$

The right side of (10) is the proportion of 0 decisions received at the fusion center. The left side of (10) is the probability that the decision arrived at the fusion center is 0. The physical meaning of (10) is that the probability that the decision is 0 should equal to the proportion of 0 decisions received at the fusion center. The calculation of (10) is less intensive than the MLE method because (10) requires only one time search of the cumulative distribution function (CDF) of a Gaussian distribution while the MLE method requires iterative search.

IV. SIMULATION SETUP

First of all, we will validate the MLE method by calculating the normalized estimation error squared (NEES) values [8]. Then, we will compare the MLE method with our DSM in terms of estimation performance and execution time. Because the MLE method will give unbiased results, we will use root-mean-square (RMS) errors as a performance criterion. The CRLB will also be provided to serve as a benchmark. We will set $\theta = 5$ and $\eta_i = 5$ for all simulations. RMS errors are calculated based on 100 Monte Carlo runs.

V. RESULTS AND ANALYSIS

For Chi-square distribution with 100 degrees of freedom, the 95% confidence interval is [0.742219, 1.29561] [11][15]. We can see that when the number of sensors was greater than 15, the NEES values fell into the confidence range (Table 1).

As for estimation performance, when the number of sensors was large, the RMS errors given by the MLE method were very close to the RMS errors given by DSM (Fig. 2). Moreover, the RMS errors given by the MLE method were also close to the CRLB. However, when the number of sensors was low, the RMS errors given by DSM were lower than the RMS errors given by the MLE method. A possible reason is that the MLE method could not converge if the number of received decisions was not enough.

Table 1: NEES values (100 runs, $\theta = 5$, and $\eta_i = 5$)

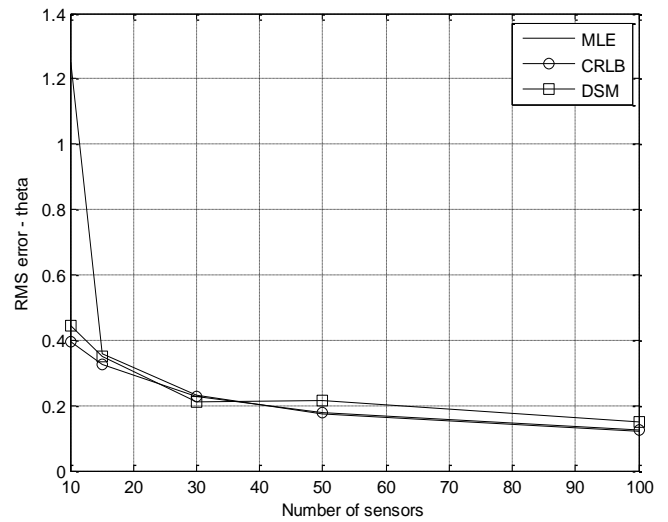| Number of sensors	10	15	30	50	100
NEES value	9.9707	1.2135	1.0343	0.9831	0.9145

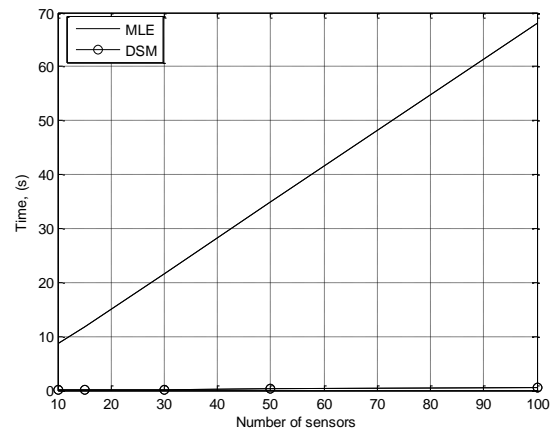Fig. 2 RMS errors given by the MLE method and DSM (100 runs, $\theta = 5$, and $\eta_i = 5$)

Fig. 3 Execution time

The execution time can indicate the complexity of calculation. The execution time of DSM was much shorter than the execution time of the MLE method (Fig. 3). Moreover, the execution time of the MLE method increased linearly as the number of sensors increased. Therefore, we can see that the MLE method is much more computationally intensive than DSM.

VI. CONCLUSION

In this paper, we presented a direct search method for distributed estimation. The estimation performance of DSM was comparable to the estimation performance of the MLE method. However, the execution time of DSM was much shorter. Therefore, DSM is possibly a better choice for distributed estimation.

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