

# Laplace and Morlet Wavelet Analysis for Gear Fault Diagnosis: A Comparative Study

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**Abstract:** The machines need to be developed with high speed and light weight to acquire market in this present competitive world and maintenance of these machines become critical and important to ensure failure free operation. Gear drives form a major component of any industrial machine and detection of faults at incipient stage is very crucial in order to reduce maintenance downtime of machine before the major failure. Vibrations emitted from faulty gears are rather non stationary and non-periodic signals and hence it is difficult to detect the gear fault by conventional FFT analysis. Therefore an effective and sophisticated signal processing method using wavelet analysis has successfully being applied. This paper investigates the application of Laplace wavelet kurtosis for gear fault diagnosis. Also, this paper presents the optimisation of wavelet parameters to maximize the kurtosis parameter in order to render the wavelet coefficients sensitive to the generated fault signals. Further, this paper compares the use of Morlet and Laplace wavelet kurtosis for automated fault detection in gears for various fault stages and also compares the Laplace and Morlet wavelet kurtosis for varying working condition.

**Index Terms:** Wavelet, Morlet wavelet, Laplace Wavelet Kurtosis, Gear,

## I. INTRODUCTION

The need for productivity and profit has resulted in the design and construction of many high speed and lightweight machines and structures resulting in dynamic instability and severe vibration problems. The maintenance of these machines becomes necessary to ensure failure free operation. Gears form a major component of any machine. Vibration analysis has been regarded as one of the strong tools for condition monitoring of faulty gears. Different techniques such as time domain, frequency domain and time-frequency domain techniques are widely used to analyse the vibration signals [1]. The traditional Fast Fourier Transform (FFT) is a type of frequency domain method which involves representing a signal as the summation of its constituent sine waves at various frequencies [2]. Vibrations emitted from faulty gears are rather non stationary and non-periodic signals. FFT analysis however does not form a strong tool to analyze transitory signals and hence it is difficult to detect the gear fault by conventional FFT analysis [3-6].

Therefore an effective and sophisticated signal processing method like wavelet analysis for feature extraction from noisy gear signal can be used [7].

The Continuous Wavelet Transform (CWT) is a time-frequency method that builds on the idea of the Short Time Fourier Transform (STFT)[8]. Morlet and Impulse wavelet find major application in fault detection of bearings. The optimization of these wavelet functions using maximum kurtosis has enhanced the fault detection method. The Morlet wavelet is a wavelet composed of a complex exponential (carrier) multiplied by a Gaussian window (envelope). The Morlet wavelet has a form very similar to the Gabor transform. The important difference is that the window function also needs to be proportioned by a scaling parameter, while the size of window in Gabor transform is known to be fixed. Laplace wavelet is a complex, single sided damped exponential which finds its application in free vibration analysis of an aircraft for aerodynamic [9] and structural testing and to diagnose the wear of the intake valve of an internal combustion engine [10].

Some of the commonly used statistical parameters for vibration signature analysis are Root Mean Square (RMS), crest factor, shape factor, peak to peak, impulse factor, kurtosis etc. Kurtosis is more commonly defined as the fourth central cumulant divided by the square of the variance of the probability distribution [11]. Spectral Kurtosis is obtained through calculating the kurtosis of each frequency line in a time-frequency diagram. The spectral kurtosis has been successfully described and applied to the vibration analyses and diagnostics of rotating machines [12-13].

This paper investigates the application of Laplace wavelet kurtosis for gear fault diagnosis. Wavelet parameters are optimised to maximize the kurtosis parameter in order to render the wavelet coefficients sensitive to the generated fault signals. Also, this paper compares the use of Morlet and Laplace wavelet kurtosis for automated fault detection in gears for various fault stages and for varying working condition.

## II. WAVELET KURTOSIS

Wavelet analysis make use of a variety of probing functions, but however the family always consists of enlarged or compressed versions of the basic function. This concept defines the equation for the continuous wavelet transform (CWT) as given in equation 1.

$$W(a, b) = \int_{-\infty}^{\infty} x(t) \frac{1}{\sqrt{|a|}} \psi^* \left( \frac{t-b}{a} \right) dt \quad (1)$$

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Where 'b' acts to translate the function across x (t) just as t. 1. does in equations above and the variable 'a' acts to vary the time scale of the probing function  $\psi$ . If 'a' is greater than 1, the wavelet function  $\psi$  is stretched along the time axis and if 'a' is less than 1 then it contracts.

The popular wavelet used for mechanical fault detection particularly gear and bearings, is the Morlet wavelet defined by the equation 2

$$\psi(t) = e^{-t^2} \cos\left(\pi \sqrt{\frac{2}{\ln 2}} t\right) \quad (2)$$

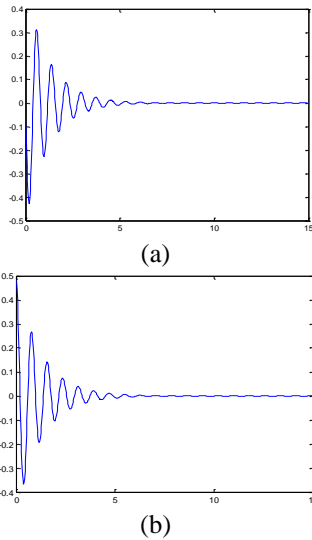
The Laplace wavelet is a complex single sided damped exponential function. The major field where this wavelet has been applied is for the vibration field analysis of an aircraft for aerodynamic and structural testing and to diagnose the wear of the intake valve of an internal combustion engine. The Laplace wavelet function is defined by

$$\psi(t) = A e^{-\left(\frac{\beta}{\sqrt{1-\beta^2}} + i\right) \omega_c t} \quad t \geq 0$$

$$\psi(t) = 0 \quad t \text{ is otherwise} \quad (3)$$

Where  $\beta$  is the damping factor, the frequency

$\omega_c$  determines the number of significant oscillations of the wavelet and A is an arbitrary scaling factor. Figure 1 depicts the real and imaginary part of the Laplace wavelet.



**Figure 1.** Laplace Wavelet a) Real part b) Imaginary part

The wavelet transform (WT) of the signal  $x(t)$  with the mother wavelet  $\psi(t)$  is the inner product of  $x(t)$  with a scaled and conjugate wavelet  $\psi^*_{a,b}$ . The result of the wavelet transform obtained will be analytical signal since the wavelet used is analytical and complex wavelet to calculate the wavelet transform as shown in equation (4) and (5).

$$WT\{x(t), a, b\} = \langle x(t), \psi_{a,b}(t) \rangle = \frac{1}{\sqrt{a}} \int_{-\infty}^{\infty} x(t) \psi^*_{a,b}(t) dt \quad (4)$$

$$= \text{Re}[WT(a, b)] + i \text{Im}[WT(a, b)] \quad (5)$$

Where  $\psi^*_{a,b}$  is a family of wavelet with  $a$  as scale parameter and  $b$  as translation parameter.

The wavelet kurtosis are calculated by the following steps

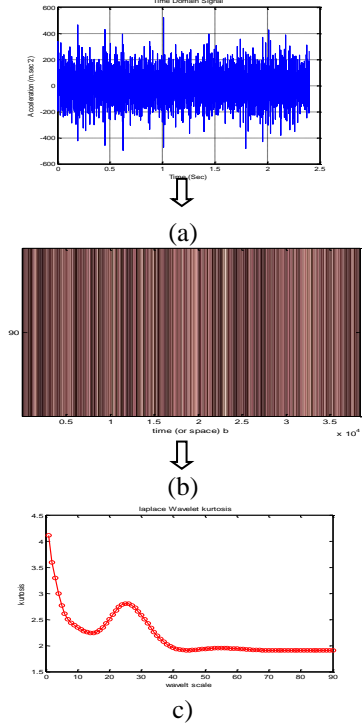
1) From the experimental setup time domain values are collected using an accelerometer and vibration data collector.

2) Time domain values are transformed into wavelet transform using Laplace wavelet function.

3) Laplace wavelet kurtosis is calculated from wavelet transform

The schematic representation above steps is depicted in

**Figure 2**



**Figure 2.** The Methodology for calculation of wavelet kurtosis (a) Vibration signal collected from experimental setup, (b) The Wavelet Transform (c) Plot of Wavelet Kurtosis vs. wavelet scale

Let  $x(n)$  be a real discrete time random process, and  $WT_a$  its  $N$  point Laplace wavelet transform at scale  $a$ . The LWK for  $x(n)$  is defined as the kurtosis of the magnitude of  $WT_a$  at each wavelet scale  $a$  as in the equation (6).

$$LWK(a) = \frac{\sum_{n=1}^N abs(WT_a^4(x(n), \psi_{\beta}, \omega_c))}{[\sum_{n=1}^N abs(WT_a^2(x(n), \psi_{\beta}, \omega_c))]^2} \quad (6)$$

### III. EXPERIMENTAL SETUP

The experimental setup is depicted in Figure 3. It consists of single stage gear box, motor, loading system, coupling and bearings. One gear was connected to 0.5 HP, 2900 RPM electric motor driven through coupling and the other gear was connected to a loading system. The gear and pinion has 46 and 23 teeth respectively. The 25mm diameter shaft connects gears with motor and loading system. The shafts are supported at its ends with bearings. The vibration data is collected from the drive end bearing of gear box using the accelerometer (model 621B40, IMI sensors, sensitivity is 1.02 mV/m/s<sup>2</sup> and frequency range up to 18 kHz) with a NI Data Acquisition Device. The healthy gear is shown in Figure 4. The vibration data collected are used as input to MATLAB for signal processing.

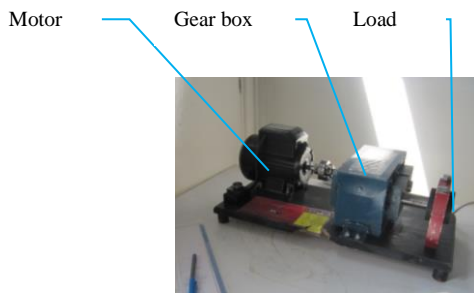


Figure 3. Fault Simulator set up.

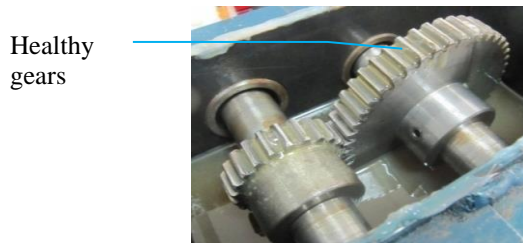


Figure 4. View of healthy gears

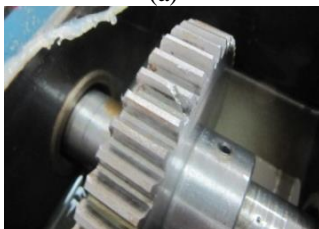
The vibration signals from a healthy gear were collected at a shaft speed of 2850 RPM. Faults were induced in four different stages as shown in Table 1 and the corresponding vibration readings were taken. The various fault stages are shown in Figure 5.

Stage	Condition of the gear	Fault description
Stage 0	Healthy gear	Without any induced fault
Stage 1	Faulty gear	A crack of 3mm is induced at the root of the tooth
Stage 2	Faulty gear	Tooth was partially broken
Stage 3	Faulty gear	Fault was further increased.
Stage 4	Faulty gear	Tooth was completely removed

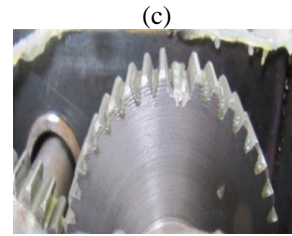
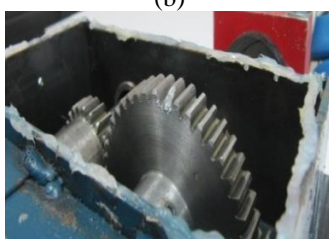
Table 1: Stages of induced fault



(a)



(b)



(d)

Figure 5: Stages of induced crack (a) Stage 1. (b) Stage 2. (c) Stage 3. (d) Stage 4

#### IV. IMPLEMENTATION

This section gives various applied examples to demonstrate the proposed approach. A rise in the magnitude of the kurtosis will indicate the presence of a defect. The magnitude of such kurtosis will increase with increase in fault magnitude. A typical time domain signal obtained from the experimental setup is shown in the figure 6

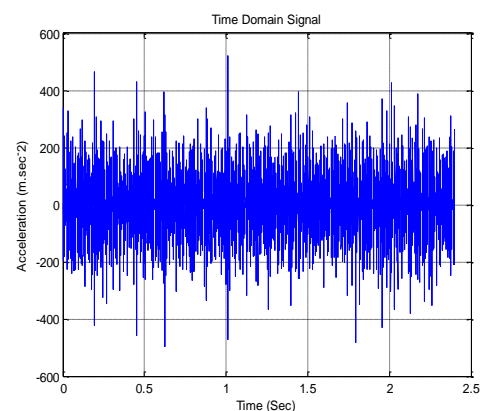


Figure 6: Time domain data

This is further processed using various signal processing techniques based on spectral kurtosis (SK) principle and wavelet kurtosis based on Laplace and Morlet wavelet function.

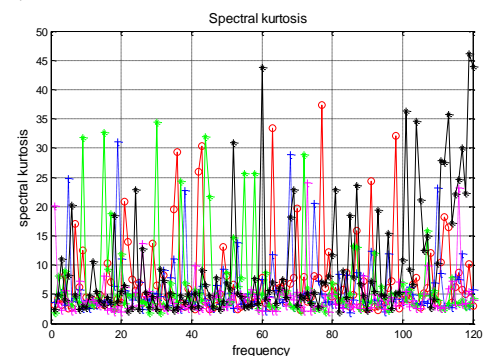


Figure 7: SK for stages of fault.

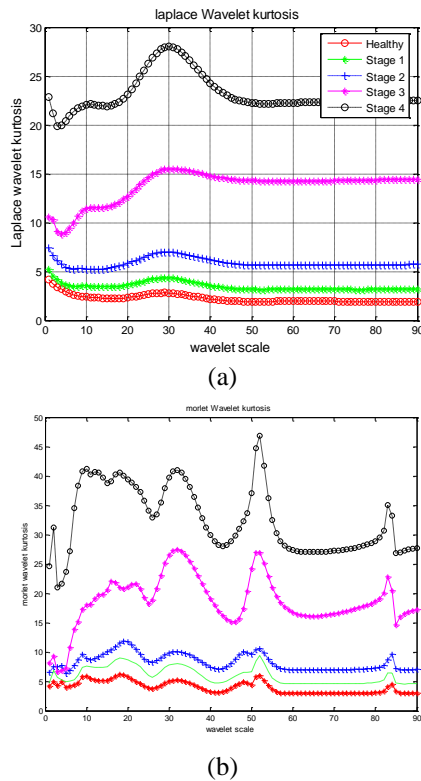
The figure 7 depicts the spectral kurtosis applied for determination of gear faults.

Healthy	Red
Stage1	Green
Stage2	Blue
Stage3	Magenta

**Table 2:** Color code for healthy and faulty gear

It is difficult to analyze the healthy and faulty signals from Figure.7 due to complexity and overlap of signals with a constant window size.

Figure 8 shows Laplace wavelet and Morlet wavelet kurtosis with wavelet scale of 90 for the gear with healthy conditions and 4 different stages of crack. The Laplace wavelet shape parameters  $\beta = 0.3$  and  $\omega_c = 8.1$  are selected based on maximum kurtosis.


**Figure 8** (a) Laplace wavelet kurtosis (b) Morlet wavelet kurtosis

The healthy and faulty conditions of the gear at different stages are shown with increasing magnitude of LWK value with distinct correlation between them. As the fault progresses in size, corresponding LWK values also increase in magnitude. The wavelet scale number and frequency relationship is given in equation (7)

$$F_a = \frac{F_0}{a * \Delta} \quad (7)$$

Where  $F_a$  is the frequency,  $F_0$  = wavelet central frequency,  $a$  = wavelet scale,  $\Delta$  = sampling frequency. The wavelet scale number of 30 is corresponding frequency of 24Hz, which is equal to gear rotation frequency.

As seen from Figure 8, Laplace wavelet kurtosis forms a strong tool for analyzing gear fault defects at various stages of the fault. Its magnitude increases with corresponding increase in the severity of the fault. However, the Morlet wavelet kurtosis fails to provide a distinction at the lower stages of the gear fault with the corresponding increasing in the magnitude of the fault. There is mixing of magnitudes at lower stages of the fault.

Figure 9 shows the Laplace wavelet kurtosis applied for various working conditions.

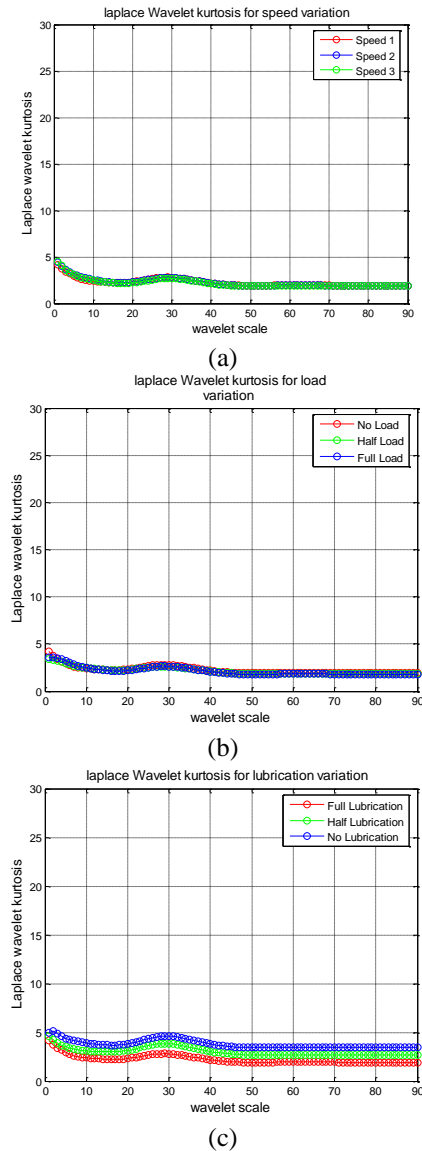
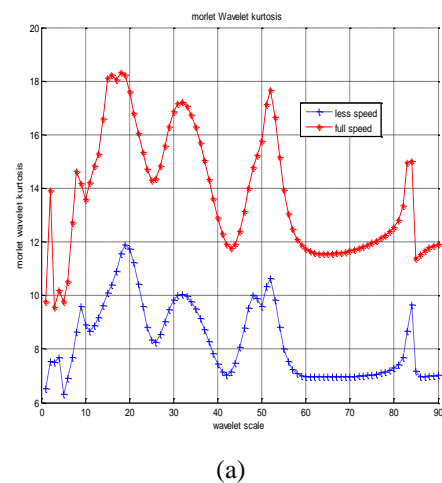
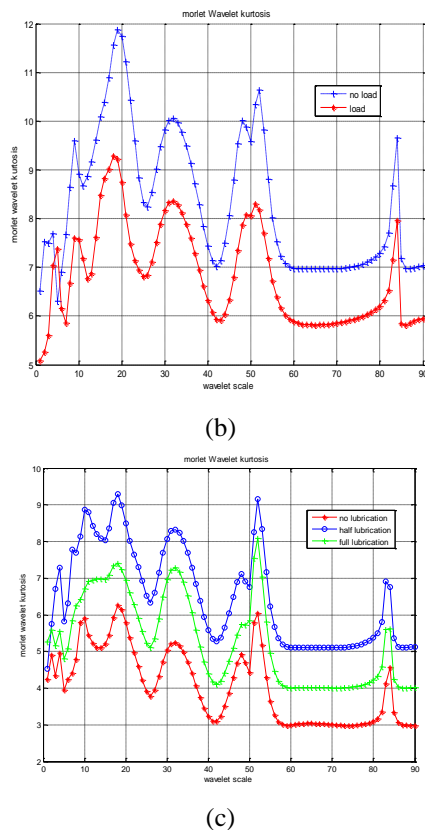

**Figure 9.** Laplace wavelet kurtosis for different working conditions (a) varying speed condition (b) varying load conditions (c) varying lubrication condition.

Figure 10 shows the Morlet wavelet kurtosis for various working conditions.



(a)



**Figure 10.** Morlet wavelet kurtosis for different working conditions (a) varying speed condition (b) varying load conditions (c) varying lubrication condition

Figure 9 depicts that the proposed technique of Laplace wavelet kurtosis has less influence for different working conditions and hence enhances the extraction of defect features of gear, whereas as seen in Figure 10 the Morlet wavelet kurtosis shows considerable variation for different working conditions.

## V. CONCLUSION

Different signal processing method adopted for gear fault diagnosis is presented in this paper. A comparison of Laplace wavelet kurtosis and Morlet wavelet kurtosis for gear fault diagnosis based on vibration signal processing are implemented and the results are compared for the various stages of induced fault conditions on the experimental setup. From the studies the Laplace wavelet kurtosis proved to be a useful tool which provides a distinct correlation between healthy and faulty gear. The study also shows that Laplace wavelet kurtosis did not show much variation on varying work condition. All these factors enhance the use of Laplace wavelet kurtosis method for gear fault diagnosis.

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