# Non-Newtonian Fluid Flow with Arrhenius Reaction between Heated Parallel Plates through a Porous Medium

**A.W. Ogun Sola, B.A. Peter**

***Abstract- The flow of a fluid of grade three between heated parallel plates is examined. It is assumed that the fluid is temperature-dependent and reacts satisfying Arrhenius law. We employed Galerkin weighted residual method to solve the resulting non-linear equations. The results show the effects of*** $Br$ ***Brinkman number, which is the parameter that controls the viscous dissipation,*** $\Lambda$ ***which is the non-Newtonian parameter. The result shows that the velocity of the flow decreases as Brinkman number,*** $M_1$ ***and*** $\Lambda$ ***increases. The temperature of the flow decreases with increase in*** $Frank-Kamenetskii$ ***parameter. We also deduce from the result that*** $Frank-Kamenetskii$ ***parameter has considerable effects on the temperature profile of the system.***

***Keywords: Non-Newtonian fluid, Weighted residual method, Third grade fluid and Arrhenius reaction.***

## I. INTRODUCTION

The study of heat transfer and thermal stability of reactive non-Newtonian fluids is extremely important for the safety and proper handling of materials during processing. Non-Newtonian fluids have received much attention than Newtonian fluids in the recent years due to its practical importance, rapid development of modern industrial materials and technological applications. It has given insight in the understanding dynamics of terrestrial heat flow through aquifer, hot fluid and ignition front displacements in the reservoir engineering, heat exchange between soil and atmosphere, flow of moisture through porous industrial materials, heat exchangers with fluid beds, fiber coating and granular insulation materials, packed-bed chemical reactors, preheating coal-water mixture, ceramic processing ,catalytic reactors, polymer solution, molten plastics, oil recovery, to mention but just a few applications. Heat transfer problem of third grade fluids without heat source has been studied by several authors: Hayat et al [1] considered partial slip effect on the flow and heat transfer characteristics in a third grade fluid. Fosdick and Rajagopal [2] performed a complete thermodynamic analysis of constitutive equations for the third grade fluid involving heat transfer process. Massoudi and Christie [3] analyzed numerically the flow of a third grade fluid in a pipe without heat source where the shear viscosity was assumed to be temperature dependent . Olajuwon [4] examined the flow and natural convection heat transfer in a power-law fluid past a vertical plate with heat generation.

**Manuscript published on 30 April 2014.**
***Correspondence Author(s)**
A.W. Ogun Sola, Molipa Road, I-ODE, Ogun, Nigeria
B.A. Peter, Molipa Road, I-ODE, Ogun, Nigeria

© The Authors. Published by Blue Eyes Intelligence Engineering and Sciences Publication (BEIESP). This is an open access article under the CC-BY-NC-ND license http://creativecommons.org/licenses/by-nc-nd/4.0/

Yurusoy et al [5] examined entropy analysis for third grade fluid flow with Vogel's models of viscosity in annular pipe. Nadeem et al [6] analytically considered the effects of partial slip on a fourth grade fluid with variable viscosity and Makinde [7] employed Hermite-Pade approximations to evaluate thermal radiation effect of inherent irreversibility in a variable viscosity channel flow. Massoudi and Christie [8] studied the effects of variable viscosity and viscous dissipation on the flow of third grade fluid in a pipe. Nadeem et al [9] studied analytical solutions for pipe flow of a fourth grade fluid with Reynolds and Vogel's models of viscosities. Nadeem et al [10] examined the influence of heat and mass transfer on peristaltic flow of a third order fluid in a diverging tube. Furthermore, Truesdell and Noll [11] analyzed the non-linear field theories of mechanics. Frank-Kamenetskii theory allowed for the temperature gradient to be taken into account, i.e. there could be a considerable resistance to heat transfer in the reacting system, or the system has reactants with low thermal conductivity and highly conducting walls. Jayeoba and Okoya [14] employed approximate analytical solutions for pipe flow of a third grade fluid with models of viscosities and heat generation/absorption. Rilvin and Ericksen [12] analyzed stress deformation relation for isotropic materials. Motivated by the work of Szeri and Rajagopal [13] which examined the effects of variable viscosity parameter and viscous dissipation parameter on the flow of a Non-Newtonian fluid between heated parallel plates. Their results show that the temperature and velocity distribution remain sensibly invariant with respect to the variable viscosity parameter. Lazarus [15] studied the effects of variable viscosity on the velocity fluid and temperature fluid using semi-implicit finite difference scheme of Laminar flow in a channel filled with saturated porous media. The results show that the velocity fluid and temperature fluid increases as variable viscosity parameter increases. Haroon et al [16] examined analysis of poiseuille flow of a reactive power-law fluid between parallel plates. The results show that the shear thinning/thickening behavior depends on the power-law index and the pressure gradient. There are manifestations of fluid behavior which cannot be adequately explained on the basis of the classical, linearly viscous model. Several constitutive equations have been suggested to characterize such non-Newtonian behavior. In this work, we considered a fully developed, steady flow, reacting flow of an incompressible, reactive Power-law fluid, homogeneous fluid of fluid of third grade, thermal conductivity which is an exponential function of the fluid mean temperature and the fluid reacts according to Arrhenius law between two heated parallel plates, due to an external pressure gradient along the plates.


*Published By:
Blue Eyes Intelligence Engineering and Sciences Publication (BEIESP)*
*© Copyright: All rights reserved.*

The plates are located in the $z=-h$ and $z=h$ planes, respectively, of an orthogonal Cartesian coordinate system.

Following Szeri and Rajagopal [13] an incompressible, homogeneous fluid of third grade is characterized by Cauchy stress $\tau$ of the following form:

$$\tau=-pI+\mu(T)A_1+\alpha_1(T)A_2+\alpha_2(T)A_1^2 + \beta_1(T)A_3+\beta_2(T)[A_1A_2+A_2A_1]+\beta_3(T)(trA_1^2)A_1. \quad (1)$$

where $-pI$ denote the indeterminate part of the stress due to the constraint of incompressibility $\mu(T)$ is the coefficient of viscosity and $\alpha_1(T),\alpha_2(T)$ are material moduli, usually referred to as normal stress coefficients. The kinematic tensors $A_1$, $A_2$ are defined by [2] through

$$A_1=(grad\,v)+(grad\,v)^\tau \quad (2)$$

$$A_n=\frac{d}{dt}A_{n-1}+A_{n-1}(grad\,v)+(grad\,v)^T A_{n-1} \qquad n=2,3. \quad (3)$$

Here $\frac{d}{dt}$ denotes material time derivative and $v$ is the velocity vector. The above model contains, as a special subclass, the classical linearly viscous model (the case when all the coefficients expect $\mu$ are set equal to zero).

The thermodynamics and stability of model (1) has been studied in detail [13]. The thermodynamic compatibility in the sense that all motions of the fluid meet the Clausius-Duhem inequality, which is generally interpreted as a statement of the second law of thermodynamics, and the assumption that the specific Helmholtz free energy of the fluid be a minimum when the fluid is in "equilibrium", places restrictions on the structure of the constitutive equations which model the fluid. It has been shown (Theorem 2 [3]) that the response functions $\Psi,\tau\, and\ q$ for specific Helmholtz free energy, the stress and the heat flux, respectively, of an incompressible, homogeneous fluid of third grade are compatible with thermodynamics only if

(i) the viscosity $\mu(T)$ is non-negative, $\mu(T)\ge 0,$ (4)

(ii) the normal stress coefficients $\alpha_1(T) and\ \alpha_2(T)$ meet the requirements, $\alpha_1(T)\ge 0$

$$-\sqrt{24\mu(T)\beta_3(T)}\le\alpha_1(T)+\alpha_2(T)<\sqrt{24\mu(T)\beta_3(T)} \quad (5)$$

(iii) the material coefficients $\beta_1(T),\beta_2(T) and\ \beta_3(T)$ satisfy $\beta_1(T)=0,\beta_2(T)=0,\beta_3(T)\ge 0$ (6)

(iv) the specific Helmholtz free energy $\Psi$ has the form

$$\Psi=\hat{\Psi}(T,L)=\hat{\Psi}(T,0)+\frac{\alpha_1(T)}{4\rho}\left|L+L^T\right|^2 \quad (7)$$

In the above expressions

$$L=grad\,v, \quad (8)$$

$\rho$ denotes the density and $|A|$ denotes the trace norm of $A$.

In our analysis we assume that the fluid is thermodynamically compatible ; hence the stress constitutive relation (1) reduces to

$$\tau=-pI+\mu(T)A_1+\alpha_1(T)A_2+\alpha_2(T)A_1^2+\beta(T)(trA_1^2)A_1. \quad (9)$$

## II. GOVERNING EQUATIONS AND METHOD OF SOLUTION

Following [14] and neglecting porosity term the governing equations are conservation of mass, conservation of momentum and conservation of energy for an incompressible fluid. For the problem under consideration, flow of a thermodynamically compatible fluid of third grade between heated plates at $z=-h\ \ and\ \ z=h,$ the lower plate is stationary and the upper plate is moving with a constant speed $U,$ respectively we seek velocity fields of the form :

$$v=u(y)\,i \quad (10)$$

where $i$ denotes the unit vector in the $x$ coordinate direction, the direction that is chosen parallel the external pressure gradients.

In the absence of body forces, the balance of linear momentum

$$div\,\tau+\rho b-\frac{\mu_{ef}v}{K}=\rho\frac{dv}{dt}$$

(Equation of Motion) (11)

where $K$ is the porous medium permeability

reduces to

$$\frac{d}{dy}\left[\mu(T)\frac{du}{dy}\right]+2\frac{d}{dy}\left[\beta(T)\left(\frac{du}{dy}\right)^3\right]-\frac{\mu_{ef}v}{K}=\frac{\partial\hat{p}}{\partial x} \quad (12)$$

$$\frac{d}{dy}\left\{[2\alpha_1(T)+\alpha_2(T)]\left(\frac{du}{dy}\right)^3\right\}=\frac{\partial p}{\partial y} \quad (13)$$

$$0=\frac{\partial p}{\partial z} \quad 14)$$

under the condition (10). In deriving the above, we made use of the fact that the fluid is incompressible and is hence constrained to satisfy

$$div\,v=0. \quad (15)$$

Defining the modified pressure

$$\hat{p}=p-[2\alpha_1(\theta)+\alpha_2(T)]\left(\frac{du}{dy}\right)^2 \quad (16)$$

equation (12) takes the simpler form


Published By:
Blue Eyes Intelligence Engineering
and Sciences Publication (BEIESP)
© Copyright: All rights reserved.

$$\frac{d}{dy}\left[\mu(T)\frac{du}{dy}\right]+2\frac{d}{dy}\left[\beta(T)\left(\frac{du}{dy}\right)^3\right]-\frac{\mu_{ef}v}{K}=\frac{\partial\hat{p}}{\partial x} \tag{17}$$

as $\hat{p}=\hat{p}(x)$ only. (18)

Having found the equation of motion (17), neglecting the reaction term [14] presented the energy equation as follows:

$$\rho\frac{de}{dt}=\tau.L-div\,q+\frac{\mu_{ef}v^2}{K}+\rho r+Q(T) \tag{19}$$

Subject to the boundary conditions

$$u(-h,t)=0,u(h,t)=U \tag{20}$$

$$T(0,t)=T_b,T(h,t)=T_w$$

Here $e$ denotes the internal energy, $L$ is the velocity gradient, $r$ is the radiant energy, both per unit mass and $Q(T)$ denotes the reacting term.

It follows from (8) and (9) that

$$\tau.L=\tfrac{1}{2}\mu(T)|A_1|^2+\tfrac{1}{4}\alpha_1(T)\frac{d}{dt}|A_1|^2+\frac{\alpha_1(T)+\alpha_{21}(T)}{2}tr\,A_1^3+\tfrac{1}{2}\beta_3(T)|A_1|^4 \tag{21}$$

We assume that the heat flux vector $q$ is given by Fourier's law

$$q=-k\,grad\,T \tag{22}$$

where the minus sign is introduced to account for the fact that the heat is conducted from high temperature to a low temperature, so that $(grad\,T)$ inherently negative; therefore the double negative indicates a positive flow of heat in the direction of decreasing temperature, $k=k(T)$ denotes the thermal conductivity. We now turn our attention to the contribution by internal energy of the energy equation. Since the internal energy is related to the specific Helmholtz free energy through

$$e=\Psi+T\eta, \tag{23}$$

where $\eta$ is the entropy, it follows that

$$e=\hat{\Psi}(T,0)+\frac{\alpha_1(T)}{4\rho}|A_1|^2+T\eta \tag{24}$$

If the fluid is thermodynamically compatible, then the specific energy is related to the specific Helmholtz free energy through [3]

$$\eta=-\Psi_T, \tag{25}$$

where the suffix denotes differentiation with respect to that variable. It follows (24) and (25) that

$$e=\hat{\Psi}(T,0)+\frac{\alpha_1(T)}{4\rho}|A_1|^2-T\,\hat{\Psi}_T \tag{26}$$

We now employ two schemes when dealing with equation (19)

(1) In the first instance we allow

$$T=T(x,y) \tag{27}$$

but keep the material properties of the fluid constant. Substituting (26) into (19) under this restriction, we find on neglecting the radiant heating

$$\mu\left(\frac{du}{dy}\right)^2+2\beta_3\left(\frac{du}{dy}\right)^4+\frac{\mu_{ef}v^2}{K}+k\left(\frac{\partial^2T}{\partial x^2}+\frac{\partial^2T}{\partial y^2}\right)+\rho\left\{T\frac{\partial^2\Psi}{\partial T^2}\right\}\frac{\partial T}{\partial x}u+Qc_0Ae^{\frac{-E}{RT}}=0 \tag{28}$$

If we now neglect heat conduction along the flow in comparison with heat conduction normal to the plates and put

$$-T\Psi_{TT}(T,A_1)=C(T,A_1)$$

where $C>0$ is the specific heat of the material, we obtain the appropriate form of the energy equation

$$\mu\left(\frac{du}{dy}\right)^2+2\beta_3\left(\frac{du}{dy}\right)^4+\frac{\mu_{ef}v^2}{K}+k\left(\frac{\partial^2T}{\partial y^2}\right)+Qc_0Ae^{\frac{-E}{RT}}=\rho C\frac{\partial T}{\partial x}u \tag{29}$$

This is a partial differential equation in $T(x,y)$, but can be reduced to an ordinary differential equation via a similarity transformation that is readily available under the assumption of constant heat flux at the walls.

Let $T_w=T_w(x)$ denote the temperature at the walls, i.e. $T(x,0)=T(x,h)=T_w(x)$, and let $T_b(x)$ represent the fluid bulk temperature, defined by

$$T_b(x)=\frac{\int_0^h CuTdy}{\int_0^h Cudy} \tag{30}$$

then the similarity transformation that reduces (29) to an ordinary differential equation is

$$T(x,y)=\bar{T}(\bar{y})[T_b(x)-T_w(x)]+T_w(x)\ . \tag{31}$$

Here $\bar{T}(\bar{y})$ is a similar temperature. The non-dimensional coordinates $\bar{y}$ and other non-dimensional quantities are defined through

$$\bar{y}=\frac{y}{h},\ \bar{u}=\frac{u}{V},Da=\frac{K}{Vh^2},$$

$$\Lambda=\frac{\beta_3V^2}{\mu h^2},\quad Br=\frac{\mu V^2}{k(T_b-T_w)} \tag{32}$$

and the characteristic velocity $V$ is given by

$$V=-\frac{h^2}{2\mu}\frac{d\hat{p}}{dx} \tag{33}$$

Substituting (32) into (29) we obtain



Published By:
Blue Eyes Intelligence Engineering
and Sciences Publication (BEIESP)
© Copyright: All rights reserved.

$$\overline{T}'' + Br\left(\overline{u}'\right)^2\left[1+2\Lambda\left(\overline{u}'\right)^2\right] + \frac{u^2}{Da} = \frac{h^2V}{\alpha\left(T_b - T_w\right)}\overline{u}\left[\overline{T}\left(\frac{dT_b}{dx} - \frac{dT_w}{dx}\right) + \frac{dT_w}{dx}\right] \quad (34)$$

If now the heat flux at the walls

$$q_w = \lambda\left(T_w - T_b\right) \quad (35)$$

is a constant and if the film heat transfer coefficient $\lambda$ is independent of $x$

$$\overline{T}'' + Br\left(\overline{u}'\right)^2\left[1+2\Lambda\left(\overline{u}'\right)^2\right] + C_1\overline{u} + \frac{u^2}{Da} = 0; C_1 = \frac{h^2V}{\alpha\left(T_b - T_w\right)}\frac{dT_w}{dx} \quad (36)$$

The appropriate form of the equation of motion, obtained by substituting (32) into (17),is

$$\overline{u}'' + 6\Lambda\left(\overline{u}'\right)^2\overline{u}'' + 2 - \frac{1}{Da} = 0 \quad (37)$$

Following [14,17] equations (36) and (37) are to be solved subject to the boundary conditions:

$$\overline{u}(-h) = \overline{T}(0) = 0, \overline{u}(1) = 1, \overline{T}(1) = 1, \text{let } h = 1, U = 1 \quad (38)$$

$$\theta(0) = 0, \ \theta(1) = 0 \quad (39)$$

to obtain solutions at constant heat flux and constant fluid properties.

We now proceed to solve equations (36) and (37) subject to (38) and (39) numerically using Galerkin-Weighted Residual Method as follows:

$$\text{let } u = \sum_{i=0}^{2} A_i e^{y}, \theta = \sum_{i=0}^{2} B_i e^{\left(-i/4\right)y} \quad (40)$$

A maple 14 pseudo code was used to perform the iterative computation and results are presented in Figures 1 and 2 as follows:

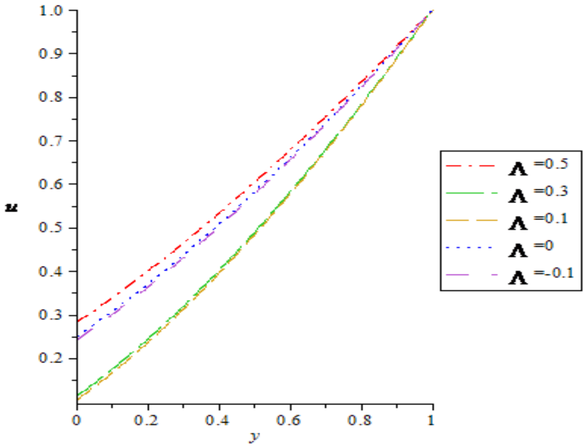


Fig.1: Graph of the velocity function $u$ against the similarity variable $y$ when $Br = 0.5, Da = 0.5$

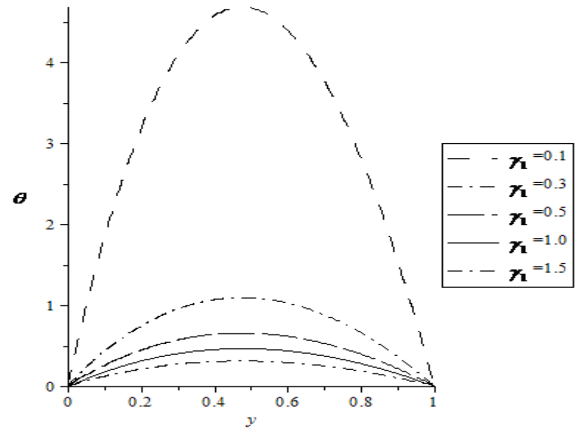


Fig.2: Graph of the temperature function $\theta$ against the similarity variable $y$ of $\Lambda = 0.5; \ Br = Da = 1.5, \varepsilon = 0; 0.25, C = 0.1, \psi > 0$.

(11) In the second instance we allow the material properties of the fluid to be temperature dependent but require the temperature to satisfy the constraint

$$T = T(y) \quad (41)$$

Then by virtue of (26) and (41)

$$\frac{de}{dt} = 0 \quad (42)$$

substituting (22) and (25) into (19), in the absence of radial heating with the inclusion of the reaction term we considered the energy equation in the following form

$$\mu\left(\frac{du}{dy}\right)^2 + 2\beta_3\left(\frac{du}{dy}\right)^4 + k\left(\frac{\partial^2 T}{\partial y^2}\right) + \frac{u^2}{Da} + Qc_0Ae^{\frac{-E}{RT}} = 0 \quad (43)$$

$$\mu\left(\frac{du}{dy}\right)^2 + 2\beta_3\left(\frac{du}{dy}\right)^4 + k\left(\frac{\partial^2 T}{\partial y^2}\right) + \frac{u^2}{Da} + QC_0Ae^{-E/RT_0}e^{\frac{E\left(T-T_0/TT_0\right)}{R}} = 0 \quad (44)$$

where $Q$ - Heat release per units mass, $E$ -Activating energy, $R$ -Universal gas constant.

To non-dimensionalize (44) we introduce the following variables and parameters

$$\varepsilon = \frac{RT_0}{E}, \theta = \frac{E\left(T - T_0\right)}{R\theta_0^2}, T = \frac{\theta RT_0^2}{E} + T_0, T - T_0 = \varepsilon T_0\theta, T = \varepsilon T_0\theta + T_0 = T_0\left(\varepsilon\theta + 1\right)$$

$$\overline{y} = \frac{y}{h}, \ \overline{u} = \frac{u}{V}, \theta = \frac{T - T_1}{T_2 - T_1},$$

$$\Lambda = \frac{\beta_* V^2}{\mu_* h^2}, \ \ Br = \frac{\mu_* V^2}{k\left(T_2 - T_1\right)}, \overline{\mu} = \frac{\mu}{\mu_*} \quad (45)$$

as contained in [14,17]

Here $\mu_* = \mu\left(\theta_*\right); \beta_* = \beta_3\left(\theta_*\right)$ and the characteristic velocity $V$ is given in (33), but with


Published By:
Blue Eyes Intelligence Engineering
and Sciences Publication (BEIESP)
© Copyright: All rights reserved.

$\mu_*$ replacing $\mu, \theta_1 = \theta(0)$ and $\theta_2 = \theta(h)$ are the wall temperatures at the lower and upper plate respectively. Substituting (45) into (44) dropping the bars we obtain

$$\left(\frac{d\theta}{dy}\right)^2 + \frac{u^2}{Da} + Br(u')^2\left[1+2Br\Lambda\left(\frac{du}{dy}\right)^2\right] + \frac{\psi}{Br}e^{\frac{\theta}{(1+\varepsilon\theta)}} = 0 \quad (46)$$

where $Br = \frac{k(T_2 - T_1)}{\mu_* V^2}, \psi = \frac{EQC_0 A}{(RT_0^2 + T_0 E)} e^{-E/RT_0}, \psi$ being $Frank - Kamenetskii$ parameter for the system. From equation (25) we seek variable thermal conductivity $k(T)$ of the form

$$k(T) = k_0 e^{-\gamma\theta} \quad (47)$$

$$\frac{d}{dy}\left(k_0 e^{-\gamma_1\theta}\frac{d\theta}{dy}\right) + \frac{u^2}{Da} + Br(u')^2\left[1+2Br\Lambda\left(\frac{du}{dy}\right)^2\right] + \frac{\psi}{Br}e^{\frac{\theta}{(1+\varepsilon\theta)}} = 0 \quad (48)$$

Equation (46) is to be solved subject to the dimensionless boundary conditions:

$$\theta(0) = 0, \ \theta(1) = 0 \quad (49)$$

We now proceed to solve equations (38) and (46) subject to (39) and (48) numerically using Galerkin-Weighted Residual Method as follows:

$$\text{let } u = \sum_{i=0}^{2} A_i e^{(-i/5)y}, \theta = \sum_{i=0}^{2} B_i e^{(-i/4)y} \quad (50)$$

A maple 14 pseudo code was used to perform the iterative computation and results are presented in Figures 3 and 4 as follows:

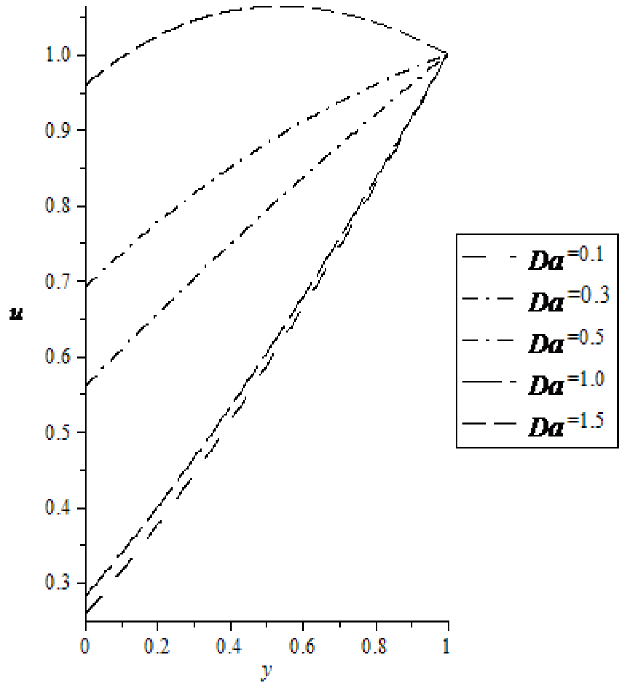


Fig.3: Graph of the velocity function $u$ against the similarity variable $y$ of when $\Lambda = 0.5, C = -0.1, Br = 0.1, \psi > 0$

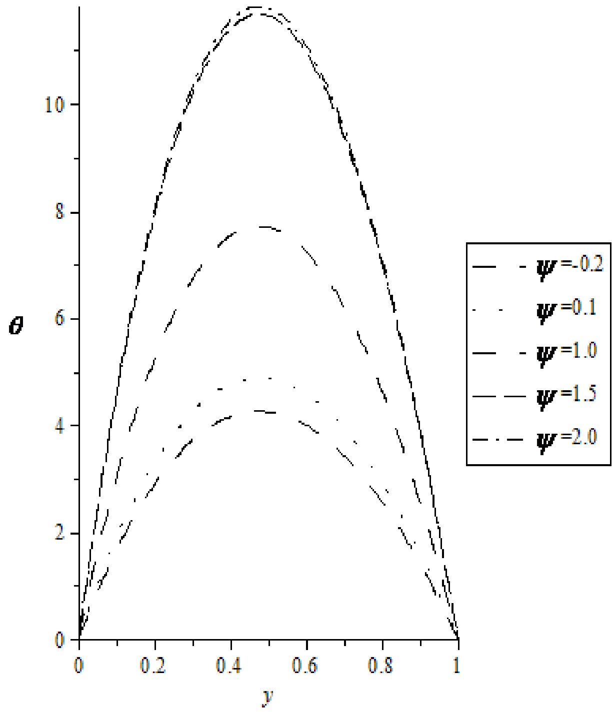


Fig.4: Graph of the temperature function $\theta$ against the similarity variable $y$ of $\Lambda = 0.5; Br = Da = 1.5, \varepsilon = 0; 0.25, C = 0.1$

## III. DISCUSSION OF RESULTS/CONCLUSION

The study of heat transfer and reactive non-Newtonian fluids is extremely important due to its wide variety of practical applications in processes such as filtration of polymer solutions and soil remediation through the removal of liquid pollutants. From Fig.1 the result shows that the velocity increases as non-Newtonian parameter increases. It is noticed from Figure 2 that the temperature profile decreases as variable thermal conductivity parameter increases. We observed from Figure 3 that the velocity profile increases as Darcy parameter increases. We noticed from Fig.4 that as $\psi$ $Frank - Kamenetskii$ parameter increases the temperature profile decreases.

## IV. CONCLUSION

A comprehensive set of graphical results for velocity profile and temperature profile are discussed. It is observed that velocity fluid and the temperature fluid decreases as $Frank - Kamenetskii$ parameter decreases. A transient increase in both the fluid velocity and temperature is observed with increase in $\Lambda$ non-Newtonian parameter, $Br$ Brinkman number and Darcy number which decreases the porosity in the system of flow.

For engineering purpose, the flow model of our problem represents the oils well and as the $\psi$ $Frank - Kamenetskii$ parameter is increasing there is quick recovery of oil from the oils well. Also, the results of this problem are of great interest in production processing, for the safety of life and proper handling of the materials during processing.


Published By:
Blue Eyes Intelligence Engineering
and Sciences Publication (BEIESP)
© Copyright: All rights reserved.

## REFERENCE


1. T. Hayat, M.A. Faroog, T.Javed and M. Sajid, Partial slip effects on the flow and heat transfer characteristics in third grade, Non-linear Analysis, Real World Applications 10725- 755,2009.
2. R.L. Fosdick and K.R. Rajagopal, Thermodynamics and stability of fluids of third grade , Proc. R. Soc. Lond. A 369 351-377,1980.
3. M. Massoudi and I. Christe, Heat transfer and flow of third grade fluid in a pipe, Math. Modeling Sci. Comput. 2 1273-1275,1993.
4. B.I. Olajuwon, Flow and natural convection heat transfer in a power-law fluid past a vertical plate with heat generation, Int. J. Non-Linear Science, 7 (1) 50-56, 2009.
5. M.Yurusoy, H.Bayrakoekem, M. Kapucu and F. Aksoy, Entropy analysis for third grade fluid flow with Vogel's models of viscosity in annular pipe, Int. J. Non-Linear Mech.43 588-599, 2008
6. S. Nadeem and M. Ali, Analytical solutions for pipe flow of a fourth grade fluid with Reynolds and Vogel's models of viscosities, Communications in Non-Linear Sci. and Numerical Simulation, 14 (5) 2073-2090,2009.
7. O.D. Makinde, Hermite-Pade approach to thermal radiation effect on inherent irreversibility in a variable viscosity channel flow, Computers and Mathematics with Applications 58 2330-2338,2009.
8. M. Massoudi and I. Christe, Effects of variable viscosity and viscous dissipation on the flow of third grade fluid in a pipe, Int. J. Non-Linear Mech.30(5) 687-699,1995.
9. S. Nadeem, T. Hayat ,N.S. Akbar and M.Y. Malik, on the influence of heat transfer in peristalsis with variable viscosity, Int. J. Heat and Mass Transfer 52 4722-4730,2009.
10. S. Nadeem, N.S. Akbar a, N. Bibi and S. Ashiq, Influence of heat and mass transfer on peristaltic flow of third order fluid in a diverging tube, Commun. Non-Linear Sci. Numeric. Simulat. 11 856-868 ,2010.
11. C. Truesdell and W. Noll, The non-linear field theories of mechanics. In Handbuch der Physik (Edited by Flugge), Vol.111/3, Springer, Berlin, 1965.
12. Rivlin, R.S. and Erickson, J.I., Stress deformation relation for isotropic materials. J rat. Mech. Analysis, Vol.4,pp 323-425, 1995.
13. A.Z. Szeri and K.R. Rajagopal, Flow of a Non-Newtonian fluid between heated parallel plates. Int. J. Non-Linear Mechanics, Vol.20.No. 2. 91-101, 1985.
14. O.J. Jayeoba and S.S. Okoya, Approximate analytical solutions for pipe flow of a third grade fluid with models of viscosities and heat generation/absorption, J. of Nigerian Mathematical Society 31 207-228, 2012.
15. Lazarus Rundora, Laminar flow in a channel filled with saturated porous media. A Ph.D. Thesis submitted to the Dept. of Mechanical Engineering, Cape Peninsula University of Technology,2013 .
16. T. Haroon, A.R. Ansari, A.M. Sidiqui and S.U. Jan, Analysis of poiseuille flow of a reactive power-law fluid between parallel plates, J. of Applied Mathematical Sciences,Vol.5, no.55,2721-2746, 2011.



*Published By:*
*Blue Eyes Intelligence Engineering*
*and Sciences Publication (BEIESP)*
*© Copyright: All rights reserved.*