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Reacting Laminar Flow with Variable Thermal Conductivity and Suction/Injection in a Channel Filled with Saturated Porous Media

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Abstract- In this work, we examined reacting laminar flow of a third grade fluid with variable thermal conductivity and suction/ injection in a channel filled with saturated porous media. It is assumed that the fluid reacts satisfying Arrhenius law. We employed Galerkin weighted residual method to solve the resulting non-linear equation. The results show the effects of variable thermal conductivity parameter, V_1 suction/ injection parameter, Brinkman number, Reynolds number, Prandtl number, Darcy number and $Frank-Kamenetskii$ parameter on the flow system.

Keywords: Non-Newtonian fluid , Weighted residual method, Laminar flow, Suction/Injection and Arrhenius reaction.

I. INTRODUCTION

The study of heat transfer and thermal stability of reactive non-Newtonian fluids is extremely important for the safety and proper handling of materials during processing. It has given insight in the understanding dynamics of terrestrial heat flow through aquifer, hot fluid and ignition front displacements in the reservoir engineering, heat exchange between soil and atmosphere, flow of moisture through porous industrial materials, heat exchangers with fluid beds, packed-bed chemical reactors, preheating coal-water mixture, ceramic processing , polymer solution, molten plastics, oil recovery, to mention but just a few applications.

In the recent years the study of third grade fluids have received much attention than Newtonian fluids due to its practical importance, rapid development of modern industrial materials and technological applications: Hayat et al [1] considered partial slip effect on the flow and heat transfer characteristics in a third grade fluid. Fosdick and Rajagopal [2] performed a complete thermodynamic analysis of constitutive equations for the third grade fluid involving heat transfer process. Massoudi and Christie [3] analyzed numerically the flow of a third grade fluid in a pipe without heat source where the shear viscosity was assumed to be temperature dependent . Olajuwon [4] examined the flow and natural convection heat transfer in a power-law fluid past a vertical plate with heat generation. Yurusoy et al [5],Nadeem et al [6] analytically considered the effects of partial slip on a fourth grade fluid with variable viscosity and Makinde [7] employed Hermite-Pade approximations to evaluate thermal radiation effect of inherent irreversibility in a variable viscosity channel flow.

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Massoudi and Christie [8] studied the effects of variable viscosity and viscous dissipation on the flow of third grade fluid in a pipe. Nadeem et al [9], Nadeem et al [10] examined the influence of heat and mass transfer on peristaltic flow of a third order fluid in a diverging tube.

Furthermore, Truesdell and Noll [11] analyzed the non-linear field theories of mechanics. Frank-Kamenetskii theory allowed for the temperature gradient to be taken into account, i.e. there could be a considerable resistance to heat transfer in the reacting system, or the system has reactants with low thermal conductivity and highly conducting walls. Jayeoba and Okoya [12] employed analytical approximation to determine the velocity and temperature fields for steady flow of a third grade fluid in a pipe. Rilvin and Ericksen [13] analyzed stress deformation relation for isotropic materials. Szeri and Rajagopal [14] examined the effects of variable viscosity parameter and viscous dissipation parameter on the flow of a Non-Newtonian fluid between heated parallel plates. Their results show that the temperature and velocity distribution remain sensibly invariant with respect to the variable viscosity parameter. Jayeoba and Okoya [15] which employed analytical approximation to determine the velocity and temperature fields for steady flow of a third grade fluid in a pipe. Lazarus [16] studied the effects of variable viscosity on the velocity fluid and temperature fluid using semi-implicit finite difference scheme of Laminar flow in a channel filled with saturated porous media. The results show that the velocity fluid and temperature fluid increases as variable viscosity parameter increases. Haroon et al [17] examined analysis of poiseuille flow of a reactive power-law fluid between parallel plates. The results show that the shear thinning/thickening behavior depends on the power-law index and the pressure gradient. Motivated by the work of [16] we consider a reacting laminar flow with variable thermal conductivity and suction/injection in a channel filled with saturated porous media

II. GOVERNING EQUATIONS AND METHOD OF SOLUTION

Following [16] and taking k to be temperature-dependent,i.e. $k(T)$ the governing equations are continuity, momentum and energy for an unsteady fluid.

$$\rho\left(\frac{\partial u}{\partial \bar{t}}+V\frac{\partial u}{\partial \bar{y}}\right)=-\frac{\partial p}{\partial \bar{x}}+\frac{\partial}{\partial \bar{y}}\left(\bar{\mu}(T)\frac{\partial u}{\partial \bar{y}}\right)+\left(6\beta_3\left(\frac{\partial u}{\partial \bar{y}}\right)^2\frac{\partial^2 u}{\partial \bar{y}^2}\right)+\alpha_1\frac{\partial^3 u}{\partial \bar{y}^2\partial \bar{t}}-\frac{\overline{\mu_{ef}}(T)u}{\rho K} \tag{2.1}$$

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$$\rho cp\left(\frac{\partial T}{\partial t}+V\frac{\partial T}{\partial \bar{y}}\right)=\frac{\partial}{\partial \bar{y}}\left(k(T)\frac{\partial T}{\partial \bar{y}}\right)+\left(\frac{\partial u}{\partial \bar{y}}\right)^2\left(\bar{\mu}(T)+2\beta_3\left(\frac{\partial u}{\partial \bar{y}}\right)^2\right)+Q_c+\frac{\overline{\mu_{ef}}(T)u^2}{K} \tag{2.2}$$

$$Q_c=QC_0K_0(T),K_0(T)=J\left(\frac{hT}{vl}\right)^m\exp\left(-\frac{E}{RT}\right) \tag{2.3}$$

Together with the initial and boundary conditions

$$u(\bar{y},0)=0,T(\bar{y},0)=T_0 \tag{2.4}$$

$$u(0,\bar{t})=0,\ -k\frac{\partial T}{\partial y}(0,\bar{t})=h_1\left[T_a-T(0,\bar{t})\right] \tag{2.5}$$

$$u(a,\bar{t})=0,\ -k\frac{\partial T}{\partial y}(0,\bar{t})=h_2\left[T(a,\bar{t})-T_a\right] \tag{2.6}$$

where Q is the heat release per units mass, E is the activating energy, R is the universal gas constant, θ -is the dimensionless temperature, K is the permeability of the porous media, k is the thermal conductivity, ρ is the density, C_p is the specific heat at constant pressure, μ is the dynamic viscosity, $\mu\left(\frac{\partial h}{\partial r}\right)^2$ is the viscous heating effect, direction, μ_{ef} is the effective viscosity, m is the dimensionless numerical exponent, e^T is the thermal expansion, T_0 is the fluid initial temperature, T_a is the ambient temperature, T is the absolute temperature within the boundary layer, $T_1, T_2,.....T_\infty$ - Temperature at the plate, h is the Boltzmann's constant, C_0 is the initial concentration of the reactant species, a is the channel width, l is the Plank's number, h_1 is the heat transfer coefficient at lower plate, h_2 is the heat transfer coefficient at the upper plate and α_1 & β_3 are the material coefficients.

$$Q_c=QC_0K_0(T),\ K_0(T)=J\left(\frac{hT}{vl}\right)^m\exp\left(-\frac{E}{RT}\right) \tag{2.7}$$

Together with the initial and boundary conditions

$$u(\bar{y},0)=0,T(\bar{y},0)=T_0 \tag{2.8}$$

$$u(0,\bar{t})=0,\ -k\frac{\partial T}{\partial y}(0,\bar{t})=h_1\left[T_a-T(0,\bar{t})\right] \tag{2.9}$$

$$u(a,\bar{t})=0,\ -k\frac{\partial T}{\partial y}(0,\bar{t})=h_2\left[T(a,\bar{t})-T_a\right] \tag{2.10}$$

where Q is the heat release per units mass, E is the activating energy, R is the universal gas constant, θ -is the dimensionless temperature, K is the permeability of the porous media, k is the thermal conductivity, ρ is the density, C_p is the specific heat at constant pressure, μ is the dynamic viscosity, $\mu\left(\frac{\partial h}{\partial r}\right)^2$ is the viscous heating effect, direction, μ_{ef} is the effective viscosity, m is the dimensionless numerical exponent, e^T is the thermal expansion, T_0 is the fluid initial temperature, T_a is the ambient temperature, T is the absolute temperature within the boundary layer, $T_1, T_2,.....T_\infty$ - Temperature at the plate, h is the Boltzmann's constant, C_0 is the initial concentration of the reactant species, a is the channel width, l is the Plank's number, h_1 is the heat transfer coefficient at lower plate, h_2 is the heat transfer coefficient at the upper plate and α_1 & β_3 are the material coefficients we introduce the following dimensionless variables and parameters

$$\varepsilon=\frac{RT_0}{E},\theta=\frac{E(T-T_0)}{R\theta_0^{\ 2}},T=\frac{\theta RT_0^{\ 2}}{E}+T_0,T-T_0=\varepsilon T_0\theta,T=\varepsilon T_0\theta+T_0=T_0(\varepsilon\theta+1)$$

$$\frac{\bar{y}}{a}=y,\frac{\bar{u}}{u_0}=u,Da=\frac{K}{a^2},Bi_2=\frac{h_2a}{k},Ha^2=\frac{\sigma B_0^2a^2}{\mu_0},\theta_a=\frac{E(T-T_0)}{RT_0^2},\Pr=\frac{\mu_0c_p}{k}$$

,

$$t=\frac{\bar{t}\mu_0}{\rho a^2},\ \mathrm{Re}=\frac{\mu_0u_0}{\rho a^2},\bar{\mu}=\frac{\mu}{\mu_*},V_1=\frac{v\alpha\rho}{\mu_0},Bi_1=\frac{h_1a}{k},S^2=\frac{1}{Da},x=\frac{\bar{x}}{a},p=\frac{\bar{p}\rho a^2}{\mu_0^2},G=-\frac{\partial\bar{p}}{\partial\bar{x}}$$

$$\gamma=\frac{\beta_3\mu_0}{\rho^2a^4},\alpha=\frac{bRT_0^2}{E},\delta=\frac{\alpha_1}{\rho a^2},$$

$$\psi=\frac{l^mh^2T_0^{\ m-2}}{v^mh^mRK\mu_*V^2}\frac{EQC_0A}{(RT_0^2+T_0E)}e^{-E/RT_0} \tag{2.11}$$

$$\Gamma=\frac{l^mv^m\mu_0^3}{T_0^mh^m}\frac{QC_0A}{QC_0Aa^4\rho}e^{E/RT_0},$$

ψ being $Frank-Kamenetskii$ parameter for the system. Substituting (2.11) into (2.1) and (2.2),considering a steady case we obtain

$$G-S^2u+\frac{1}{\mathrm{Re}}\frac{d^2u}{dy^2}-\theta'u'+6\gamma u''(u')^2-V_1u'=0 \tag{2.12}$$

$$\frac{1}{\Pr}\frac{d^2\theta}{dy^2}+\Gamma\left[S^2u^2+(u')^2\left(1+2\gamma u'^2\right)\right]+\psi(1+\varepsilon\theta)^me^{\frac{\theta}{(1+\varepsilon\theta)}}-V_1\theta'=0 \tag{2.12}$$

Following [16] Equations (2.11) and (2.12) are to be solved subject to the boundary conditions:

$$u(0)=u(1)=0,Bi_1\theta(0)-Bi_1\theta_a,Bi_2\theta_a-Bi_2\theta(1) \tag{2.13}$$

We now proceed to solve equations (2.1) and (2.12) subject to (2.13) numerically using Galerkin-Weighted Residual Method as follows:

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let $u=\sum_{i=0}^{2}A_i e^{y}, \theta=\sum_{i=0}^{2}B_i e^{\left(-\frac{i}{4}\right)y}$ (2.14)

A maple 14 pseudo code was used to perform the iterative computation and results are presented in Figures 1 -2 as follows:

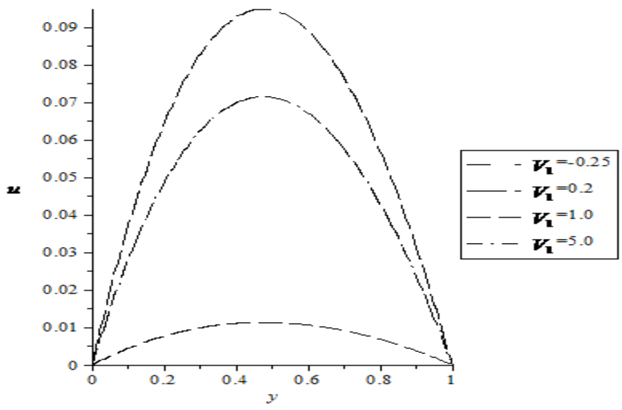

Fig.1: Graph of the velocity function u against the similarity variable y when $Br=0.5, G=2.0, S=0.01, \mathrm{Re}=0.25$

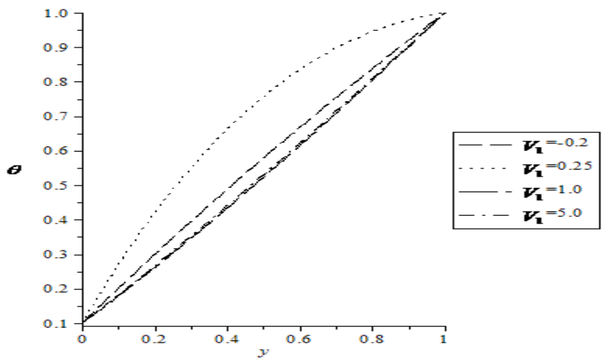

Fig.2: Graph of the temperature function θ against the similarity variable y of

when $Br=0.5, \mathrm{Re}=\mathrm{Pr}=1.0, \varepsilon\geq 0, \psi\geq 0, \gamma, m\geq 0$

From equation (2.12) we seek variable thermal conductivity $k(T)$ of the form

$$k(T)=k_0 e^{-\gamma_1\theta} \quad (2.15)$$

Substituting (2.15) into (2.12) we obtain

$$G-S^2u+\frac{1}{\mathrm{Re}}\frac{d^2u}{dy^2}-\theta' u'+6\gamma u''(u')^2-V_1u'=0 \quad (2.16)$$

$$\frac{1}{\mathrm{Pr}}\frac{d}{dy}\left(e^{-\gamma_1\theta}\theta'\right)+\Gamma\left[S^2u^2+(u')^2\left(1+2\gamma u'^2\right)\right]+\psi(1+\varepsilon\theta)^m e^{\frac{\theta}{(1+\varepsilon\theta)}}-V_1\theta'=0 \quad (2.17)$$

Equations (2.16) and (2.17) are to be solved subject to the boundary conditions:

$u(0)=u(1)=0, Bi_1\theta(0)-Bi_1\theta_a, Bi_2\theta_a-Bi_2\theta(1)$ (2.18)

We now proceed to solve equations (2.16) and (2.17) subject to (2.18) numerically using Galerkin-Weighted Residual Method as follows:

let $u=\sum_{i=0}^{2}A_i e^{y}, \theta=\sum_{i=0}^{2}B_i e^{\left(-\frac{i}{4}\right)y}$ `(2.19)

A maple 14 pseudo code was used to perform the iterative computation and results are presented in Figures 3 -4 as follows:

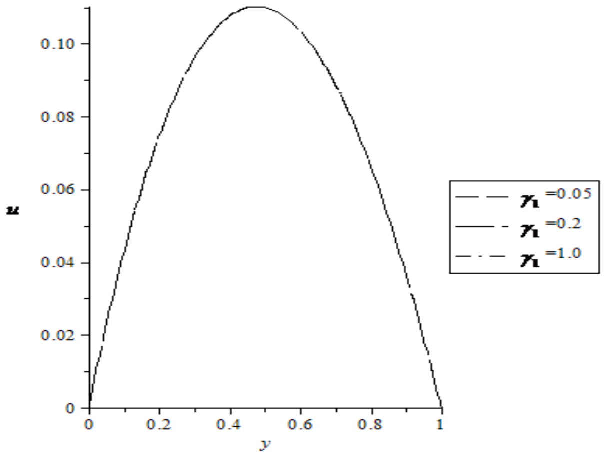

Fig.3: Graph of the velocity function u against the similarity variable y when

$\Gamma=0.5, m\geq 0.1, S=0.5, G=-3.0, \mathrm{Re}=0.25\ \alpha$

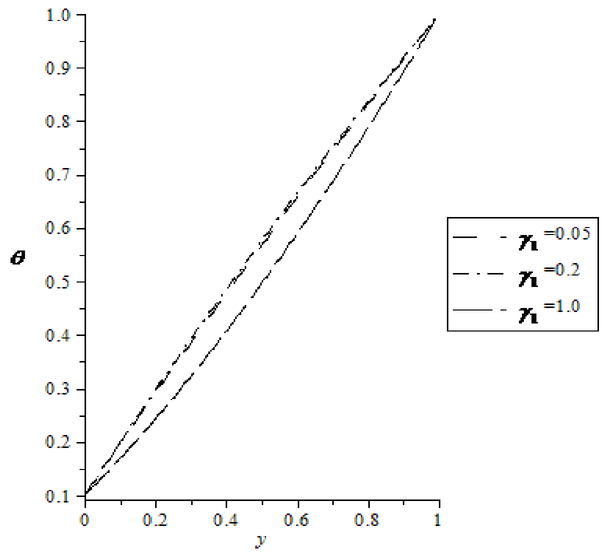

Fig.4: Graph of the temperature function θ against the similarity variable y when

$\Gamma=0.5, \psi=0.25, \mathrm{Pr}=0.75, \mathrm{Re}=0.2, \gamma=0.01, m=3.0, \varepsilon\geq 0, Bi_1=0.5, Bi_2=1.0, \theta_a=1.2, S=0.01.$

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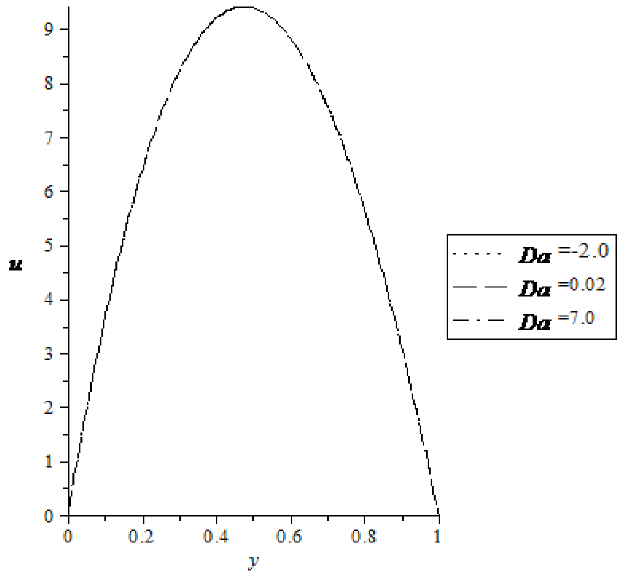

Fig.5: Graph of the velocity function u against the similarity variable y when

$$\Gamma = 0.5, m \geq 0.1, S = 0.5, \gamma = 1.2, G = -3.0, \mathrm{Re} = 0.25$$

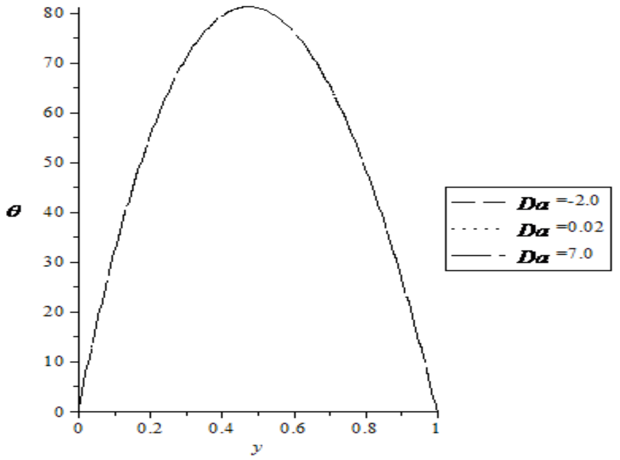

Fig.6: Graph of the temperature function θ against the similarity variable y when

$$\Gamma = 0.5, \psi = 0.25, \Pr = 0.75, \mathrm{Re} = 0.2, \gamma = 0.01, m = 3.0, \varepsilon \geq 0, Bi_1 = 0.5, Bi_2 = 1.0, \theta_a = 1.2, S = 0.01.$$

3.0 Discussion of Results/Conclusion

From Figs.1 and 3 the results show that the velocity profile decreases for an increase in the values of γ_1, V_1 variable thermal conductivity and suction/injection parameters with increase in each of $\varepsilon \geq 0; G, S, \Pr, \mathrm{Re}, Br, \ \gamma, m, \delta$ parameters . From Figs.2 and 4 the results show that the temperature increases as γ_1, V_1 variable thermal conductivity and suction/injection parameters increases.

CONCLUSION

A comprehensive set of graphical results for velocity profile and temperature profile are discussed. It is observed that the temperature fluid increases as variable thermal conductivity parameter, suction/injection and $Frank-Kamenetskii$ parameter increases. We observed that there is a transient decrease in fluid velocity fluid with an increase in the fluid variable thermal conductivity parameter (which decreases the viscosity). A transient increase in the fluid temperature is observed with increase in γ_1 variable thermal conductivity parameter, V_1 suction/injection parameter, Λ non-Newtonian parameter, Br Brinkman number and Darcy number which decreases the porosity in the flow.

For engineering purpose, the flow model of our problem represents the oils well and as the γ_1 variable thermal conductivity parameter and V_1 suction/injection parameter is increasing the is viscosity of oil in the reservoir is being reduced. Also, the results of this problem are of great interest in production processing, for the safety of life and proper handling of the materials during processing.

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