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Distributed Estimation in Wireless Sensor Networks with Heterogeneous Sensors

Zhenxing Luo

Abstract*—*In this paper, a robust distributed estimation method in wireless sensor networks (WSNs) with heterogeneous sensors is presented. Particularly, a single parameter is estimated based on decisions from heterogeneous sensors, which have different signal gains. The sensor gains follow uniform distributions. Using the distributions of sensor gains, we calculated the probability density function of the signal received by sensors. Then, the overall likelihood function for a given decision vector is derived and a maximum likelihood estimation (MLE) method is used to estimate the unknown parameter. To evaluate estimation performance, the Cramer-Rao lower bound (CRLB) is also derived. Simulation results showed that if the range of sensor gains was narrow, the RMS errors were close to CRLB. If the range of sensor gains was wide, the RMS errors deviated from CRLB.

Index Terms*—*Distributed estimation, maximum likelihood estimation, wireless sensor networks.

I. INTRODUCTION

Wireless sensor networks (WSNs) have become a popular research topic [1-9][17-23]. Usually, WSNs can have two different structures. One structure of WSNs consists of a large number of sensors and a head node [10]. The other structure of WSNs consists of a large number of sensors and a fusion center [8]. In the second structure, sensors can gather environmental information and send the information to the fusion center. The fusion center, after receiving information from sensors, can process the information and perform many tasks, such as estimation, tracking and detection [8].

In this paper, we only consider the second structure. For the estimation problem, the fusion center can estimate some unknown parameters based on information from sensors. The measurement process is carried out by sensors and the measurement model can be either a nonlinear model, such as the measurement model in [8][9], or a linear model, such as the measurement model in [11]-[15]. Using a nonlinear measurement model, in [8][9], the fusion center employed a maximum likelihood estimation(MLE) method to estimate three unknown parameters. Using a linear measurement model, in [11][12], the fusion center estimates a parameter based on decisions transmitted through constrained communication channels. Similarly, for a linear measurement model, a nonparametric estimation method was presented in [13]. However, in [11]-[15], the sensor gains were assumed to be identical. In practice, sensor gains may not be identical because of manufacturing errors or environmental influence.

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Zhenxing Luo, Department of Electrical and Computer Engineering, the University of Alabama at Birmingham, Birmingham, AL, USA.

To address this problem, we propose to model the probability density functions (PDFs) of sensor gains as uniform distributions and incorporate the PDFs into the MLE scheme. A similar idea to model factors, which can affect estimation performance, was presented in [16]. However, in [16], the measurement model was a nonlinear model and the sensor position uncertainty was modeled.

The main contribution of this paper is the modeling of sensor gains and incorporating the sensor gains into the MLE scheme. Particularly, the sensor gains are modeled using uniform distributions. Moreover, the Cramer-Rao lower bound (CRLB) corresponding to the robust estimation method is derived. Finally, simulation results are presented to prove that the robust estimation method could provide RMS estimation errors close to the CRLB.

The rest of the paper is organized in the following way. Section 2 presents the distributed estimation method based on decisions transmitted by heterogeneous sensors. The CRLB is presented in Section 3. The simulation setup was provided in Section 4, followed by results and analysis in Section 5. Concluding remarks are delivered in Section 6.

II. DISTRIBUTED ESTIMATION BASED ON DECISIONS TRANSMITTED BY HETEROGENEOUS SENSORS

As shown in Figure 1, an unknown but fixed parameter, θ, is estimated using a total number of N sensors. Sensors are not identical and have different signal gains. The gain of the ith sensor is denoted as G_i, which follows the uniform distribution $G_i \in u[a,b]$.

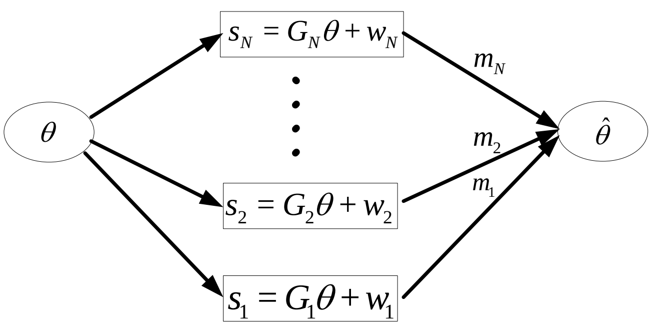

Figure 1: Distributed estimation system diagram

The signal received by the ith sensor from θ is a_i, which can be denoted as:

$$a_i = G_i\theta . \tag{1}$$

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Because the signal emitting from θ will be corrupted by noises, the signal finally measured by the ith sensor is s_i. The measurement process can be denoted as:

$$s_i = a_i + w_i = G_i\theta + w_i\,. \qquad (2)$$

The noise w_i is a Gaussian noise, which follows the distribution $N(0,\sigma^2)$.

The PDF of G_i is

$$f(G_i) = \frac{1}{b-a},\ \ G_i \in [a,b]\,. \qquad (3)$$

From $f(G_i)$, we can have the PDF of

$$f(a_i) = \frac{1}{(b-a)\theta},\ \ a_i \in [\theta a,\theta b]\,. \qquad (4)$$

It is well-known that the PDF of the sum of two independent random variables can be derived by calculating the convolution of their PDFs [16]. Therefore, we have

$$f(s_i) = \int_{\theta a}^{\theta b} \frac{1}{(b-a)\theta}\frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(s_i-\tau)^2}{2\sigma^2}} d\tau,\ \ a_i \in [\theta a,\theta b]\,. \qquad (5)$$

After a sensor measures the signal, the sensor will compare the signal received with a set of pre-determined thresholds $\vec{\gamma}_i$, which is

$$\vec{\gamma}_i = [\gamma_{i0},\gamma_{i1},...,\gamma_{iL}]\,. \qquad (6)$$

By comparing the received signal with $\vec{\gamma}_i$, the ith sensor will produce a decision m_i. The process is denoted by

$$m_i = \begin{cases} 0 & -\infty < s_i < \gamma_{i1} \\ 1 & \gamma_{i1} < s_i < \gamma_{i2} \\ \vdots & \vdots \\ L-2 & \gamma_{i(L-2)} < s_i < \gamma_{i(L-1)} \\ L-1 & \gamma_{i(L-1)} < s_i < \infty \end{cases} \qquad (7)$$

In (9), $\gamma_{i0} = -\infty$ and $\gamma_{iL} = \infty$. Given a θ, the probability that m_i assumes value l is

$$p_{il}(\vec{\gamma}_i,\theta) = R(\gamma_{il}) - R(\gamma_{i(l+1)}) = \int_{\gamma_{il}}^{\gamma_{i(l+1)}} f(s_i)ds_i \qquad (8)$$

where $R(x)$ is defined as

$$R(x) = \int_x^{\infty} f(s_i)ds_i \qquad (9)$$

For a given decision vector

$$\mathbf{M} = [m_1,\ m_2,\ \cdots, m_{N-1}\ m_N]\,, \qquad (10)$$

the fusion center estimates θ by maximizing

$$\ln p(\mathbf{M}|\theta) = \sum_{i=1}^{N}\sum_{l=0}^{L-1} \delta(m_i - l)\ln\left[p_{il}(\vec{\gamma}_i,\theta)\right] \qquad (11)$$

where

$$\delta(x) = \begin{cases} 1, & x = 0 \\ 0, & x = 0 \end{cases}. \qquad (12)$$

The maximum likelihood estimator is

$$\hat{\theta} = \max_{\theta} \ln p\left(\mathbf{M}|\theta\right). \qquad (13)$$

For an unbiased estimate of θ, the CRLB is given by

$$E\{[\hat{\theta}(\mathbf{M}) - \theta][\hat{\theta}(\mathbf{M}) - \theta]^T\} \geq \mathbf{J}^{-1} \qquad (14)$$

$$\mathbf{J} = -E\left[\nabla_\theta \nabla_\theta^T \ln p\left(\mathbf{M}|\theta\right)\right]. \qquad (15)$$

III. CRAMER-RAO LOWER BOUND

If the estimate is unbiased, the root-square-mean (RMS) errors are the most important indicator of estimation performance. The CRLB is the lowest bound of the RMS errors. Therefore, we will derive the CRLB for the maximum likelihood estimator in (13).

For the maximum likelihood estimator in (13), the (1, 1) element of matrix $\mathbf{J}$ can be derived using an approach similar to that employed in [8]

$$\frac{\partial^2 \ln p\left(\mathbf{M}|\theta\right)}{\partial P_0^2} = \sum_i\sum_l -\frac{\delta(m_i - l)}{p_{il}^2(\vec{\gamma}_i,\theta)}\left[\frac{\partial p_{il}(\vec{\gamma}_i,\theta)}{\partial P_0}\right]^2 + \frac{\delta(m_i - l)}{p_{il}(\vec{\gamma}_i,\theta)}\frac{\partial^2 p_{il}(\vec{\gamma}_i,\theta)}{\partial P_0^2}. \qquad (16)$$

Because $E\left[\delta(m_i - l)\right] = p_{il}(\vec{\gamma}_i,\theta)$, the expected value of (16) is

$$E\left[\frac{\partial^2 p\left(\mathbf{M}|\theta\right)}{\partial P_0^2}\right] = \sum_i\sum_l -\frac{1}{p_{il}(\vec{\gamma}_i,\theta)}\left[\frac{\partial p_{il}(\vec{\gamma}_i,\theta)}{\partial \theta}\right]^2. \qquad (17)$$

The expression of $p_{il}(\vec{\gamma}_i,\theta)$ is

$$p_{il}(\vec{\gamma}_i,\theta) = \int_{\gamma_{il}}^{\gamma_{i(l+1)}} \frac{1}{\theta(b-a)}\int_{\theta a}^{\theta b} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(s_i-\tau)^2}{2\sigma^2}} d\tau ds_i. \qquad (18)$$

The derivative of $p_{il}(\vec{\gamma}_i,\theta)$ is

$$\Delta = \frac{\partial p_{il}(\vec{\gamma}_i,\theta)}{\partial \theta} = \int_{\gamma_{il}}^{\gamma_{i(l+1)}} \left[\frac{-\int_{\theta a}^{\theta b}\frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{(s_i-\tau)^2}{2\sigma^2}}d\tau}{(b-a)\theta^2} + \frac{be^{-\frac{(s_i-\theta b)^2}{2\sigma^2}} - ae^{-\frac{(s_i-\theta a)^2}{2\sigma^2}}}{\sqrt{2\pi}\sigma(b-a)\theta}\right] ds_i. \qquad (19)$$

Finally, we have

$$\mathbf{J}(1,1) = E\left[\frac{\partial^2 \ln p\left(\mathbf{M}|\theta\right)}{\partial P_0^2}\right] = \sum_i\sum_l -\frac{\Delta^2}{p_{il}}. \qquad (20)$$

Other elements of $\mathbf{J}$ can be derived similarly.

IV. SIMULATION SETUP

To validate the robust estimation method, normalized estimation error squared (NEES) values are presented first. We set $a=1$, $\theta=5$, $\gamma=5$ and varied b value to generate NEES values. All NEES values were calculated based on 100 runs. Results are shown in Table 1. The sensor gains follow the same uniform distribution, which is defined by the lower bound a and upper bound b. To show the effect of b values on the RMS errors and CRLB, we set $a=1$, $\theta=5$, $\gamma=5$ and varied b values.

Results are shown in Figure 2. To show the effect of a values on the RMS errors and CRLB, we set $b=1$, $\theta=5$, $\gamma=5$ and varied a values from -3 to 1. Results are shown in Figure 3. Threshold is also important in the robust estimation method. To see the effect of threshold γ on RMS errors and CRLB, we set $a=1$, b=2, $\theta=5$, and varied γ from 2 to 6. All RMS errors in this paper were calculated using 100 runs.

V. RESULTS AND ANALYSIS

For unbiased MLE method, the estimation error of θ is approximately Gaussian and the NEES follows Chi-square distribution [8]. The 95% confidence interval for Chi-square with 100 degrees of freedom is [0.742219, 1.29561]. Comparing the NEES values in Table 1 with the confidence range, we can see that when $b=1$ and $b=2$, the NEES values were within the confidence range. When b was greater than 2, the NEES values were outside the confidence range. This illustrated that the robust estimation method was valid when the range of uniform distribution was narrow. When the range of the uniform distribution was wide, the NEES values also increased. This means that when the sensor gains are so different from each other, the estimation performance will degrade.

Table 1: NEES values for different b values ($a=1$, $\theta=5$, $\gamma=5$ and 100 runs)

	$a=1$ $b=1$	$a=1$ $b=2$	$a=1$ $b=3$	$a=1$ $b=4$	$a=1$ $b=5$
NEES value	0.9501	1.1886	30.3260	30.7748	110.1886

The b value can also significantly affect the estimation errors and CRLB. When the b value was close to 1, which means the range of sensor gains was narrow, the RMS errors were small and the RMS errors were close to the CRLB. When the b value increased, the RMS errors also increased and deviated from CRLB (Figure 2).

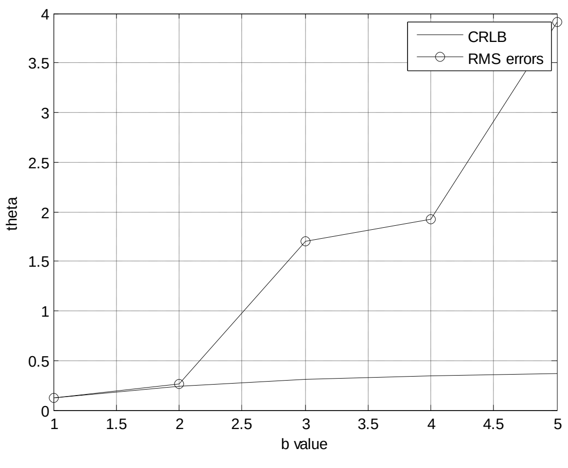

Figure 2: RMS estimation errors compared to the CRLB ($a=1$, $\theta=5$, $\gamma=5$, 100 runs, and different b values)

Similarly, when the a value was close to 1, which means the range of sensor gains was narrow, the RMS errors were low and the RMS errors were close to the CRLB. When the a value deviated from 1, the RMS errors also increased and were much greater than the CRLB (Figure 3).

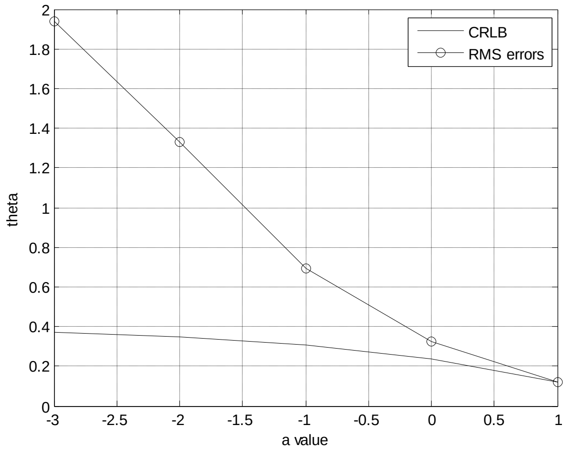

Figure 3: RMS estimation errors compared to the CRLB ($b=1$, $\theta=5$, $\gamma=5$, 100 runs, and different a values)

The results in Figure 2 and Figure 3 proved that the robust estimation method could provide satisfactory results when the range of sensor gains was narrow. Large range of sensor gains will definitely introduce greater estimation errors.

Sensors quantize the received signal according to the threshold. Therefore, threshold is also important in the robust estimation method. When the threshold was close to the real θ value, the RMS errors were small (Figure 4). However, if the threshold was far away from the real θ value, the RMS errors increased significantly. This is because the environmental noise is a Gaussian noise with 0 mean. If the threshold is close to the real θ value, about half sensors will be fired and half sensors will not be fired. If threshold is significantly lower than the real θ value, many sensors will be fired and sensors are saturated. Information from sensors will not be useful. Similarly, if the threshold is too high, not enough sensors will be fired and identifiability becomes a problem.

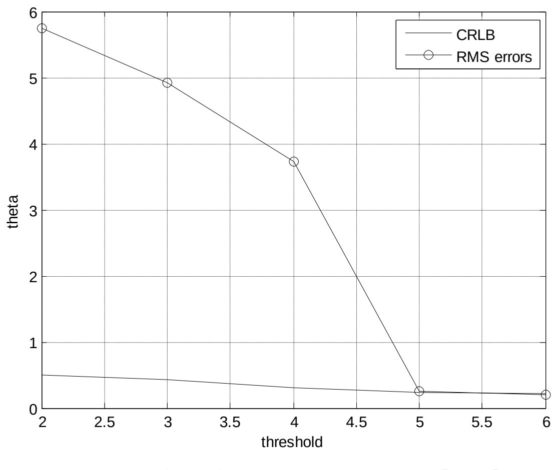

Figure 4: RMS estimation errors compared to the CRLB ($a=1$, $b=2$, $\theta=5$, 100 runs, and different γ values)

VI. CONCLUSION

In this paper, a robust estimation method is presented to deal with heterogeneous sensors with different sensor gains in WSNs. Simulation results showed that this method could alleviate the problem caused by sensor gains when the range of sensor gains was narrow. In practice, sensors may have different gains due to manufacturing errors or environmental influences. Therefore, this method is useful in many real applications.

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