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Implementing Kalman Filter in GPS Navigation

Kanika Gupta, Apurva, Priya Jindal, Vishakha Snehi

Abstract*: This paper describes about the increase in efficiency of the GPS Navigation System when conventional tracking loops are replaced by the Kalman Filter. The Kalman Filter is a recursive algorithm that helps in reducing the square root of the error in the non-linear and noisy dynamic systems. The approach is also called Digital Filtering, more precisely - Adaptive Filtering. The paper highlights various errors in the GPS Systems and describes how Kalman Filter can effectively reduce them. The various kinds of errors are ionospheric error, tropospheric error, onboard clock error, that is, error in the satellite's clock, receiver clock error, ephemeris data errors, that is, small error in the position of the satellite. We aim at reducing such errors by using the Extended Kalman Filter. The Kalman Gain coefficient is the most important component of the entire algorithm. It will we multiplied with the error residuals iteratively, which will reduce the error value in the final readings eventually. Also, by replacing conventional looping, which provides accurate readings after 3rd or 4th iteration, with the Kalman Filter will provide the accurate readings before so many iterations which will reduce the delay. As a result, the new GPS Navigation system will provide much accurate and faster readings to the user.

***Index Terms*: Ephemeris Errors, GPS Navigation, Kalman Filter, Tracking Loops.**

I. GPS AT A GLANCE

The Global Positioning Satellite System is a space based navigation system that is formed, presently, by using a constellation of 30 healthy satellites around the earth. It helps in providing us with details regarding the position and time information of the user in all weather conditions. The satellites have their own orbits and take 12 hours to orbit the earth completely. There are five ground stations that are responsible for ensuring proper functioning of the satellites. The broadcasting rate for each satellite is 50 bits/s. The satellite sends the message in three parts. The time-of-day, GPS week number and satellite health information is transmitted in the first part of the message whereas the ephemeris data and the almanac are transmitted as second and third part respectively. The position calculated by a GPS receiver requires the current time, the position of the satellite and the measured delay of the received signal and then by using the triangulation rule the position of the receiver is determined. Major sources of error in GPS are produced due to the atmospheric effect (ionospheric, tropospheric), clock errors of the satellite, multipath effect, satellite orbits errors and calculation-rounding errors. There are 6 orbits dispersed equally for the complete GPS system as of now.

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***Correspondence Author(s)**
Prof. Kanika Gupta, IT Department, ABES Engineering College, Ghaziabad, India.
Ms. Apurva, IT Department, ABES Engineering College, Ghaziabad, India.
Ms. Priya Jindal, IT Department, ABES Engineering College, Ghaziabad, India.
Ms. Vishakha Snehi, IT Department, ABES Engineering College, Ghaziabad, India.

Each orbit has minimum 4 satellites functional at all points of time. The orbits are inclined at an angle of 55° relative to the equator and separated from each other by multiples of 60° right ascension. The orbits are non-geostationary and almost circular with radii of 26,560 km.

For the determination of its position on earth, the time of sending the message from the satellite and receiving the message by the GPS receiver is compared. This time difference is used for calculating the distance between receiver and satellite can be calculated. Trilateration is required for calculating the present position from the data collected from various satellites. It calculates the distance from three satellites, therefore, there are minimum three satellites required to get any reading in a GPS. This process is also known as 2D-Position Fix. The most accurate reading can be obtained by using four satellites and it is called 3D-Position Fix. A 3D-Position Fix can even help determine the height above the earth's surface.

Table I: Various Error Sources

Error Source	Error
Ionospheric effects	± 5 meter
Shifts in the satellite orbits	± 2.5 meter
Clock errors of the satellites' clocks	± 2 meter
Multipath effect	± 1 meter
Tropospheric effects	± 0.5 meter
Calculation- und rounding errors	± 1 meter

II. ERROS IN GPS NAVIGATION SYSTEMS

The most accurate GPS System will also have errors. The errors are mainly caused by environmental factors. There are various types of errors which are given as follows:

Atmospheric Conditions: The environmental conditions and the components of the earth's atmosphere cause the GPS signals to refract and even weaken. The two exterior parts of the atmosphere, that is, the ionosphere and the troposphere, both have different speeds for the same GPS signals.

*Clock Drift***:** The GPS messages sent by the satellite contain the ephemeris data in their second part. The ephemeris data contains data regarding the orbital position of the satellite. There are clock drifts that result from the satellite's motion. This leads to inconsistency in the clock time and causes errors. This type of error can induce a 1-5 meters positional error.

Noise: There is ample possibility of noise/distortion in the channel. Since, there is wireless communication involved, the data is exposed to maximum noise. The distorted data due to noise can introduce around 1 to 10 meters of positional error.

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Multipath: The surface of the earth has a varying terrain. There are even man-made obstacles like very high tower-like buildings, etc which cause hindrance in the channel. The GPS signals often get reflected by such obstacles causing the signal to bounce off from one point to another. This causes the multiple path error. Since GPS receivers should be very accurate, this type of error is a serious concern for developershe Global Positioning Satellite System is a space based navigation system that is formed, presently, by using a constellation of 30 healthy satellites around the earth. It helps in providing us with details regarding the position and time information of the user in all weather conditions. The satellites have their own orbits and take 12 hours to orbit the earth completely. There are five ground stations that are responsible for ensuring proper functioning of the satellites. The broadcasting rate for each satellite is 50 bits/s.

The measured pseudo range is equal to the true pseudo range plus various perturbing factors due to different error source, as shown below:

$$\rho_i = \rho_{Ti} + c\left(\delta_i^s - \delta_R\right) = \rho_{Ti} - c\delta_R + c\left(\Delta T_i + \Delta I_i + \Delta v_i + \Delta b_i\right) \quad \ldots(1)$$

Where:

ΔT_i : Troposphere error

ΔI_i : Ionosphere error

Δv_i : Relativistic time correction

Δb_i : Satellite bias clock error

A. Ionospheric Error

As GPS satellite signals traverse the ionosphere, they are delayed by an amount proportional to the number of free ions encountered.

John A. Klobuchar developed a simple analytical model for ionospheric time delay. We have used the same model for the ionospheric error correction. The model requires data regarding the user's approximate position along with the elevation angle and the azimuth. The calculation proceeds as follows:

Calculate the Earth-centered angle ψ

$$\psi = 0.0137/(E + 0.11) - 0.022 \quad (semicircles)$$

Compute the subinospheric latitude, ϕ_I

$$\phi_I = \phi_U + \psi \cos A \quad (semicircles)$$

If $\phi_I \geq 0.416$, then $\phi_I = 0.416$. If $\phi_I \leq -0.416$, then $\phi_I = -0.416$.

Compute the subionospheric longitude

$$\lambda_I = \lambda_U + \left(\psi \frac{\sin A}{\cos\phi_I}\right) \quad (semicircles)$$

Find the geomagnetic latitude, ϕ_m , of the subionospheric location looking toward each GPS satellite. It is found by

$$\phi_m = \phi_I + 0.064\cos(\lambda_I - 1.617) \quad (semicircles)$$

Find the local time, t, at the subionospheric point

$$t = 4.32 * 10^4 \lambda_I + Time_{GPS} \quad (Second)$$

If t>86400, use t=t-86400. If t<0, add 86400.

To convert to slant time delay, the slant factor, F, is computed.

$$F = 1 + 16(0.53 - E)^3$$

The ionospheric time delay T_{iono} is then computed by first computing x

$$x = \frac{2\pi(t - 50400)}{\underbrace{\sum_{n=0}^{3} \beta_n \phi_m^n}_{PER}}$$

If PER< 72000 then PER=72000.

If $|x| > 1.57$ then

$$T_{iono} = F \times \left(5 \times 10^{-9}\right)$$

Otherwise

$$\Delta I_i = T_{iono} = F \times \left[5 \times 10^{-9} + \sum_{n=0}^{3} \alpha_n \phi_m^n \times \left(1 - \frac{x^2}{2} + \frac{x^4}{24}\right)\right] \quad \ldots(2)$$

B. Tropospheric Error

The prime motive behind analysing the tropospheric error is to estimate the delay in the signal propagation. The wet tropospheric delay can be very helpful for weather and climate depicting models as they can help estimate the amount of water vapour in the troposphere. Because of this the Hopfield model is dedicated to tropospheric delay modelling and estimation only. In the model, Hopfield has split the troposphere delay into two parts which ate "Dry" and "Wet". The zenith delay of the dry component is given by:

$$K_d = 1.55208E{-4} \times P_{amb} \times (40136 + 148.72 \times T_{amb}) / (T_{amb} + 273.16)$$

Where T_{amb} is ambient temperature and P_{amb} is ambient air pressure. The zenith delay of the wet component is given by

$$K_w = -0.282 \times \frac{P_{vap}}{(T_{amb} + 273.16)} + 8307.2 \times \frac{P_{vap}}{(T_{amb} + 273.16)^2}$$

The zenith delay's is multiplied with their mapping functions to correct the readings for elevations lower than 90 Deg. And add the components to obtain the SV's tropospheric delay correction

$$\Delta T = \frac{K_d}{\sin\left(\sqrt{El^2 + 1.904E - 3}\right)} + \frac{K_w}{\sin\left(\sqrt{El^2 + 0.6854E - 3}\right)} \quad (Meter) \quad \ldots(3)$$

C. Clock Drift Error

The GPS receiver is needed to correct the satellite clock errors. The GPS receiver must present the user with the accurate representation of the user's position and time at the time of transmission. It should allow the user to see the satellite's sending time as well as the user's receiving time also. The satellite clock correction Δt_{sv} is obtained using coefficients that are broadcasted from the satellite after being uploaded by the GPS control segment. Relativistic correction must be computed by the GPS receiver. A first order effect described in the GPS gives the relativistic correction for an Earth-centered,

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Earth-fixed (ECEF) observer and a GPS satellite of eccentricity e. this relativistic correction varies as the *sine* of the satellite eccentric anomaly E_k as follows:

$$\Delta t_R = Fe\sqrt{A}\sin(E_k) \quad \ldots(4)$$

Where,
F:-4.442807633E-10s/m.
E_k: eccentric anomaly of the satellite orbit
A: semi major axis of the satellite orbit

III. INTRODUCTION TO TRACKING LOOPS

The central part of a GPS receiver is the tracking loop. It extracts the positioning data from the ephemeris data send by the satellite. They work on pseudo range measurements that allow the user's position to be calculated. The tracking loops duplicate the gold codes, that is, a type of binary sequence used in CDMA telecommunications. They also duplicate the carrier signal broadcast by each satellite. The main objective of tracking loops is to produce a replica of identical phase and frequency as what is received.

The major drawback of tracking loops is their limited ability to track the dynamics of the relative motion between the receiver and satellites. This can cause the tracking loops to become unlocked during high dynamic maneuvers. In order to improve the response of the tracking loops to platform dynamics, the bandwidth of the loop filter must be increased. However, this also increases the amount of noise passed by the filter.

A second shortcoming of classical tracking loops is that they employ fixed band- widths and gains, which operate on the received data regardless of high or low SNR values. The signal to noise ratio (SNR) can change significantly when tracking satellites. Under these conditions, the loop operates off noise rather than ignoring the measurements. For example, when a satellite is blocked, the tracking loops are in a state of random walk [Dierendonck, 1996]. Depending upon the duration of the signal loss and SNR, the loop may need to be re-initialized to reacquire the signal. If the satellite outage is for a prolonged time, the acquisition process may have to be repeated.

A. Position Determination using Tracking Loops

The data message sent from each satellite is recovered by the tracking loops and is used to calculate the position of the satellites. In addition, the Delay Lock Loop's produce pseudorange measurements to each satellite. Once the positions of at least four satellite's are known and pseudorange measurements are produced, the receiver can determine its position. Each individual pseudorange measurement can be expressed as shown below in Equation.

$$\rho_i = \|s_i - U\| + ct_u \quad \ldots(5)$$

$$\rho_i = \sqrt{(x_i - x_u)^2 + (y_i - y_u)^2 + (z_i - z_u)^2} + ct_u \quad \ldots(6)$$

The above model ignores the errors caused by the ionosphere, troposphere, and satellite clock offset. The pseudorange ρ_i to the ith satellite is a function of the user's coordinates (x_u, y_u, z_u) and clock bias t_u.

Obtaining the user's position and clock bias requires solving the second order, non- linear equations shown in equations below.

$$\rho_1 = \sqrt{(x_1 - x_u)^2 + (y_1 - y_u)^2 + (z_1 - z_u)^2} + ct_u$$
$$\rho_2 = \sqrt{(x_2 - x_u)^2 + (y_2 - y_u)^2 + (z_2 - z_u)^2} + ct_u$$
$$\vdots \;=\; \vdots$$
$$\rho_i = \sqrt{(x_i - x_u)^2 + (y_i - y_u)^2 + (z_i - z_u)^2} + ct_u$$

These equations can be solved by using analytical solutions, iterative numerical techniques, or the extended Kalman filter.

IV. KALMAN FILTER

The extended Kalman filter (EKF) is an alternate method to solve for the user's position and clock bias. The recursive nature of the EKF naturally lends itself to this task. The EKF formulation incorporates knowledge of the previous measurements into the current estimate of the user states. In contrast, the least squares solution is a point solution. The EKF method also allows the user's clock and platform dynamics to be modelled. The EKF architecture generally used to calculate user position and clock bias differs from that used in other applications. The difference is that the EKF used in GPS tracks the residuals of the states, not the states themselves. The purpose of filtering is to extract the required information from a signal, ignoring everything else. How well a filter performs this task can be measured using a cost or loss function. Indeed we may define the goal of the filter to be the minimisation of this loss function. If we are sure about the measurements, the measurements error covariance will decrease to zero and the Kalman gain also decreases.

A. State Space Representation State equation:

$$\mathbf{x}(k+1) = \mathbf{F}(k)\mathbf{x}(k) + \mathbf{G}(k)\mathbf{u}(k) + \mathbf{v}(k) \quad k = 0,1,\ldots \quad \ldots(7)$$

where ***x(k)*** is the *nx* dimensional *state* vector, ***u(k)*** is the *nu* dimensional *known* input vector, ***v(k)*** is (unknown) zero mean white *process* noise with covariance

$$E[\mathbf{v}(k)\mathbf{v}(k)'] = \mathbf{Q}(k) \quad \ldots(8)$$

Measurement equation:

$$\mathbf{z}(k) = (k)\,(k) + (k) \quad k = 1,\ldots. \quad \ldots(9)$$

w(k) is unknown zero mean white *measurement* noise with known covariance

$$E[\mathbf{w}(k)\mathbf{w}(k)'] = \mathbf{R}(k) \quad \ldots(10)$$

B. Kalman Gain

The gain takes *a priori* estimate covariance, our observation model, and our residual variance in order to decide how much we should change our model parameters before the next prediction.

C. Methodology

The Global Positioning System (GPS) is a satellite based navigation system that provides accurate positioning information anywhere on the globe. The Kalman filter provides optimal estimate of the system state vector, and has been widely applied to the fields of navigation such as GPS receiver position/velocity determination. The Kalman Filter algorithm for GPS Navigation is summarized below:-

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1. The navigation filter receives the raw pseudo-range and pseudo-range rate measurements from the GPS receiver.

2. Initialize state vector and state covariance matrix: - $\hat{x}_k^-$ and P_k^-.

3. Compute Kalman gain matrix from state covariance and estimated measurement covariance:

$$K_k = P_k^- H_k^T (H_k P_k^- H_k^T + R_k)^{-1} \quad \ldots(11)$$

4. The prediction error vector is multiplied by the Kalman gain to get state correction vector and update state vector:

$$\hat{x}_k = \hat{x}_k^- + K_k (z_k - H_k \hat{x}_k^-) \quad \ldots(12)$$

5. Update error covariance:-

$$P_k = (I - K_k H_k) P_k^- \quad \ldots(13)$$

6. Predict new state vector and state covariance matrix:-

$$\hat{x}_{k+1}^- = A_k \hat{x}_k + B u_k \quad \ldots(14)$$

$$P_{k+1}^- = A_k P_k A_k^T + Q_k \quad \ldots(15)$$

V. SIMULATIONS FROM MATLAB DEPICTING ERROR SEQUENCES USING TRACKING LOOPS

In this part of the paper the simulations from MATLAB files are presented. It uses the Ephemeris data to calculate the satellite position and finally plots the different results.

The 3-D model of the GPS satellites and receiver position is shown in Figure 1. The experiment uses reading of 6 satellites. A minimum of 4 satellites is required to estimate the position of a receiver. Each satellite has its own orbit and takes 12 hours to orbit the Earth.

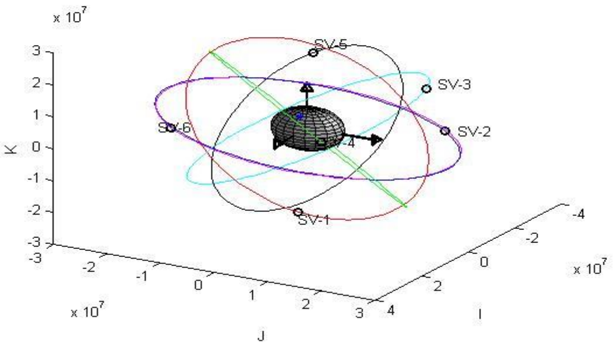

Figure I: GPS 3-D Model

The algorithm iteratively solves for delta x, delta y and delta z which is the difference between the true solution and initial guess.

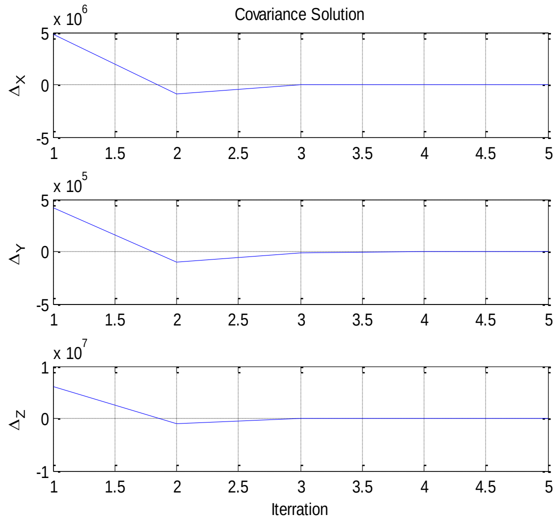

Figure II: Iterative Method for GPS Navigation-Delta x

The above figure is the plot of the covariance solution. It shows that after the 5th iteration the solution converges, however the number of iterations can be reduced by using Kalman Filter.

The x, y, z coordinates of the GPS receiver position are computed using the covariance matrix iteratively and the results are plotted in the following figure:

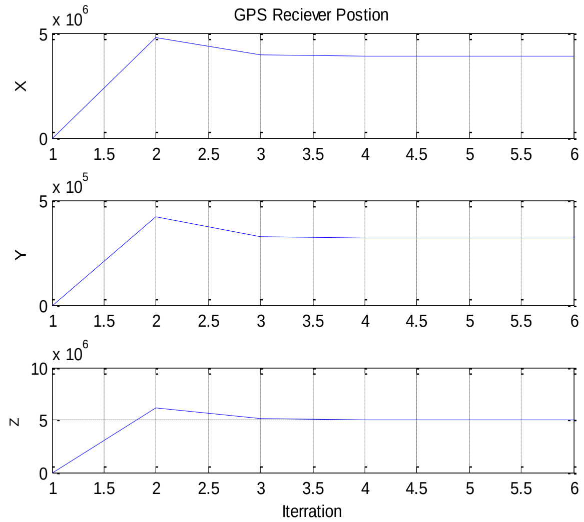

Figure III: Iterative Method for GPS Navigation in Finding Position

The following figure shows the convergence of the algorithm and represents the norm of $[\Delta x \quad \Delta y \quad \Delta z]$.

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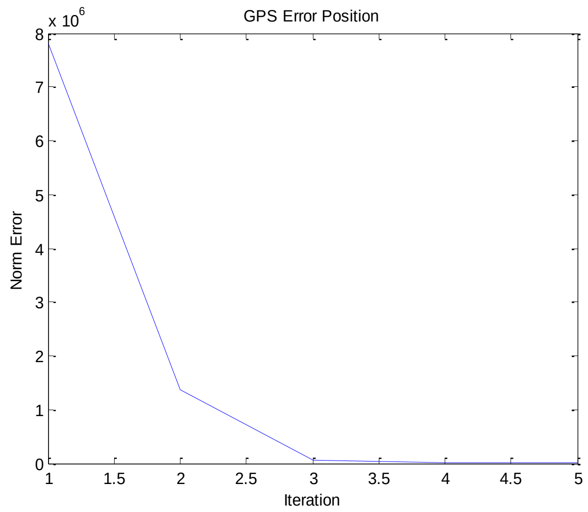

Figure IV: Iterative Method for GPS Navigation –Norm Error

The graph shows that the norm error becomes constant and reduces to zero after the 4th iteration. The number of iterations can be reduced using the iterative Kalman Filter algorithm.

VI. APPLICATIONS

A. Aided Inertial Navigation System

The combination of an IMU and a computer running navigation equations is called an Inertial Navigation System (INS). Due to errors in the gyros and accelerometers, an INS will have unlimited drifting velocity, position and attitude. So based on the measurements and sensor error models, the Kalman filter estimates errors in the navigation equations and all sensor errors and therefore can be suitably reduced.

B. Inertial Measurement Unit(IMU)

Several inertial sensors such as accelerometers and Gyros are often assembled to form an Inertial Measurement Unit (IMU).Typically the unit has 3 accelerometers and 3 gyros (*x*, *y* and *z*). Again the Kalman Filter estimates the measurement error in Gyros and accelerometers and it is suitably reduced then using other error reduction algorithm.

C. Missile attitude determination using Kalman Filter

The Kalman filter utilizes a series of MEMS sensors to accurately estimate the attitude angles for MAVs. The filter uses the 3 variables of the direction cosine matrix as state elements. The rigid body motion equations are introduced to correct the gravitational acceleration measurements during high speed maneuvers. The geomagnetic field is used only to determine the yaw angle. Therefore, magnetic disturbances can not affect the output of the pitch and roll. The pitch and roll angle can then be calculated with the Kalman filter

VII. CONCLUSION

The GPS Navigation System with Kalman Filter will be a faster and more accurate way to generate user's position. As it Kalman Filter takes into account the dynamic and non-linear nature of the GPS system, it focuses on error removal much more efficiently than other systems. The Kalman Filter can be modified further for enhancing the functionality of the system to greater extents. The wide applications of the same are also beneficial. Most importantly, use of Kalman Filter reduces the database management part effectively as it works in a recursive manner; there is no need for maintaining a database. This is making the system even more space efficient.

VIII. FUTURE WORK

Prof. Kanika Gupta is a Assistant Professor at ABES Engineering College, Ghaziabad. She did the post graduation in M.Tech (IT). She has completed her graduation in B.Tech (Information Technology) from Indraprastha Engineering College, Ghaziabad and has worked on various projects.She is having experience of 7 years. Her Area Of interest is Digital Image Processing,Computer Networks, Data Mining. She has published various papers in reputable national and international journals.

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AUTHOR PROFILE

Prof. Kanika Gupta is an Assistant Professor at ABES Engineering College, Ghaziabad. She did the post graduation in M.Tech (IT). She has completed her graduation in B.Tech (Information Technology) from Indraprastha Engineering College, Ghaziabad and has worked on various projects. She has an experience of 7 years. Her Area Of interest is Digital Image Processing, Computer Networks, Data Mining. She has published various papers in reputable national and international journals.

Ms. Apurva is a student at ABES Engineering College, Ghaziabad. She is currently pursuing B.Tech in the field of Information Technology. Her recent project works include: Designing of the Protocol used in Mobile Ad-Hoc Networks (MANET) using Network Simulator.

Ms. Priya Jindal is a student at ABES Engineering College, Ghaziabad. She is currently pursuing B.Tech in the field of Information Technology. Her recent project works include: Designing of the Protocol used in Mobile Ad-Hoc Networks (MANET) using Network Simulator.

Ms. Vishakha Snehi is a student at ABES Engineering College, Ghaziabad. She is currently pursuing B.Tech in the field of Information Technology. Her recent project works include: Designing of the Protocol used in Mobile Ad-Hoc Networks (MANET) using Network Simulator.

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