# Performance Evaluation of Heart from its Mathematical Model using Matlab Simulink

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***Abstract- Mathematical modeling of the biological systems plays a vital role in understanding of their activities and abnormalities associated with them. Various approaches are there in existence for the mathematical representation of the systems. This paper discusses mathematical modeling of the heart based on the activity of the heart by carotid baroreflex control mechanism as a stimulant. With the mathematical modeling, understanding the complex behavior of the system has become simple. The model shows the normal pressure variations of the heart's chambers within the standard ranges. The model can then further be used for various clinical and research applications.***

***Index Terms— Heart; Mathematical modeling; Simulation.***

## I. INTRODUCTION

Modeling of a cardiovascular system has been a part of study since many years as it plays an important role in medicine. Different perspectives have been developed towards this research field till now. According to that, methods of modeling involve statistical methods, ARMA modeling, neuro fuzzy inference, etc. But all these methods are found less effective as they involved number of assumptions. Later on, M.S.R. Shoaib and M. A. Haque showed a new mathematical model derived by considering the fluid dynamics which involved tedious derivation of differential equations. Cardiovascular system involves various subsystems such as heart and circulatory system which interact with each other in a complex way to perform different activities[3,4]. A developed model can then be used to describe how the system functions. Mathematical modeling also shows relationship between various parameters of the system. Such mathematical models have significant applications such as scientific understanding of the complex biological systems, testing the effect of changes in the system, etc.

The model discussed here is based on the regulation of heart by carotid baroreceptors. The function of baroreceptors is to control short term pressure changes[2]. They are sensitive to stretching and can show sudden increase or decrease in pressure. The studies of baroreflex provide a complete description of the different components in both dynamic and static conditions. Mathematical modeling based on non-linear theory helps to understand the cardiovascular system. Several such models of baroreflex have been proposed in past years. Ursino presented a model of short term carotid baroregulation that emphasizes the role of active changes in venous capacity in maintaining cardiac output during volume perturbations. Here, model development using baroreceptor mechanism is based on the variation in elastance of heart chambers during their activity, the afferent carotid baroreceptor pathway, and sympathetic and vegal efferent activities.

Manuscript Received on November, 2012

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## II. MODEL DESCRIPTION

The following aspects are considered while designing the mathematical model of the heart. The heart is shown as a pulsatile pump. Elastance variable represents the activity of the heart's chambers. The control of heart rate is dependent on the sympathetic and vegal activity.

### *A. Vascular Compartments of Model:*

The vascular system includes the compartments of the heart such as two atriums and two ventricles. Left atrium and left ventricle is indicated by the subscript *la* and *lv*. Similarly, *ra* and *rv* represent right atrium and right ventricle. The resistance *R* offered by each compartment accounts for the pressure energy losses in a particular compartment. The amount of stressed blood volume stored at a given pressure is indicated by compliance *C* whereas, the unstressed volume inside the heart chambers is represented by $V_u$ .

Equations showing the relationship between pressure and flow at all the points in the vascular system have been stated by considering preservation of mass[1]. The total amount of blood contained in the system is taken as 5,300 ml.

### *B. Modeling Pumping activity of the heart:*

The right and left heart function exactly in synchronization with each other. Therefore, the models for right and left heart are similarly developed with different values of parameters associated with them. The upper chamber of the heart i.e atria is considered as the chamber with linear capacity. It is modeled with the constant values of compliance and unstressed volume. When the pressure inside the atria exceeds the ventricular pressure, atrioventricular valve gets opened and blood runs from atria into ventricles. This passage of blood from atrium to the ventricle is shown as the series arrangement of an ideal unidirectional valve in series with the constant resistance. The time varying elastance is used to model contraction and expansion of the ventricles. Elastance is dependent on the isometric pressure-volume function and the resistance. The resistance reflects the viscosity of the chamber. Variation in the elastance during the cardiac cycle has been used to show the contractile activity of the ventricle. Diastolic activity of the heart's chambers i.e when the muscle fibers are relaxed and the blood is filling up the ventricle, is indicated by exponential pressure volume function. The heart period is denoted by *T* and it varies as a result of baroreflex control action.

### *C. Baroreflex Control System:*

The operation of carotid baroreflex is considered by taking into account the afferent pathway, the efferent sympathetic and parasympathetic pathways, and the action of several effectors. These parameters show the response of heart period to both sympathetic and vegal activities.

Afferent Pathway:

The carotid sinus baroreceptors are sensitive to


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the intrasinus pressure value and also its rate of change. This property is used for the representation of carotid baroreceptor and afferent pathway as a first order linear differential equation.

Efferent Sympathetic and Parasympathetic Pathways:

The autonomic nervous system innervated the heart and vessels through an integrated mechanism which is made up of parasympathetic and sympathetic nervous systems. This interconnected mechanism maintains the stable cardiovascular tone through multiple instantaneous simulation-inhibition reactions. These reactions regulate blood pressure level, heart rate, venous return, contractile status under varying conditions during daily activity such as rest, exercise, stress, etc.

The baroreceptor reflex includes the baroreceptor corpuscles, cranial nerves, the bulber vasomotor center, the sympathetic and parasympathetic efferent pathways, the blood vessels and their receptors, somatic and autonomic afferent pathways, and heart including its excitoconduction system and muscle fibers.

The baroreceptor corpuscles are not pressure receptors but the small bodies which are sensitive to stretching and to changes in vascular diameter resulting from the changes in intravascular pressure. They are situated inside the walls of blood vessels and in the heart. But, are concentrated more heavily in the carotid sinus and the aortic arch. They are also found in right and left atria, in the walls of superior and inferior vena cavae at their junction with the heart, in the pulmonary veins and throughout the pulmonary circulation. The cranial nerves constitute the afferent pathway for the baroreceptor corpuscles and are responsible for conducting the information about the intravascular pressure levels which is carried by these bodies, with each heart beat to the bulber vasomotor center.

## III. PARAMETER VALUES

The parameters describing the right and left heart are as specified in the table I.

Table I: Values for pumping chambers

| | Left Heart | Right Heart |
|---|---|---|
| 1. | $C_{la}$ = 19.23 ml/mmHg | $C_{ra}$ = 31.25 ml/mmHg |
| 2. | $V_{ula}$ = 25 ml | $V_{ura}$ = 25 ml |
| 3. | $R_{la} = 2.5 . 10^{-3}$ mm.Hg.s.ml$^{-1}$ | $R_{ra} = 2.5 . 10^{-3}$ mm.Hg.s.ml$^{-1}$ |
| 4. | $P_{0lv}$ = 1.5 mmHg | $P_{0rv}$ = 1.5 mmHg |
| 5. | $K_{Elv}$ = 0.014 ml$^{-1}$ | $K_{Erv}$ = 0.011 ml$^{-1}$ |
| 6. | $V_{ulv}$ = 16.77 ml | $V_{urv}$ = 40.8 ml |
| 7. | $E_{maxlv}$ = 2.95 mmHg/ml | $E_{maxrv}$ = 1.75 mmHg/ml |
| 8. | $K_{Rlv} = 3.75.10^{-4}$ s/ml | $K_{Rrv} = 1.4.10^{-3}$ s/ml |

The parameters $K_E$ and $P_0$ describe the end-diastolic pressure-volume function of the ventricle. $E_{max}$ represents slope of end diastolic relationship and $K_R$ represents dependence of ventricle resistance on isometric pressure. *C* and *R* represent compliance and resistance of the chamber.

The vascular compartments of the heart i.e. left atrium, left ventricle, right atrium and right ventricle possess the values of compliance *C*, inertance *L*, resistance *R* as shown by the table II.

Table II. Parameter values for vascular system

| | Compliance (ml/mmHg) | Unstressed volume (ml) | Hydraulic Resistance (mmHg.s.ml$^{-1}$) | Inertance (mmHg.ml.s$^{-2}$) |
|---|---|---|---|---|
| 1 | $C_{sa}$=0.28 | $V_{usa}$=0 | $R_{sa}$=0.06 | $L_{sa}=0.22.10^{-3}$ |
| 2 | $C_{pa}$=0.76 | $V_{upa}$=0 | $R_{pa}$=0.023 | $L_{pa}=0.18.10^{-3}$ |

where, *sa* represents systemic aorta and *pa* represents pulmonary aorta

Parameters of the regulatory mechanism are shown in the table III.

Table III. Values of parameters for regulatory mechanism

| Parameter | Value |
|---|---|
| 1. For Carotid Sinus Afferent Pathway | |
| $P_n$ | 92 mmHg |
| $f_{min}$ | 2.52 spikes/sec |
| $f_{max}$ | 47.78 spikes/sec |
| $K_a$ | 11.758 mmHg |
| $\tau_z$ | 6.37 sec |
| $\tau_p$ | 2.076 sec |
| 2. For Sympathetic efferent Pathway | |
| $f_{es\infty}$ | 2.10 spikes/sec |
| $f_{es0}$ | 16.11 spikes/sec |
| $f_{esmin}$ | 2.66 spikes/sec |
| $K_{es}$ | 0.0675 sec |
| 3. Vagal Efferent Pathway | |
| $f_{ev0}$ | 3.2 spikes/sec |
| $f_{ev\infty}$ | 6.3 spikes/sec |
| $f_{cs0}$ | 25 spikes/sec |
| $K_{ev}$ | 7.06 spikes/sec |
| 4. Effectors | |
| $G_{Emaxlv}$= 0.475 mmHg/ml.v | $G_{Emaxrv}$ = 0.282 mmHg/ml.v |
| $\tau_{Emaxlv}$ = 8sec | $\tau_{Emaxrv}$= 8sec |
| $D_{Emaxlv}$ = 2sec | $D_{Emaxrv}$ = 2sec |
| $E_{maxlv0}$ = 2.392 mmHg/ml | $E_{maxrv0}$ = 1.412 mmHg/ml |

The parameter *G* in table III represents mechanism strength, *f* represents frequency, $\tau$ time constant and *D* represents time delay and *v* represents neural efferent rate in spikes/sec. Subscript 0 indicates parameter in absence of innervation i.e. when vegal and sympathetic activities are zero. $P_n$ is the intrasinus pressure.

## IV. RESULTS

The performance of mathematical model of the heart's chambers is evaluated by integrating the differential equations by using fourth order Runge-Kutta method. Simulation of the model was executed with the MATLAB SIMULINK. During the simulation, step size was set at 0.1 sec. Also it was assumed that, during the simulation, initially, pressure and volume parameters of the system start rising from zero.

### *A. Left Heart:*

Fig.1 shows the variation of pressure during the cardiac cycle at a point in the systemic aorta for a short time duration. During the ejection period, i.e. during contraction of left ventricle, pressure in the aorta starts gradually increasing and reaches to its maximum value. During diastole, it gradually decreases and reaches to its minimum value.

Fig.2 shows the intraatrial pressure. Gradual increase in the atrial pressure can be observed due to filling of blood in atria. Normal values of pressure in the left atrium range from 0-2 mmHg upto 7-8 mmHg.

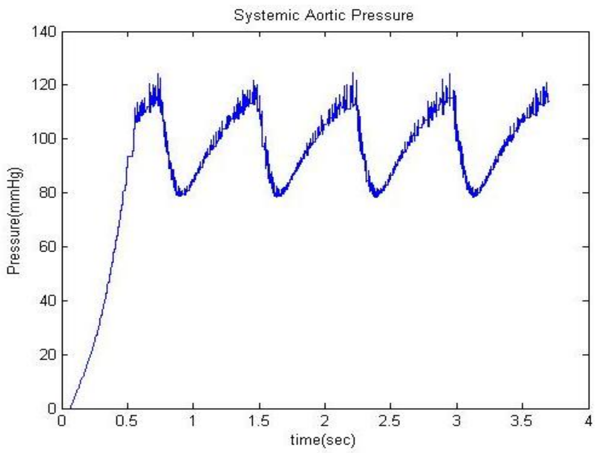


Fig.1: Pressure variations at Systemic Aorta

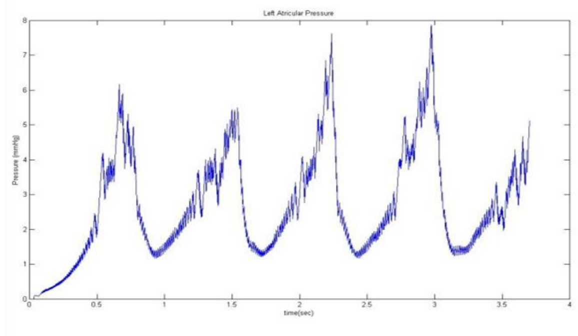

Fig.2: Left Atrial Pressure

### *B. Right Heart:*

On the similar lines, fig.3 shows the pressure variations at a point in pulmonary aorta. Both left and right ventricles are observed working in the similar fashion with systolic and diastolic values of pressure.

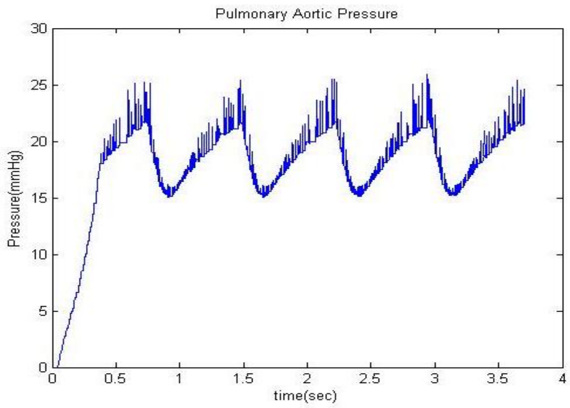


Fig.3 : Pressure variations at pulmonary aorta

As per the normal range of values, mathematical model of the right atrium indicates the value of atrial pressure as shown by the fig.4. The standard maximum value of which is 5-6 mmHg and minimum is 0-2 mmHg.

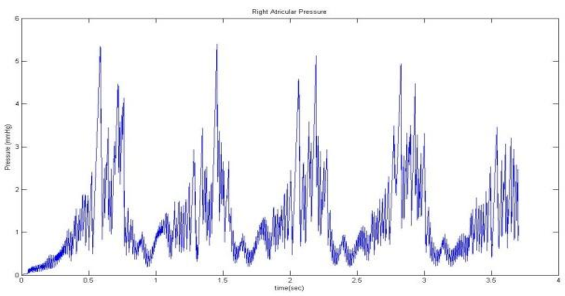

Fig.4 : Right atrial pressure

## V. CONCLUSION

The design of computational model of human organs is a new research field. It has opened various possibilities for medical analysis and simulation of various therapies.

Although some assumptions have been taken into account, the model shows periodicity during the functioning of heart chambers. The model can represent each of the heart chamber individually by changing the parameters associated with it.

The possible improvements in the mathematical model of the system would include representation of different types of abnormalities associated with heart such as blockage, leakage, etc. in which the blood flow rate and pressure gradient changes. Also, more realistic model can be developed by including more detailed anatomical structure such as valve parameters, size of the heart chambers, etc.

## VI. APPENDIX : MODEL EQUATIONS

Pressure and volume at different points in the system are stated by using conservation of mass[1]. *P* is the intravascular pressure. *Vu* is the corresponding unstressed volume, i.e. volume at zero pressure. *F* is the blood flow. *C*, *L* and *R* are compliances, inertances, and hydraulic resistances respectively. $F_{ol}$ and $F_{or}$ are cardiac output from left and right ventricle respectively.

### *A. Pumping Chambers:*

Left Heart Blood flow entering the left ventricle depends on the opening of atrioventricular valve. Hence,

$$F_{il} = 0 \qquad \text{if } P_{la} \le P_{lv}$$

$$= \left( \frac{P_{la} - P_{lv}}{R_{la}} \right) \qquad \text{if } P_{la} > P_{lv} \tag{1}$$

Therefore, volume of the left ventricle $V_{lv}$ is calculated as,

$$\frac{dV_{lv}}{dt} = F_{il} - F_{ol} \tag{2}$$

Where, $F_{ol}$ is the cardiac output from the left ventricle. Cardiac output depends on the opening of aortic valve which is ideally expressed as,

$$F_{ol} = 0 \qquad \text{if } P_{maxlv} \le P_{la}$$

$$= \left( \frac{P_{\max lv} - P_{sa}}{R_{lv}} \right) \qquad \text{if } P_{maxlv} > P_{sa} \tag{3}$$

where, $P_{maxlv}$ is the isometric left ventricular pressure. The viscous resistance of the left ventricle $R_{lv}$ is assumed to be proportional to the isometric pressure. Therefore, equation for $R_{lv}$ forms as,

$$R_{lv} = K_{Rlv} * P_{\max lv} \tag{4}$$

Where, $K_{Rlv}$ is a constant.

The instantaneous value of the pressure in the left ventricle is calculated by taking the difference between isometric pressure and viscous losses.

$$\therefore P_{lv} = P_{\max lv} - R_{lv} . f_{ol} \tag{5}$$

It is assumed that the isometric pressure volume function is exponential at diastole i.e. when the ventricle is relaxed. and linear at end systole i.e. when the ventricle is maximally contracted. The isometric pressure is calculated as,


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$$P_{\max lv}(t)=\varphi(t).E_{\max lv}.(V_{lv}-V_{ulv})+\left[1-\varphi(t)\right].P_{olv}.(e^{K_{Elv}.V_{lv}}-1) \quad \text{for } 0\le\varphi(t)\le 1 \tag{6}$$

Where, $E_{\max lv}$ is the elastance of ventricle at the instant of maximum contraction. φ(t) is the ventricle activation function, the value of which is 1 at maximum contraction and 0 at complete relaxation. It is calculated by the expression,

$$\varphi(t)=\sin^2\left[\frac{\pi.T(t)}{T_{sys}(t)}.u\right] \qquad 0\le u\le\frac{T_{sys}}{T}$$
$$=0 \qquad \frac{T_{sys}}{T}\le u\le 1 \tag{7}$$

where, $T$ is the heart period, $T_{sys}$ is the duration of systole and u is dimensionless quantity ranging from 0 to 1. It represents the fraction of cardiac cycle.

Right Heart

Blood flow entering the right ventricle,

$$F_{ir}=0 \qquad \text{if } P_{ra}\le P_{rv}$$
$$=\frac{P_{ra}-P_{rv}}{R_{ra}} \qquad \text{if } P_{ra}>P_{rv} \tag{8}$$

Volume of right ventricle

$$\frac{dV_{rv}}{dt}=F_{ir}-F_{or} \tag{9}$$

Cardiac output from the right ventricle

$$F_{or}=0 \qquad \text{if } P_{\max rv}\le P_{pa}$$
$$=\frac{P_{\max rv}-P_{pa}}{R_{rv}} \qquad \text{if } P_{\max rv}>P_{pa} \tag{10}$$

Viscous resistance of right ventricle

$$R_{rv}=K_{Rrv}.P_{\max rv} \tag{11}$$

Instantaneous right ventricle pressure,

$$P_{rv}=P_{\max rv}-R_{rv}.F_{or} \tag{12}$$

Isometric pressure in the right ventricle

$$P_{\max rv}(t)=\varphi(t).E_{\max rv}.(V_{rv}-V_{urv})+\left[1-\varphi(t)\right].P_{0rv}.(e^{K_{Erv}.Vrv}-1) \quad \text{for } 0\le\varphi(t)\le 1 \tag{13}$$

***B. Carotid baroreflex control system:***

Afferent pathway:

$$f_{cs}=\left[f_{\min}+f_{\max}.\exp\left(\frac{P-P_n}{K_a}\right)\right]\Big/\left[1+\exp\left(\frac{P-P_n}{K_a}\right)\right] \tag{14}$$

$P_{cs}$ is a carotide sinus pressure. $f_{cs}$ is the frequency spikes in the afferent fibers. $f_{max}$ and $f_{min}$ are the upper and lower saturation frequencies of discharge. $P_n$ represents the intrasinus pressure.

Efferent sympathetic Pathway :

The relation between the activity in the afferent and efferent neural pathways is expressed as,

$$f_{es}=f_{es,\infty}+\left(f_{es,0}-f_{es,\infty}\right)e^{-K_{es}\cdot f_{cs}} \tag{15}$$

where, $f_{es}$ is the frequency of spikes in the efferent sympathetic nerves.

Efferent vegal pathway:

The efferent vegal activity is related with the activity in the sinus nerve and is expressed as,

$$f_{ev}=\frac{\left[f_{ev,0}+f_{ev,\infty}.\exp\left(\frac{f_{cs}-f_{cs,0}}{K_{ev}}\right)\right]}{\left[1+\exp\left(\frac{f_{cs}-f_{cs,0}}{K_{ev}}\right)\right]} \tag{16}$$

where, $f_{ev}$ is the frequency of spikes in efferent vegal fibers.

## REFERENCES


1. Mauro Ursino, “Interaction between carotid baroregulation and the pulsating heart: a mathematical model”, American Journal of Physiology, H1733-H1747, 1998.
2. Mette S. Olufsen, Johnny T. Ottesen, Hien T. Tran, Laura M. Ellwein, Lewis A. Lipsitz, Vera Novak; Blood pressure and blood flow variation from sitting to standing: Model development and validation, Journal of applied physiology, 1523-1537, 2005.
3. Gerard J. Tortora; Principles of Anatomy and Physiology, 12th edition, 2009.
4. K.Sembulingam ; Essentials of Medical Physiology, Second Edition, 2000.
5. B.D.Chaurasia; Human Anatomy, Volume one, Third edition, 2000.
6. Dyuti Trivedi, Simulation of a complete cardiovascular loop: Development of a simulink based pressure flow model to obtain the origin of electrical impedance cardiogram, Thesis of Master of Science, May 2009.