Signal Denoising with Interval Dependent Thresholding using DWT and SWT

Ramesh Kumar, Prabhat Patel

Abstract*— Degradation of signals by noise is an omnipresent problem in almost all fields of signal processing. Therefore, in practical applications, before analyzing the received signal it is necessary to de-noise it. In this paper we have used the interval dependent threshold selection rule for threshold calculation and analyzed its performance with the help of Discrete and Stationary Wavelet Transform. The results show that Mean Square Error (MSE) of the interval dependent threshold selection rule is less than that of conventional fix form threshold selection rule for signal de-noising. Moreover using the interval dependent thresholding with Stationary Wavelet Transform (SWT) the denoising capacity of the SWT increases significantly. Therefore, it proves the superiority of proposed signal denoising algorithm. Moreover the result obtained with is compared with the denoising capability of interval dependent threshold selection rule using Discrete Wavelet Transform(DWT) and it is found that SWT gives better performance than DWT due to its property of translational invariance.

***Index Terms*— Interval dependent threshold selection, Discrete Wavelet Transform (DWT). Stationary Wavelet Transform (SWT). Mean Square Error (MSE), Signal Denoising..**

I. INTRODUCTION

In present era of communication and medical advancement, the transmission and processing of communication and biomedical signal has become a must need. During transmission, processing and reception, these signals get corrupted by noise. Therefore, it has been very necessary to device an effective method to remove these unwanted noises from these signals of importance. For this purpose various signal de-noising techniques such as linear methods (Fourier transform de-noising, Wiener filtering) and non-linear methods (wavelet transform de-noising) have been proposed by the researchers. Linear methods of signal de-noising have been widely used for noise removing up to 1990, because of their relative simplicity. However, since these methods are based upon the assumption that the signals are stationary, their effectiveness is generally acceptable but limited. But in reality, real-world signals have typically non-stationary statistical characteristics. Therefore, nonlinear methods like wavelet transform have been an active area of research for last two decades because of their ability to elucidate, simultaneously, the spectral and temporal information in a signal [1]. Wavelet transform has also an added advantage of multi resolution [1] capability which allows the transform to analyze the signal at different level of decomposition.

In general, there are three wavelet transform [2] based approaches which are used to denoised signal in wavelet domain. First one is based on the principle of modulus maxima [3] with wavelet transform and second one is grounded on the different correlation properties between the wavelet coefficient of the noise and the original signal. The third approach, i.e. the threshold de-noising given by Dohono [4], is the most commonly used de-noising approach because of its implementation simplicity. In threshold denoising technique, the threshold selection is very important. Therefore to choose the appropriate threshold, Donoho and Johnstone [4, 5] have introduced the minimax and universal thresholding schemes, and discussed both hard and soft thresholds in a general context that included ideal denoising in both the wavelet and Fourier domains. Among these threshold selection rule the universal threshold selection rule is the most widely used rule. The universal threshold gives the fix threshold value. But in practical application the variance of the noise signal changes with time and therefore there are several different variance values on several time intervals and hence there the threshold should be different for these different time interval. Keeping this point in mind we have used the interval dependent threshold selection rule [7] using SWT for signal denoising and compared the result with DWT.

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A. Wavelet Threshod Denoising

A. Discrete wavelet transform

The discrete wavelet transform (DWT) [2] of a sequence *x(n)* is given by ,

$$W(j,b) = \sum_n \frac{1}{2^j} x(n)\psi^*\left(\frac{n-b}{2^j}\right) \tag{1}$$

where ψ represents a wavelet function, which is dilated and contracted by the integer scale factor j, and delayed in time by parameter b. For an N point sequence the scale factor j assumes the values j = 0, 1,…, log2(N), producing a multi-resolution decomposition of the input signal. The delay values *b* are related to the scale by $b = K \times 2^j$ for K an integer.

In practice, the DWT is computed by passing a signal successively through a high-pass and a low pass filter. For each decomposition level, the high-pass filter forming the wavelet function produces the approximations coefficients, (a). The complementary low-pass filter representing the scaling function produces the details coefficients, d . This can be described as follows

$$d_1(n)=\sum g_1[k]\, S[2n-k] \tag{2}$$

$$a_1(n) =\sum h_1[k]\, S[2n-k] \tag{3}$$

where $d_1(n) and\ a_1(n)$ are the detailed and approximation coefficients at first level of decomposition. In the DWT case the lengths of $a_1(n)$ and $d_1(n)$ is *N/*2 where N is the length of the signal. At the next level of the DWT, $a_1(n)$ is used to generate $a_2(n)$ and $d_2(n)$ with modified filter $h_2[n]$ and $g_2[n]$, which are obtained by up sampling or down sampling $h_1[n]$ and $g_1[n]$, respectively. This process is continued recursively. Mathematically, for $j = 1,2,\dots\dots,J_0-1,\ j \le j_0$.

$$a_{j+1}(n)=\sum h_{j+1}[k]\, a_j\,[2n-k] \tag{4}$$

$$d_{j+1}(n) = \sum g_{j+1}[k]\, a_j\, [2n-k] \quad (5)$$

B. Stationary wavelet transform

The Stationary Wavelet Transform [1] is similar to the Discrete Wavelet Transform in that the high-pass and low-pass filters are applied to the input signal at each level. However, in the SWT [6], the output signal is never sub sampled (not decimated). Instead, the filters are up sampled at each level.

Suppose we are given a signal $S[n]$ of length N where $N = 2^J$ for some integer J. Let $h_1(n)$ and $g_1(n)$ be the impulse responses of the low-pass filter and the high-pass filter. The impulse responses are chosen such that the outputs of the filters are orthogonal to each other. At the first level of SWT, the input signal $S[n]$ is convolved with $h_1(n)$ to obtain the approximation coefficients $a_1(n)$ and with $g_1(n)$ to obtain the detailed coefficients, $d_1(n)$ i.e

$$a_1(n) = h_1(n) * S(n) = \sum h_1[n-k]\, \mathrm{S}[k] \quad (6)$$

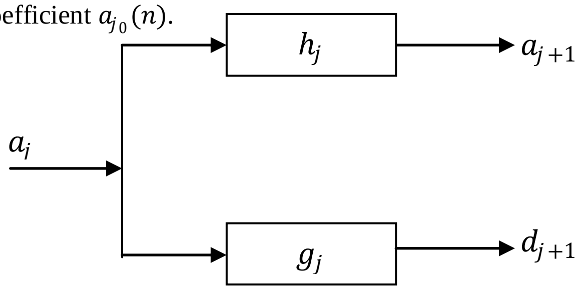
$$d_1(n) = g_1(n) * S(n) = \sum g_1[n-k]\, \mathrm{S}[k] \quad (7)$$

Since no sub-sampling is used in SWT, $a_1(n)$ and $d_1(n)$ are the sequences of length N. On the contrary, in the DWT case the lengths of $a_1(n)$ and $d_1(n)$ is $N/2$ resulting in loss of information [11]. At the next level of the SWT, $a_1(n)$is used to generate $a_2(n)$ and $d_2(n)$ with modified filter $h_2[n]$ and $g_2[n]$, which are obtained by up sampling $h_1[n]$ and $g_1[n]$, respectively. This process is continued recursively. Mathematically, for $j = 1, 2, \ldots\ldots, J_0 - 1$, $j \le j_0$

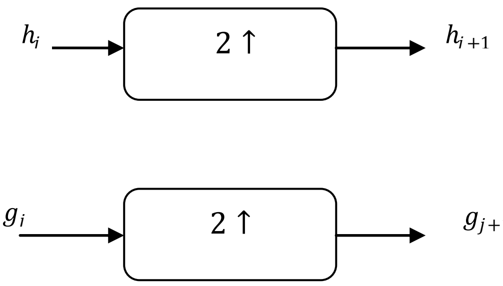
$$a_{j+1}(n) = h_{j+1}(n) * a_j(n) = \sum h_{j+1}[n-k]\, a_j\, [k] \quad (8)$$

$$d_{j+1}(n) = g_{j+1}(n) * a_j(n) = \sum g_{j+1}[n-k]\, a_j\, [k] \quad (9)$$

where $h_{j+1}(n)$ = up sampled $h_j(n)$ and $g_{J+1}(n)$ = up sampled $g_J(n)$. The output of the SWT is then the detailed coefficients$d_1(n)$,$d_2(n)$,……,$d_{j_0}(n$;and the approximation coefficient $a_{j_0}(n)$.



FILTER COMPUTATION



C . Thr Here $\boxed{2\uparrow}$ denotes up samples

A non stationary one dimensional signal corrupted with white Gaussian noise can be modeled as follows:

$$y(k) = x(k) + n(k) \quad (10)$$

Where, $y(k)$ is the noisy signal, $x(k)$ is the original signal and $n(k)$ is the white Gaussian noise. After the wavelet transformation of the noisy signal has been performed the resultant coefficients contains not only the coefficients of original signal but also the wavelet coefficients of noise. Fig. 3 shows the noise free signal and its wavelet transform and Fig. 4 shows the signal added with Gaussian noise, along with its wavelet transform. Fig. 3 and Fig. 4 shows that the noise is distributed as small coefficients in transform domain [6]

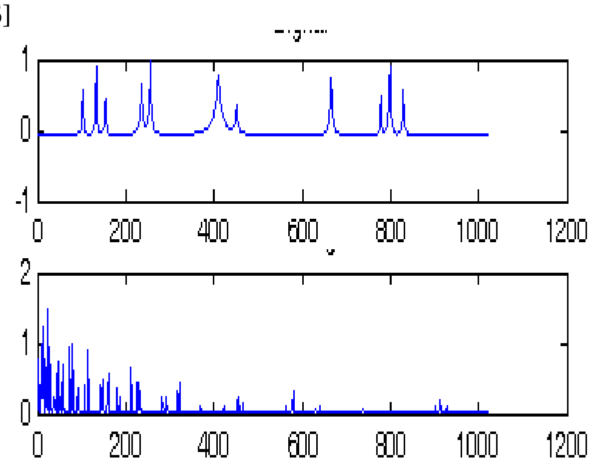


Fig 2. Signal and its wavelet coefficients

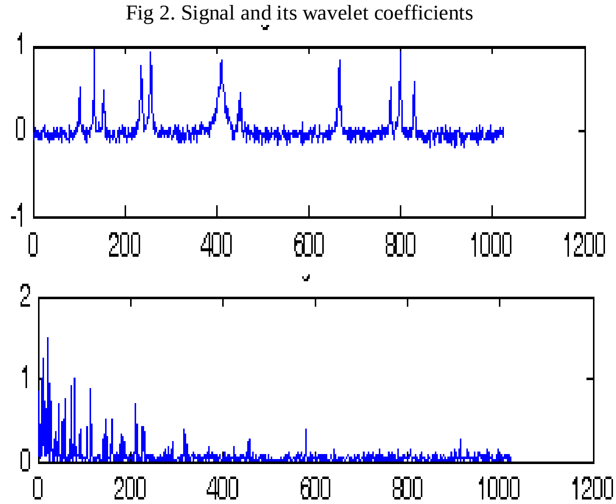


Fig 3. Noisy signal and its wavelet coefficients

Therefore the main idea is to remove these small coefficients responsible for the noise in the signal. The denoising of the noisy signal using wavelet transform is obtained in three basic steps as depicted in the following block diagram:

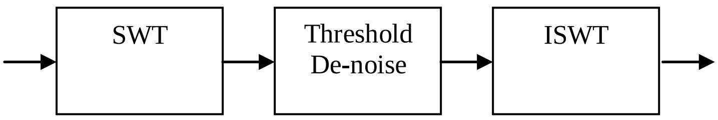


Fig 4. Block diagram of signal denoising

C. Signal Denoising Algorithm

- Decomposition: This step involves choosing a mother wavelet and a maximum decomposition level and then computing the decomposition coefficients at each level.

- Thresholding: This step involves computing threshold values for each level and applying threshold to the coefficients at each level.
- Reconstruction: In this step, the reconstruction of the signal with the modified coefficients through inverse wavelet transform is carried out.

D. Soft And Hard Thresholding

If λ denotes the given threshold, the soft threshold can be defined by [5]

$$\hat{w} = \begin{cases} sign(w).(|w| - \lambda), & |w| \geq \lambda \\ 0, & otherwise \end{cases} \tag{11}$$

and hard thresholding can be written as:

$$\hat{w} = \begin{cases} w, & |w| \geq \lambda \\ 0, & \text{otherwise} \end{cases} \tag{12}$$

Hard thresholding can be described as the process of setting to zero the elements whose absolute values are lower than the threshold. Soft thresholding is called shrink or kill which is an extension of hard thresholding. It is based on first setting the elements with the absolute values lower than the threshold to zero, and then shrinking the other coefficients. The drawbacks of the hard and soft thresholding are that the hard threshold is not continuous at threshold whereas the soft threshold is not differentiable at this value; a pre-requisite for any optimization problem.

E. Threshold Selection

Donoho and Johnston developed a universal thresholding rule [5] which can effectively remove the Gaussian random noise. The universal thresholding rule is as follows:

$$\boldsymbol{\lambda} = \sigma\sqrt{2logN} \tag{13}$$

where N is the length of the coefficients, σ is the standard deviation of noise with $\sigma = \frac{MAD}{0.6745}$, MAD represents median absolute deviation of the coefficients, and λ denotes the threshold.

The expression given in (13) considers the value of variance of noise, σ, as constant for different time interval. But the noise variance can vary with time resulting in several different variance values of noise variance in different time intervals. Therefore, in this paper, the variance adaptive threshold selection for wavelet coefficients, which takes care of varying noise variance into account, is used.

IV. RESULTS

In order to illustrate performance of the proposed threshold selection, a signal x= sin(t) is taken and the signal is corrupted by noise at different level(viz. 5db, 10db and 15db) of signal to noise ration(SNR). These noisy signals are denoised using SWT and DWT with the help of fixed form of threshold selection using universal thresholds selection given in equ.(13) and interval dependent threshold selection rule. The denoising process is implemented in MATLAB. Mean Square Error (MSE) [6] is used as performance measure for denoising. It is given by

$$MSE = \frac{1}{N}(\hat{x}(n) - x(n))^2 \tag{14}$$

Where $x(n)$ is original signal and $\hat{x}(n)$ is estimated signal after de-noising. Db3 (Daubechies wavelets) is chosen for wavelet decomposition and reconstruction. Fig 6 depict the noisy signal and Figure 7 and 8 depicts the signal denoised by universal threshold and modified threshold using SWT whereas fig. 9 and 10 depicts the signal denoised by the same using DWT respectively. Soft thresholding is used for denoising in both universal and interval dependent threshold selection. After de-noising the Mean square error (MSE) has been calculated using (14) and the result is tabulated in Table I. Bar graph of the different signals vs. MSE is plotted in Fig. 11(for DWT) and 12(for SWT).

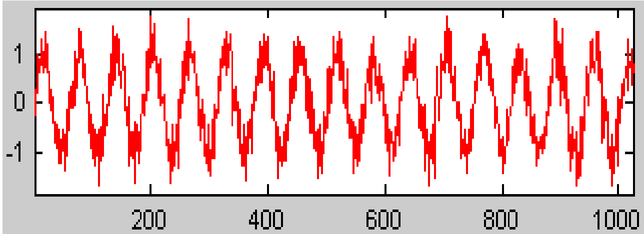


Fig 6. noisy signal (at SNR 10db)

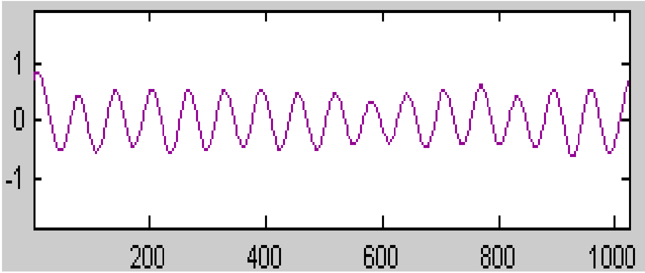


Fig 7. denoised signal by universal threshold(SWT case)

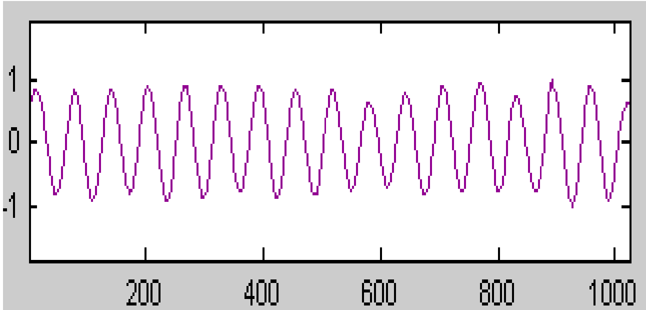


Fig 8. denoised signal by interval dependent threshold(SWT case)

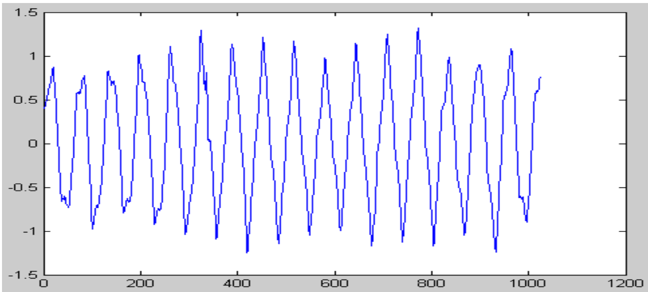


Fig 9. Denoised signal(at SNR 10db) by universal threshold(DWT case)

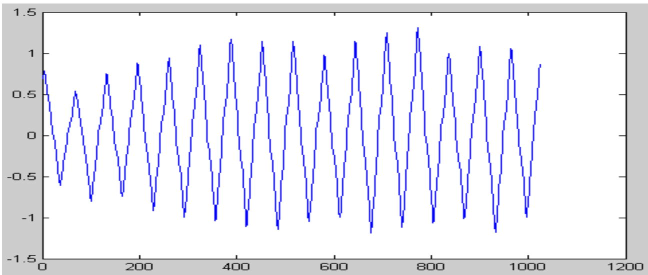


Fig 10. denoised signal(at SNR 10db) by interval dependent threshold(DWT case)

From Table I it can be clearly seen that the values of MSE for the de-noising technique using the interval dependent threshold selection rule is less when compared to the universal threshold selection rule for both DWT and SWT case due to noise variance adaptive property for threshold selection of the interval dependent threshold . The table I also conveys the fact that SWT performs better than DWT for signal denoising. The improvement in the result is due to the shift invariance property of SWT over DWT.

Table I : Performance Comparison of Universal and interval dependent Threshold Selection using DWT and SWT

S. No.	MSE(DWT)		MSE(SWT)	
	Universal threshold	Interval dependent threshold	Universal threshold	Interval dependent threshold
Signal (5db)	0.3214	0.0816	0.1533	0.0624
Signal (10db)	0.1891	0.1093	0.1451	0.0241
Signal (15db)	0.1501	0.0935	0.1374	0.0086

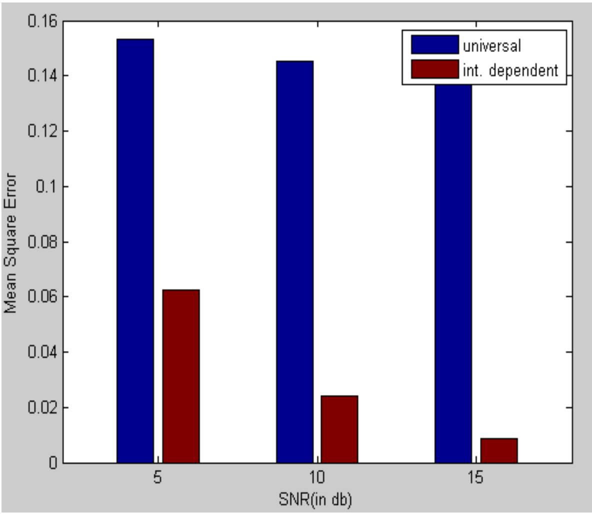


Fig 11. plot of MSE VS different noisy signals for SWT

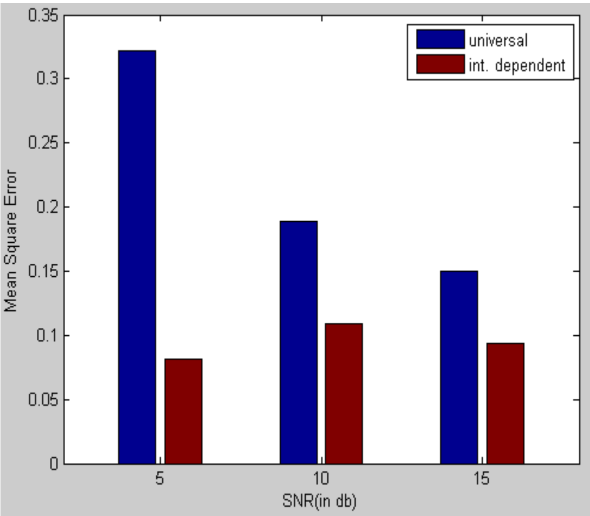


Fig 12. plot of MSE VS different noisy signals for DWT

V. CONCLUSION

The work presented in this paper put forward a new signal denoising algorithm combining the interval dependent threshold selection rule and Stationary Wavelet Transform and the result is compared with the denoising performance of interval dependent threshold with Discrete Wavelet Transform (DWT). It is found that the MSE obtained for the SWT case with interval dependent threshold is significantly lower than the DWT case with interval dependent threshold. Moreover the use of stationary wavelet transform instead of discrete wavelet transforms also increase the denoising performance due to shift invariance property of SWT. Thus, it can be concluded that the proposed denoising algorithm performs much better than traditional fix threshold selection rule.

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