Free Convection MHD Flow Past a Vertical Plate with Constant Suction

Dinesh Verma, Monika Kalra

ABSTRACT-The special effects of fluctuating gravitational field on free convection MHD flow past a homogeneously moving infinite erect porous plate with constant suction velocity in a porous medium have been analyzed. A constant heat flux is prearranged on the plate. The gravitational field is implicit in the form $g = g_0 + g_1 \cos \omega^* t^*$. The governing equations are solved by perturbation method. Fluid velocity and fluid temperature shows remarkable alter with alteration in gravitational field. Small increase in gravity modulation parameter shows considerable increase in amplitude of skin friction and has insignificant decreasing effect on phase of skin friction.

Keywords: MHD, porous media, free convection, suction, unsteady, skin friction, Gravity modulation.

I. INTRODUCTION

The study of porous media is extensively used in high temperature, heat exchangers, turbine blades and jet nozzles. Porous media are constructive in retreating the natural free convection which would be concentrated on a vertical heated surface. To create heat insulation of surface extra effective, it is necessary to learn the free convection flow through a porous medium. Unsteady oscillatory free convection flows participate an important part in chemical engineering, turbo machinery and aerospace technology. Rajvanshi and Baljender have studied the free convection MHD flow past a moving vertical porous surface with gravity modulation at constant heat flux [1]. Saini and Sharma have studied heat and mass transfer in MHD flow past a vertical plate embedded in porous medium [2]. The free convection effect on the flow of an ordinary viscous fluid past an infinite, vertical porous plate with constant suction and constant heat flux was investigated [3]. A finite element study of double-diffusive convection driven by g-jitter in a microgravity environment is studied by Shu et. al., [4]. The two-dimensional, time-dependent Navier-Stokes equations are numerically included by a time-split method using direct matrix solvers. Numerical analysis of thermosolutal flows in a cavity with gravity modulation effects have been studied by Jue and Ramaswamy [5]. Sharidan and Pop, have studied g-jitter fully developed combined heat and mass transfer by mixed convection flow in a vertical channel [6]. The effect of gravity modulation and magneto hydrodynamics on a free convection flow is useful in space technology. It needs special attention on forces involving vibrations that occur due to interaction of several phenomena.

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In space vehicle, there are transient perturbations to the gravity field at a point. An excellent account of this physical feature has been described by Li [7]. Magnetic field effects on the free convection and mass transfer flow through a porous medium with periodic suction and constant heat flux [8]. Soundlgekar and Patil ,have studied Stokes problem for infinite vertical plate with constant heat flux. They observed that with increasing time and Grashof number the velocity of the fluid increases[9]. . A computational study for the investigation of gravity modulation effects in thermally driven cavity flows at terrestrial and microgravity environments have been analyzed by Biringen [10]. The effect of time-periodic temperature gravity modulation at the inception of magneto-convection in weak electrically conducting fluids with internal angular momentum is investigated by Siddheshwar and Pranesh [11].

The aim of the paper is to investigate the effect of gravity modulation with different amplitudes and frequencies on free convection of a viscous fluid past a uniformly moving infinite vertical porous plate with constant heat flux and constant suction velocity. Governing equations have been solved with regular perturbation method. The variation in gravity modulation and magnetic parameter make significant change in skin-friction.

II. MATHEMATICAL ANALYSIS

We deem MHD incompressible viscous fluid past a homogeneously moving infinite vertical porous plate. Plate is surrounded by a porous medium of time depended permeability and constant suction velocity. The x^* - axis is in use along the plate and y^* - axis is in use normal to the plate. A uniform magnetic field normal to the direction of flow is introduced. The magnetic Reynolds number is in use to be very small so that the induced magnetic field can be neglected in comparison to the applied magnetic field. The temperature difference between the wall and the medium develops buoyancy force which induces the basic flow. Initially the plate as well as fluid is assumed to be at the same temperature.

Since the plate is considered infinite in the x^* -direction. And all physical quantities are independent of x^* and are function of y^* and t^* only. Under these assumptions flow is governed by the following set of equations

$$\frac{\partial V^*}{\partial y^*} = 0 \tag{1}$$

$$\frac{\partial u^*}{\partial t^*} + V^* \frac{\partial u^*}{\partial y^*} = g\beta\left(T^* - T_\infty^*\right) + \nu \frac{\partial^2 u^*}{\partial y^{*2}} - \nu \frac{u^*}{K^*} - \frac{\sigma}{\rho^*} B_0^2 u^* \tag{2}$$

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$$\frac{\partial T^*}{\partial t^*}+V^*\frac{\partial T^*}{\partial y^*}=\frac{\kappa}{\rho^* C_p}\left(\frac{\partial^2 T^*}{\partial y^{*2}}\right)+\frac{\nu}{C_p}\left(\frac{\partial u^*}{\partial y^*}\right)^2 \qquad (3)$$

Because the suction velocity is assumed to be constant so it can be imposed in the form

$$V^* = -V_0^* \qquad (4)$$

The relevant boundary conditions are

at $y^*=0$, $u^*=U^*$, $V^*=-V_0^*$, $\frac{\partial T^*}{\partial y^*}=-\frac{q}{\lambda}$ (5a)

as $y^*\to\infty$, $u^*\to 0$, $T^*\to T_\infty^*$ (5b)

The ($*$) stands for dimensional quantities. The subscript (∞) denote the free stream condition.

The permeability of the porous medium is taken in the form

$$K^* = K_0\left(1+\varepsilon e^{i\omega^* t^*}\right) \qquad (6)$$

The time dependent gravitational acceleration is assumed in the form $g=g_0+g_1\cos\omega^* t^*$, where g_0 the constant gravity level in the environment is, g_1 is the amplitude of the oscillating component of acceleration and ω^* is the frequency of gravitational oscillation.The gravitational accelerations is rewritten in the form

$$g=g_0+g_1 e^{i\omega^* t^*} \qquad (7)$$

Where, $g_1=\varepsilon\alpha g_0$

The following non dimensional quantities are introduced

$$y=\frac{y^* V_0^*}{\nu},\ t=\frac{t^* V_0^{*2}}{4\nu},\ \omega=\frac{4\nu\omega^*}{V_0^{*2}},\ u=\frac{u^*}{V_0^*},\ U=\frac{U^*}{V_0^*},$$

$$T=\frac{\left(T^*-T_\infty^*\right)V_0^*\lambda}{q\nu},\ G_r=\frac{\nu^2\beta g_0 q}{V_0^{*4}\lambda}$$

$$P_r=\frac{\nu}{\kappa/\rho C_p},\ M=\frac{B_0}{V_0^*}\sqrt{\sigma\nu/\rho^*},\ E_c=\frac{V_0^{*3}\lambda}{C_p q\nu},$$

$$K=\frac{V_0^{*2}K^*}{\nu^2} \qquad (8)$$

Equations (1) through (8) take the following non-dimensional form

$$\frac{\partial V}{\partial y}=0 \qquad (9)$$

$$\frac{1}{4}\frac{\partial u}{\partial t}-\frac{\partial u}{\partial y}=\frac{\partial^2 u}{\partial y^2}+\left(1+\varepsilon\alpha e^{i\omega t}\right)\left(\theta G_r\right)-M^2 u-\frac{u}{K_0\left(1+\varepsilon e^{i\omega t}\right)} \qquad (10)$$

$$\frac{1}{4}\frac{\partial T}{\partial t}-\frac{\partial T}{\partial y}=\frac{1}{P_r}\frac{\partial^2 T}{\partial y^2}+E_c\left(\frac{\partial u}{\partial y}\right)^2 \qquad (11)$$

The boundary conditions in the dimensionless form are

$$u=U,\ \frac{\partial T}{\partial y}=-1 \quad at \quad y=0$$
$$u\to 0,\theta\to 0, \quad as\, y\to 0 \qquad (12)$$

III. SOLUTION OF THE GOVERNING EQUATIONS

When the amplitude of oscillations $\left(0\prec\varepsilon\ll 1\right)$ is very small, we can assume the solutions of flow velocity u and the temperature field T in the neighborhood of the plates as

$$u(y,t)=u_0(y)+\varepsilon e^{i\omega t}u_1(y)$$
$$T(y,t)=T_0(y)+\varepsilon e^{i\omega t}T_1(y) \qquad (13)$$

Where u_0 and T_0 are respectively the mean velocity and mean temperature.

On using (13) into equations (10) and (11), equating harmonic and non-harmonic terms and neglecting ε, the following set of equations are obtained:

$$u_0''+u_0'-\left(M^2+\frac{1}{K_0}\right)u_0=-G_r T_0 \qquad (14)$$

$$T_0''+P_r T_0'=-E_c u_0'^2 \qquad (15)$$

$$u_1''+u_1'-\left(M^2+\frac{1}{K_0}+\frac{i\omega}{4}\right)u_1=-u_0''-\left(T_0+T_1\right)G_r-\alpha G_r T_0+M^2 u_0 \qquad (16)$$

$$T_1''-\frac{i\omega}{4}P_r T_1=-P_r T_1'-2E_c u_0' u_1' \qquad (17)$$

Where primes denote differentiation with respect to y. The modified boundary conditions are:

$$u_0=U, u_1=0, T_0=-1, T_1=0 \quad at \quad y=0$$
$$u_0\to 0, u_1\to 0, T_0\to 0, T_1\to 0\, as\, y\to\infty \qquad (18)$$

The equation (14) to (17) are still coupled for the variables u_0, u_1, T_0 and T_1 . To Solve them, it is to be noted that $E_c\ll 1$ for all incompressible fluid and assumed that:

$$F(y)=F_0(y)+E_c F_1(y)+o\left(E_c^{\,2}\right) \qquad (19)$$

Where F stands for u_0, u_1, T_0 and T_1.

On using (19) into equation (14) to (17) and equating the like powers of E_c , the following equations are obtained:

$$u_{00}''+u_{00}'-\left(M^2+\frac{1}{K_0}\right)u_{00}=-G_r T_{00} \qquad (20)$$

$$u_{01}''+u_{01}'-\left(M^2+\frac{1}{K_0}\right)u_{01}=-G_r T_{01} \qquad (21)$$

$$u_{10}''+u_{10}'-\left(M^2+\frac{1}{K_0}+\frac{i\omega}{4}\right)u_{10}=-u_{00}''-\left(T_{00}+T_{10}\right)G_r-\alpha G_r T_{00}+M^2 u_{00} \qquad (22)$$

$$u_{11}''+u_{11}'-\left(M^2+\frac{1}{K_0}+\frac{i\omega}{4}\right)u_{11}=-u_{01}''-\left(T_{00}+T_{11}\right)G_r-\alpha G_r T_{01}+M^2 u_{01} \qquad (23)$$

$$T_{00}''+P_r T_{00}'=0 \qquad (24)$$

$$T_{01}''+P_r T_{01}'=-u_{00}'^2 \qquad (25)$$

$$T_{10}''-\frac{i\omega}{4}P_r T_{10}=-P_r T_{10}' \qquad (26)$$

$$T_{11}''-\frac{i\omega}{4}P_r T_{11}=-P_r T_{11}'-2P_r u_{00}' u_{10}' \qquad (27)$$

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Subject to the boundary conditions;

$$u_{00} = U, u_{01} = u_{10} = u_{11} = 0$$
$$T_{00} = -1, T_{01} = T_{10} = T_{11} = 0 \quad at \;\; y = 0 \tag{28}$$

$$u_{00} = u_{01} = u_{10} = u_{11} = 0,$$
$$T_{00} = T_{01} = T_{10} = T_{11} = 0 \quad as \;\; y \to \infty \tag{29}$$

In view of the boundary conditions (28) and (29), the solutions of the boundary condition (20) to (27) are;

$$T_{00} = -e^{-P_r y} \tag{30}$$

$$T_{10} = 0 \tag{31}$$

$$u_{00} = Be^{-m_4 y} + R_1 e^{-P_r y} \tag{32}$$

$$u_{10} = -B_1 e^{-m_6 y} + R_2 e^{-m_4 y} + R_3 e^{-P_r y} \tag{33}$$

$$T_{01} = -B_2 e^{-P_r y} + R_4 e^{-2m_4 y} + R_5 e^{-2P_r y} + R_6 e^{-(m_4+P_r)y} \tag{34}$$

$$T_{11} = -B_3 e^{-m_8 y} + R_7 e^{-(m_4+m_6)y} + R_8 e^{-2m_4 y} + R_9 e^{-(m_4+P_r)y} + R_{10} e^{-(m_6+P_r)y} - R_{11} e^{-2P_r y} \tag{35}$$

$$u_{01} = B_4 e^{-m_{10} y} + R_{12} e^{-P_r y} - R_{13} e^{-2m_4 y} + R_{14} e^{-2P_r y} - R_{15} e^{-(m_4+P_r)y} \tag{36}$$

$$u_{11} = -B_5 e^{-m_{12} y} + R_{16} e^{-m_{10} y} + R_{17} e^{-P_r y} + R_{18} e^{-2m_4 y} + R_{19} e^{-2P_r y} + R_{20} e^{-(m_4+P_r)y} + R_{21} e^{-m_8 y} - R_{22} e^{-(m_4+m_6)y} - R_{23} e^{-(P_r+m_6)y} \tag{37}$$

Where,

$$m_1 = \frac{-P_r + \sqrt{P_r^2 + i\omega P_r}}{2}, \; m_2 = \frac{-(P_r + \sqrt{P_r^2 + i\omega P_r})}{2}, \; m_3 = \frac{-1 + \sqrt{1 + 4(M^2 + \frac{1}{K_0})}}{2}, \; m_4 = \frac{-\left[1 + \sqrt{1 + 4(M^2 + \frac{1}{K_0})}\right]}{2}$$

$$R_1 = G_r \left[P_r^2 - P_r - \left(M^2 + \frac{1}{K_0} \right) \right]^{-1}, \; R_2 = \frac{(R_1 - U)(M_4^{\,2} - M^2)}{m_4^{\,4} - m_4 - A}, \; R_3 = \frac{R_1 (M^2 - P_r^2) + G_r (1 + \alpha)}{P_r^2 - P_r - A}$$

Where,

$$A = M^2 + \frac{1}{K_0} + \frac{i\omega}{4}, \; B = U - R_1, \; B_1 = R_2 + R_3, \; B_2 = R_4 + R_5 + R_6, \; B_3 = R_7 + R_8 + R_9 + R_{10} - R_{11}$$

And the other constants like m_5 *to* m_{10}, B_4 *to* B_5, R_4 *to* R_{23} are not presented here for the sake of brevity.

IV. SKIN FRICTION

The skin- friction in the non dimensional form on the plate y =0 is given by $\tau = \dfrac{\tau^*}{\rho V_0^{*2}} = \left(\dfrac{\partial u}{\partial y}\right)_{y=0}$ (38)

Splitting the equation (38) into real and imaginary parts and taking real parts only; $\tau = \tau_m + \varepsilon |N| \cos(\omega t + \phi)$ (39)

Where, $|N| = \sqrt{N_r^{\,2} + N_i^{\,2}}$, $\phi = \tan^{-1}\left(\dfrac{N_i}{N_r}\right)$,

$N_r = \mathrm{Re}(N)$, $N_i = \mathrm{Im}(N)$, $\tau_m = u_0'(0)$.

V. RESULT AND DISCUSSION

As it is very difficult to explain each and every fluid parameter involved in the problem. So we have tried to discuss a few of them. In the present problem we have discussed the velocity field, temperature field, and skin friction by assigning numerical values to the various parameters.

In the velocity profile we can observe that its component varies more rapidly near the plate and then decreases exponentially far away from the plate. The magnitude of the velocity decreases with increases in Schmidt number S_c and increases with Grashof number G_r .Also the same is the case of temperature profile as it increases with Grashof number and decreases with schmith number.

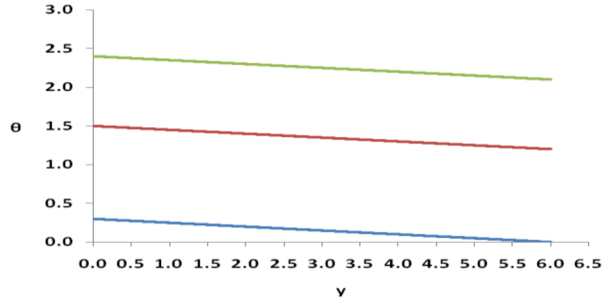

Fig. I. Temperature profile versus y

Fig I shows the comparison of temperature profile for different values of y and for different values to different fluid parameters. From the graph it is clear that the temperature increases with Grashof number and it decreases with Schmith number.

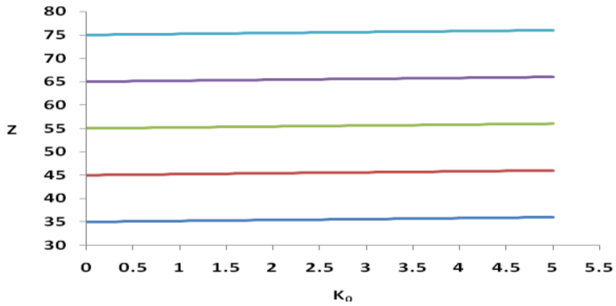

Fig.II. Skin friction co-efficient versus K_0

Fig. II shows the comparison of skin friction Z for different values of K_0 for different values of fluid parameters. From the graph one can observe that skin friction is directly proportional to gravity modulation parameter but it is inversely proportional to the magnetic parameter M, Schmidt number S_c .

VI. NOMENCLATURE

B_0 = Magnetic field intensity
E_c =Eckert number
G_r = Grash of number
K = Dimensionless permeability of porous media
K_0 = permeability constant of porous media
K^* = permeability of porous media
M = Magnetic parameter
p = pressure
p_r = prandtl number
q = heat flux per unit area
T^* = Temperature

T_∞^* = Temperature of fluid in free stream
t = dimensionless time
t^* = time
u = dimensionless velocity of fluid
u^* = velocity of fluid
U = dimensionless velocity of the moving vertical porous plate
U^* = velocity of the moving vertical porous plate
V = Suction velocity
V_0 = constant Suction velocity
β = coefficient of thermal expansion
β^* = coefficient of thermal expansion with concentration
λ = thermal conductivity
ε = a constant
ϕ = phase difference for skin friction
κ = thermal conductivity
μ = viscosity
υ = kinematics viscosity
ρ = density of fluid
σ = electric permeability
θ = dimensionless temperature
ω = dimensionless frequency of gravitational oscillation
ω^* = frequency of gravitational oscillation
τ = skin friction
τ_m = mean skin friction
g = acceleration due to gravity
g_0 = constant gravity level
g_1 = the amplitude of oscillating component of acceleration due to gravity.

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