

# Spectrum Efficiency for Rate-Adaptive MIMO OSFBC-OFDM Systems over Various Adaptation Policies

CH.Siva Rama Krishna, P.Venkateswara Rao

**Abstract**—In this paper, closed-form expressions for capacities per unit bandwidth for multiuser MIMO-OFDM systems employing Orthogonal Space-Frequency Block Coding (OSFBC) over multipath frequency-selective fading channels are derived for optimal power adaptation, optimal rate adaptation with constant transmit power, channel inversion with fixed rate, and truncated channel inversion adaptation policies. A Signal to Noise Ratio (SNR) based user selection scheme is considered. Closed-form expressions are derived for OSFBC-OFDM system. Optimal power adaptation policy provides the highest capacity over the other adaptation policies. Capacity gains are more prominent for optimal rate adaptation with constant transmit power policy as compared to other adaptation policies.

**Keywords:** Orthogonal space-frequency block coding; optimal power adaptation; optimal rate adaptation with constant transmit power; channel inversion with fixed rate; truncated channel inversion; outage probability.

## I. INTRODUCTION

Multiple-Input–Multiple-Output (MIMO) technology has been recognized as a key approach for improving system performance and the channel capacity of wireless communication systems [1]. On the other hand, Orthogonal Frequency Division Multiplexing (OFDM) has been considered as a promising technique in future broadband wireless communications. In particular, the MIMO-OFDM system is considered as an attractive solution for broadband wireless communications [2]. A significant advantage of MIMO-OFDM systems is that they allow rate and power allocation (through adaptive modulation) and dynamic resource allocation to the system [3], [4]. The work that has been done for variable-rate and variable-power allocation in these systems introduces high system complexity, particularly with using the well-known water-filling technique [5]–[9]. In [10], the authors proposed performance analysis of scheduling schemes for Rate-Adaptive MIMO OSFBC-OFDM systems. The channel capacity per unit bandwidth for different adaptation policies over various fading channels with different diversity schemes is discussed in [11], [12]. This paper derives closed-form expressions for the capacities per unit bandwidth for multiuser MIMO-OFDM systems employing OSFBC over multipath frequency-selective fading channels for optimal power and rate adaptation, optimal rate adaptation with constant transmit power,

channel inversion with fixed rate, and truncated channel inversion adaptation policies. An SNR based user selection scheme is assumed. This paper organized as follows. Section 2 derives closed form expressions for capacities per unit bandwidth of a MIMO OSFBC-OFDM system for the optimal power and rate adaptation policy, optimal rate adaptation with constant power policy, channel inversion with fixed rate policy, and truncated channel inversion with fixed rate policy respectively. Section 3 presents numerical results for the comparison of capacities per unit bandwidth for various adaptation policies. Finally, Section 4 presents the conclusion.

## II. SPECTRUM EFFICIENCY FOR OSFBC-OFDM SYSTEM

In MIMO-OFDM systems, OFDM converts the frequency-selective fading channel into several parallel flat-fading sub-channels; therefore, the subchannel faded value, i.e.,  $|H_{j,i}[k, n]|$ , can be assumed to be Rayleigh flat fading, and

hence  $|H_{j,i}[k, n]|^2$  for each user in each subchannel is a Chi-squared (central) distributed random variable. The closed form solution for the Probability Density Function (PDF) of the received SNR for a subchannel of each user in an OSFBC-OFDM system is given by Eq. (3) on page 947 of [10] as

$$f_{\gamma}(\gamma) = \frac{\gamma^{n_T n_R - 1}}{(n_T n_R - 1)!} \left( \frac{n_T R_c}{\gamma} \right)^{n_T n_R} \exp\left(-\frac{\gamma n_T R_c}{\gamma}\right) \quad (1)$$

where  $n_T$  = number of transmit antennas,  $n_R$  = number of receive antennas,  $R_c$  = OSFBC code rate,  $\bar{\gamma}$  average received SNR,  $\gamma$  instantaneous SNR.

### 2.1 Optimal power and rate adaptation policy

Given an average transmit power constraint, the channel capacity of a fading channel with received SNR distribution,  $f_{\gamma}(\gamma)$ , and optimal power and rate adaptation

$\langle C \rangle_{\text{opra}}$  [bps] as [11]

$$\langle C \rangle_{\text{opra}} = B \int_{\gamma_o}^{\infty} \log_2 \left( \frac{\gamma}{\gamma_o} \right) f_{\gamma}(\gamma) d\gamma \quad (2)$$

where  $B$  [Hz] is the channel bandwidth and  $\gamma_o$  is the optimal cutoff SNR level below which data transmission is suspended. This optimal cutoff must satisfy [13]

Manuscript published on 30 June 2013.

\*Correspondence Author(s)

**Siva Rama Krishna** received his B.TECH. degree in Electronics and Communication Engineering from Mother Teresa Institute of Science & Technology, JNTUH University, India

**Venkateswara Rao.P** received his B.TECH. degree in Electronics and Communication Engineering from Sri Sarathi Institute of Engineering & Technology, JNTUK University, India

© The Authors. Published by Blue Eyes Intelligence Engineering and Sciences Publication (BEIESP). This is an open access article under the CC-BY-NC-ND license <http://creativecommons.org/licenses/by-nc-nd/4.0/>

$$\int_{\gamma_0}^{\infty} \left( \frac{1}{\gamma_0} - \frac{1}{\gamma} \right) f_{\gamma}(\gamma) d\gamma = 1 \quad (3)$$

To achieve the capacity in (3), the channel fade level must be tracked at both the receiver and transmitter, and the transmitter to adapt its power and rate accordingly, allocating higher power levels and rates for good channel conditions (large), and lower power levels and rates for unfavorable channel conditions (small). Since no data is sent when  $\gamma < \gamma_0$ , the optimal policy suffers a probability of outage,  $P_{\text{out}}$ , equal to the probability of no transmission, given by

$$P_{\text{out}} = P[\gamma \leq \gamma_0] = \int_0^{\gamma_0} f_{\gamma}(\gamma) d\gamma = 1 - \int_{\gamma_0}^{\infty} f_{\gamma}(\gamma) d\gamma. \quad (4)$$

Substituting (1) into (3), and simplifying, we find that  $\gamma_0$  must satisfy

$$\frac{\Lambda^c(m, n_T R_C \gamma_0)}{\gamma_0} - \left( \frac{n_T R_C}{\gamma} \right) \Lambda^c(m-1, n_T R_C \gamma_0) - \Gamma(m) \quad (5)$$

where  $m = n_T n_R$ ,  $\Gamma(x) = \int_0^{\infty} t^{x-1} e^{-t} dt$ , is the gamma function (from [14], section 8.35, page 890), and  $\Lambda^c(x, a) = \int_a^{\infty} t^{x-1} e^{-t} dt$  is the complementary incomplete gamma function (from [14], section 8.35, page 890).

Let  $x = \gamma_0$  in (5), we have

$$g(x) = \frac{\Lambda^c(m, n_T R_C x)}{x} - \left( \frac{n_T R_C}{x} \right) \Lambda^c(m-1, n_T R_C x) - \Gamma(m) \quad (6)$$

Note that

$$\frac{dg(x)}{dx} = -\frac{1}{x^2} \Lambda^c(m, n_T R_C x) < 0 \forall x > 0. \text{ Moreover, from (6)}$$

$$\lim_{x \rightarrow 0} g(x) = +\infty \quad \text{and} \quad \lim_{x \rightarrow \infty} g(x) = -\Gamma(m) < 0.$$

Thus, it can be concluded that there is a unique  $\gamma_0$  for which  $g(\gamma_0) = 0$ , which satisfies (5). Substituting (1) into (2), we have

$$\frac{\langle C \rangle_{\text{opra}}}{B} = \int_{\gamma_0}^{\infty} \log_2 \left( \frac{\gamma}{\gamma_0} \right) \frac{\gamma^{n_T n_R - 1}}{(m-1)!} \left( \frac{n_T R_C}{\gamma} \right)^m \exp \left( -\frac{n_T R_C}{\gamma} \right) d\gamma \quad (7)$$

Making change of variables in the integral of (7), where

$$t = \frac{\gamma}{\gamma_0} \quad \text{and} \quad dt = \frac{d\gamma}{\gamma_0}, \text{ we have}$$

$$\frac{\langle C \rangle_{\text{opra}}}{B} = \mu^m \frac{\log_2(e)}{\Gamma(m)} \mathfrak{I}_m(\mu) \quad (8)$$

$$\text{where } \mu = \frac{n_T R_C \gamma_0}{\gamma}.$$

Eq. (8) gives the spectrum efficiency,  $\langle C \rangle_{\text{opra}}/B$  [bits/Hz], under OPRA policy for MIMO-OFDM systems employing OSFBC over multipath frequency-selective fading channels.

Substituting (1) into (4), the expression for outage probability is given as

$$P_{\text{out}} = 1 - \int_{\gamma_0}^{\infty} \frac{\gamma^{n_T n_R - 1}}{(m-1)!} \left( \frac{n_T R_C}{\gamma} \right)^m \exp \left( -\frac{n_T R_C}{\gamma} \right) d\gamma \quad (9)$$

$$= 1 - \frac{1}{\Gamma(m)} \left( \frac{n_T R_C}{\gamma} \right)^m \left( \int_0^{\infty} \gamma^{m-1} \exp \left( -\frac{n_T R_C}{\gamma} \right) d\gamma - \int_0^{\gamma_0} \gamma^{m-1} \exp \left( -\frac{n_T R_C}{\gamma} \right) d\gamma \right) \quad (10)$$

Making change of variables in the integral of (10), where

$$t = \frac{n_T R_C \gamma}{\gamma} \quad \text{and} \quad dt = \frac{n_T R_C d\gamma}{\gamma}, \text{ we have,}$$

$$P_{\text{out}} = \frac{1}{\Gamma(m)} \Lambda(m, \mu), \quad (11)$$

where  $\Lambda(x, a) = \int_0^a t^{x-1} e^{-t} dt$  is the incomplete gamma function (from [14], section 8.35, page 890).

## 2.2 Optimal rate adaptation with constant transmit power policy

Adapting the code rate to channel conditions with a constant transmit power, the channel capacity,  $\langle C \rangle_{\text{ora}}$  [bits/s] is given as [11]

$$\langle C \rangle_{\text{ora}} = B \int_0^{\infty} \log_2(1 + \gamma) f_{\gamma}(\gamma) d\gamma. \quad (12)$$

Substituting (1) into (12) and defining the integral [12]

$$I_n(\zeta) = \int_0^{\infty} t^{n-1} \log_e(1+t) \exp(-\zeta t) dt; \zeta > 0, \quad (13)$$

the channel capacity  $\langle C \rangle_{\text{ora}}$  of a OSFBC-OFDM system can be expressed as

$$\begin{aligned} \frac{\langle C \rangle_{\text{ora}}}{B} &= \int_0^{\infty} \log_2(1 + \gamma) \frac{\gamma^{m-1}}{(m-1)!} \left( \frac{n_T R_C}{\gamma} \right)^m \exp \left( -\frac{n_T R_C}{\gamma} \right) d\gamma \\ &= \frac{1}{\Gamma(m)} \left( \frac{n_T R_C}{\gamma} \right)^m \log_2(e) \int_0^{\infty} \log_e(1 + \gamma) \gamma^{m-1} \exp \left( -\frac{n_T R_C}{\gamma} \right) d\gamma \quad (14) \end{aligned}$$

Simplifying and rearranging (14), the channel capacity per unit bandwidth  $\langle C \rangle_{\text{ora}}$  of the OSFBC-OFDM system is given by

$$\frac{\langle C \rangle_{\text{ora}}}{B} = \frac{\zeta^m \log_2(e)}{\Gamma(m)} I_m(\zeta), \quad (15)$$

$$\text{where } \zeta = \frac{n_T R_C}{\gamma}.$$

### 2.3 Channel inversion with fixed rate policy

The channel capacity with this technique,  $\langle C \rangle_{\text{cifr}}$  [bps] is derived from the capacity of an AWGN channel and is given as [11]

$$\langle C \rangle_{\text{cifr}} = B \log_2 \left( 1 + \frac{1}{\int_0^\infty \frac{f_\gamma(\gamma)}{\gamma} d\gamma} \right). \quad (16)$$

Channel inversion with fixed rate suffers a large capacity penalty relative to the other techniques, since a large amount of the transmitted power is required to compensate for the deep channel fades. A better approach is to use a modified inversion policy which inverts the channel fading only above a fixed cutoff fade depth  $\gamma_0$ . The capacity with this truncated channel inversion and fixed rate policy is given as [11]

$$\langle C \rangle_{\text{tifr}} = B \log_2 \left( 1 + \frac{1}{\int_{\gamma_0}^\infty \frac{f_\gamma(\gamma)}{\gamma} d\gamma} \right) (1 - P_{\text{out}}) \quad (17)$$

where  $P_{\text{out}}$  is given by (4).

By substituting the PDF from (1) into (16), it can be found that the capacity per unit bandwidth for total channel inversion policy is an OSFBC-OFDM system is given as

$$\frac{\langle C \rangle_{\text{cifr}}}{B} = \log_2 \left( 1 + \frac{1}{\int_0^\infty \frac{\gamma^{m-1} \left( \frac{n_T R_C}{\gamma} \right)^m \exp\left(-\frac{n_T R_C}{\gamma}\right)}{\gamma} d\gamma} \right) \quad (18)$$

Making change of variables in the integral of (18), where

$$t = \frac{n_T R_C}{\gamma} \text{ and } dt = -\frac{n_T R_C}{\gamma^2} d\gamma, \text{ we have,}$$

$$\frac{\langle C \rangle_{\text{cifr}}}{B} = \log_2 \left( 1 + \frac{\Gamma(m)}{\left( \frac{n_T R_C}{\gamma} \right)^m \int_0^\infty t^{m-2} \exp(-t) dt} \right) = \log_2 \left( 1 + \left( \frac{\gamma}{n_T R_C} \right)^m (m-1) \right) \quad (19)$$

With truncated channel inversion, the capacity per unit bandwidth (bps/Hz) can expressed in terms of  $\bar{\gamma}$  and  $\gamma_0$  by substituting (1) into (17) as

$$\frac{\langle C \rangle_{\text{tifr}}}{B} = \log_2 \left( 1 + \frac{1}{\int_{\gamma_0}^\infty \frac{\gamma^{m-1} \left( \frac{n_T R_C}{\gamma} \right)^m \exp\left(-\frac{n_T R_C}{\gamma}\right)}{\gamma} d\gamma} \right) \cdot (1 - P_{\text{out}}) \quad (20)$$

$$\frac{\langle C \rangle_{\text{tifr}}}{B} = (1 - P_{\text{out}}) \log_2 (1 +$$

$$\left. \frac{\Gamma(m) \left( \frac{\bar{\gamma}}{n_T R_C} \right)^m}{\int_0^\infty \gamma^{m-2} \exp\left(-\frac{n_T R_C}{\gamma}\right) d\gamma - \int_0^{\gamma_0} \gamma^{m-2} \exp\left(-\frac{n_T R_C}{\gamma}\right) d\gamma} \right) \quad (21)$$

Making change of variables in the integral of (21), where

$$t = \frac{n_T R_C}{\gamma} \text{ and } dt = -\frac{n_T R_C}{\gamma^2} d\gamma, \text{ we have,}$$

$$\frac{\langle C \rangle_{\text{tifr}}}{B} = \log_2 \left( 1 + \frac{\Gamma(m) \left( \frac{\bar{\gamma}}{n_T R_C} \right)^m}{\int_0^\infty t^{m-2} \exp(-t) dt - \int_0^{\gamma_0} t^{m-2} \exp(-t) dt} \right) (1 - P_{\text{out}}) \quad (22)$$

$$= \frac{1}{\Gamma(m)} \log_2 \left( 1 + \frac{\Gamma(m) \left( \frac{\bar{\gamma}}{n_T R_C} \right)^m}{\Gamma(m-1) - \Lambda(m-1, \mu)} \right) [\Gamma(m) - \Lambda(m, \mu)] \quad (23)$$

where  $P_{\text{out}}$  is given by (11).

### III. NUMERICAL RESULTS

Fig. 1 show that calculated channel capacity per unit bandwidth as a function of  $\bar{\gamma}$  for various adaptation policies for a MIMO OSFBC-OFDM system. These curves are obtained using closed form expressions, (9), (15), (19) and (23). From Fig. 1, it can be observed that OPRA policy yields a significant increase in capacity as compared to ORA and Cifr policies. The spectral efficiency curves obtained using TIFR policy lie in between the curves obtained for OPRA policy and Cifr policy. The number of transmit and receive antennas used in Fig. 1 is  $n_T = 2$  and  $n_R = 2$  (or  $M = 4$ ). Fig. 2 shows channel capacity per unit bandwidth curves under high SNR region of MIMO OSFBC-OFDM system with OPRA police using (9). For OPRA policy under high SNR region, when number of receive antennas are increased from  $n_R = 2$  to  $n_R = 3$ , capacity is improved by 0.327 bps/Hz, and that from  $n_R = 2$  to  $n_R = 4$ , it is 1.134 bps/Hz, and that from  $n_R = 2$  to  $n_R = 5$ , it is 1.352 bps/Hz.

Fig. 3 shows channel capacity per unit bandwidth curves under high SNR region of MIMO OSFBC-OFDM system with ORA police using (15). For ORA policy under high SNR region, when number of receive antennas are increased from  $n_R = 2$  to  $n_R = 3$ , capacity is improved by 0.372 bps/Hz, and that from  $n_R = 2$  to  $n_R = 4$ , it is 0.631 bps/Hz, and that from  $n_R = 2$  to  $n_R = 5$ , it is 1.854 bps/Hz.

Fig. 4 shows channel capacity per unit bandwidth curves under high SNR region of MIMO OSFBC-OFDM system with Cifr police using (19). For Cifr policy under high SNR region, when number of receive antennas are increased from  $n_R = 2$  to  $n_R = 3$ , capacity is improved by 0.705 bps/Hz, and that from  $n_R = 2$  to  $n_R = 4$ , it is 1.002 bps/Hz, and that from  $n_R = 2$  to  $n_R = 5$ , it is 1.475 bps/Hz.



Fig. 5 shows channel capacity per unit bandwidth curves under high SNR region of OSFBC-OFDM system with TIFR policy using (23). For TIFR policy under high SNR region, when number of receive antennas are increased from  $n_R=2$  to  $n_R=3$ , capacity is improved by 0.703 bps/Hz, and that from  $n_R=2$  to  $n_R=4$ , it is 1.006 bps/Hz, and that from  $n_R=2$  to  $n_R=5$ , it is 1.485 bps/Hz.

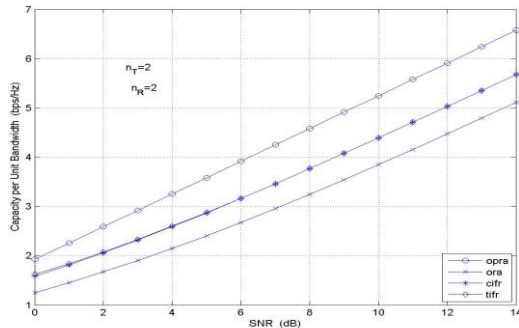


Fig. 1: Capacity per unit bandwidth of a MIMO OSFBC-OFDM system for various adaptation policies.

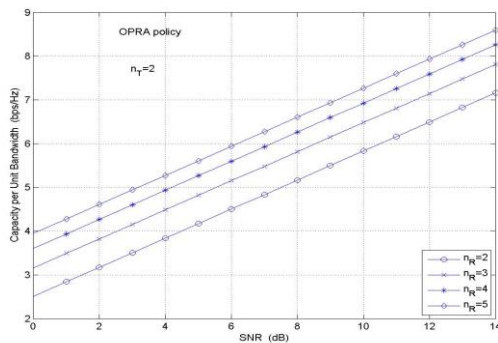


Fig. 2: Capacity per unit bandwidth of a MIMO OSFBC-OFDM system for various receive antennas under OPRA policy.

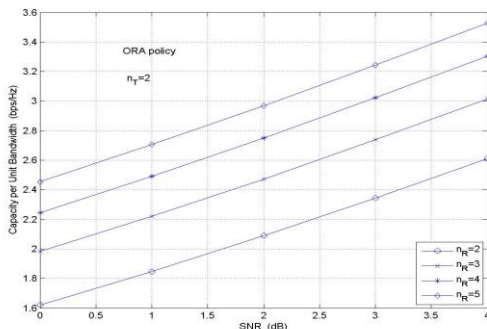


Fig. 3: Capacity per unit bandwidth of a MIMO OSFBC-OFDM system for various receive antennas under ORA policy.

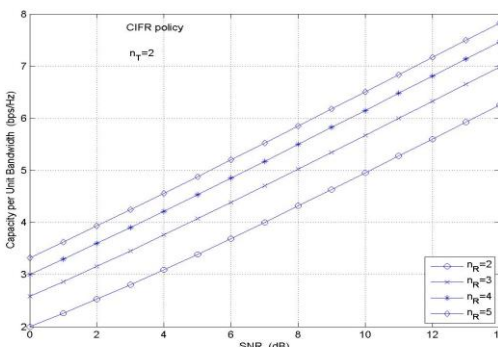


Fig. 4: Capacity per unit bandwidth of a MIMO OSFBC-OFDM system for various receive antennas under CIFR policy.

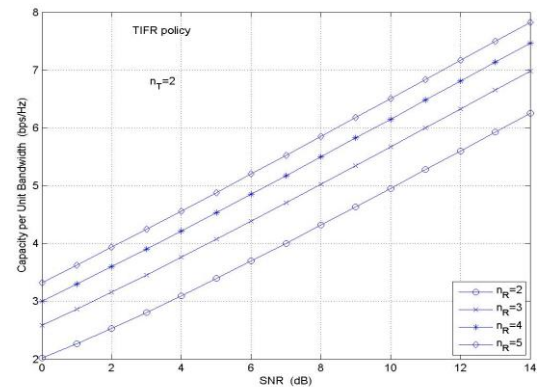


Fig. 5: Capacity per unit bandwidth of a MIMO OSFBC-OFDM system for various receive antennas under TIFR policy.

#### IV. CONCLUSIONS

Closed-form expressions for the spectrum efficiencies for the four adaptation policies are derived for an OSFBC-OFDM system. Optimal power and rate adaptation policy provides the highest capacity over the other adaptation policies. Capacity improves with an increase in the receive antennas and an increase in the average SNR. ORA policy shows the least spectrum efficiency as compared to other policies. Thus, OPRA policy is the best suited for this OSFBC-OFDM system. The transmitter has to adapt its power and rate accordingly, allocating high power levels and rates for good channel conditions ( $\gamma$  large), and lower power levels and rates for unfavorable channel conditions ( $\gamma$  small).

#### REFERENCES

1. Duman, T. M., and Ghayeb, A., *Coding for MIMO Communication Systems*, John Wiley & Sons Ltd, West Sussex, England, 2007.
2. Yang, H., "A road to future broadband wireless access: MIMO-OFDM Based air interface," *IEEE Communication Magazine*, vol. 43, no. 1, pp. 53–60, Jan. 2005.
3. Liew, T., and Hanzo, L., "Space-time trellis and space-time block coding versus adaptive modulation and coding aided OFDM for wideband channels," *IEEE Transactions on Vehicular Technology*, vol. 55, no. 1, pp. 173–187, Jan. 2006.
4. Jiang, M., and Hango, L., "Multiuser MIMO-OFDM for next generation wireless systems," *Proceedings of the IEEE*, vol. 95, no. 7, pp. 1430–1469, March 2007.
5. Niyato, D., Hossain, E., and Bhargava, V., "Scheduling and admission control in power-constrained OFDM wireless mesh routers: Analysis and optimization," *IEEE Transactions on Wireless Communications*, vol. 6, no. 10, pp. 3738–3748, Oct. 2007.
6. Chiochan, S., and Hossain, E., "Adaptive radio resource allocation in OFDMA systems: A survey of the state-of-the-art approaches," *Wireless Communications and Mobile Computing*, vol. 9, no. 4, pp. 513–527, April 2009.
7. Niyato, and D., Hossain, E., "Adaptive fair subcarrier/rate allocation in multirate OFDMA networks: Radio link level queuing performance analysis," *IEEE Transactions on Vehicular Technology*, vol. 55, no. 6, pp. 1897–1907, Nov. 2006.
8. Zhang, Y. J., and Letaief, K. B., "Multiuser adaptive subcarrier-and-bit allocation with adaptive cell selection for OFDM systems," *IEEE Transactions on Wireless Communications*, vol. 3, no. 5, pp. 1566–1575, Sep. 2004.
9. Zhang, Y. J., and Letaief, K. B., "An efficient resource-allocation scheme for spatial multiuser access in MIMO/OFDM systems," *IEEE Transactions on Communications*, vol. 53, no. 1, pp. 107–116, Jan. 2005.

10. Torabi, M., Ajib, W., and Haccoun, D., "Performance Analysis of scheduling schemes for Rate-adaptive MIMO OSFBC-OFDM Systems," *IEEE Transactions on Vehicular Technology*, vol. 54, no. 5, pp. 2363-2379, June 2010.
11. Bhaskar, V., "Spectrum Efficiency Evaluation for MRC Diversity schemes Under Different Adaptation Policies Over Generalized Rayleigh Fading channels," *International Journal of Wireless Information Networks*, vol. 14, no. 3, pp. 191-203, Sep. 2007.
12. Bhaskar, V., "Capacity evaluation for equal gain diversity scheme over Rayleigh fading channels," *International Journal of Electronics and communications*, vol. 63, no. 3, pp. 235-240, Sep. 2008.
13. Alouini, M. S., and Goldsmith, A. J., "Capacity of Rayleigh Fading Channels Under Different Adaptive Transmission and Diversity-Combining Techniques," *IEEE Transactions on Vehicular Technology*, vol. 48, no. 4, pp. 11653-1181, July 1999.
14. Gradshteyn, I and Ryzhik, I., *Table of Integrals, Series and Products*, 6<sup>th</sup> edition, Academic press, London, 2000.

## AUTHOR PROFILE



**Siva Rama Krishna** received his B.TECH. degree in Electronics and Communication Engineering from Mother Teresa Institute of Science & Technology, JNTUH University, India in 2009, M.TECH. degree in Communication Systems from S.R.M. University, Chennai in 2011. He worked as an Assistant Professor in the Department of Electronics and Communication Engineering at SVIST Engineering College, Tiruvuru, Since 2011. He is currently working as an Assistant

Professor in the Department of Electronics and Communication Engineering at Dhaneekula Institute of Engineering & Technology College, Ganguru. His research interests include wireless communications, error control coding, diversity combining and MIMO.

<sup>1</sup>Email: sivaram\_chavalam@yahoo.co.in



**Venkateswara Rao.P** received his B.TECH. degree in Electronics and Communication Engineering from Sri Sarathi Institute of Engineering & Technology, JNTUK University, India in 2010, M.TECH. degree in Systems & Signal Processing from Laki Reddy Bali Reddy college of Engineering (Autonomous) Mylavaram in 2012. He is currently working as an Assistant Professor in the Department of Electronics

and Communication Engineering at Lakireddy Bali Reddy College Of Engineering Mylavaram, India. Since December 2012. His research interests include Signal Processing, Communications.

Email: venki.mega@gmail.com