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Order Reduction using Modified Pole Clustering and Factor Division Method

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Abstract*— *The authors proposed a mixed technique for reducing the order of the high order dynamic systems. In this technique, the denominator polynomial of the reduced order model is determined by using the modified pole clustering while the coefficients of the numerator are obtained by Factor Division Method. This technique is simple and gives stable reduced models for the stable high-order system. C.B. Vishwakarma, modified pole clustering technique is suggested, which generates the more effective cluster .If a cluster contains r number of poles, then IDM criterion is repeated r times with the most dominant pole available in that cluster. The Factor division algorithm has been successfully used to find reduced order approximants of high order systems. The proposed method is described by solving a numerical example taken from the literature.

Index Terms*—*Modified pole clustering, Order reduction, Factor Division, Transfer function

I. INTRODUCTION

In many practical situations the model of the original system is a fairly complex and is of high order. The complexity often makes understanding of the behavior of the system difficult if not impossible. The analysis of most of the high order systems is both tedious as well as costly; it poses a great challenge to both the system analyst and control engineer. The design and optimization of such systems becomes relatively easy if a low order linear equation can be derived which provides an acceptable approximation to the system. Several order reduction techniques for linear dynamic systems in the frequency domain are available the literature [1-4]. Further, some methods have also been suggested by combining the features of two different methods [5-7]. In the clustering technique suggested by Sinha and Pal [8], the poles and zeroes are separately grouped to form clusters and then these clusters are replaced by their cluster centres by using an inverse distance measure (IDM) criterion. In the literature [9], only poles are grouped together to generate cluster centres and then denominator polynomial of the cluster centres and then denominator polynomial of the reduced model is synthesized from these cluster centres, C.B. Vishwakarma[10] a modified pole clustering technique is suggested, which generates the more effective cluster centres than cluster centres obtained by authors [8, 9]. If a cluster contains *r* number of poles, then IDM criterion is repeated *r* times with the most dominant pole available in that cluster.

The Factor division algorithm [11] can be used to determine the reduced numerator. The Factor division algorithm has been successfully used to find reduced order approximants of high order systems. It avoids finding time moments and solving the Pade equation, whilst the reduced models still retains the initial time moments of full systems.

In the proposed method, the denominator polynomial of the reduced order model is obtained by using modified Cauer form while the coefficients of the numerator are obtained by Factor division method. The proposed method is compared with the other well-known order reduction techniques available in the literature.

II. STATEMENT OF THE PROBLEM

Let the transfer function of high order original system of the order ‘ n ’ be

$$G_n(s) = \frac{N(s)}{D(s)} = \frac{c_0+c_1s+c_2s^2+\cdots\cdots+c_{n-1}s^{n-1}}{d_0+d_1s+d_2s^2+\cdots\cdots+d_ns^n} \tag{1}$$

Where c_i (i varies 0 to n-1) and d_j (j varies 0 to n) are constants.

$$R_k(s) = \frac{N_k(s)}{D_k(s)} = \frac{e_0+e_1s+e_2s^2+\cdots\cdots+e_{k-1}s^{k-1}}{f_0+f_1s+f_2s^2+\cdots\cdots+f_ks^k} \tag{2}$$

where e_i (i varies 0 to k-1) and f_j (j varies 0 to k) are unknown scalar constants.

The objective of this paper is to realize the k^{th} -order reduced model in the form of (2) from the original system (1) such that it retains the important features of the original high -order system.

III. REDUCTION METHOD

The reduction procedure for getting the k^{th} -order reduced models consist of the following two steps:

Step-1: Determination of the denominator polynomial for the k^{th} -order reduced model using modified pole clustering:

The following rules are used for clustering the poles of the original system given in frequency domain:

- Separate clusters should be made for real and complex poles.
- Clusters of the poles in the left half s -plane should not contain any pole of the right half s -plane and vice-versa.
- Poles on the jw -axis have to be retained in the reduced order model.
- Poles at the origin have to be retained in the reduced order model.

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The brief algorithm for realizing the denominator polynomial by using the modified pole clustering is as follows:
Let there be r real poles in i[th] cluster are
$p_1, p_1, p_1, \ldots\ldots\ldots\ldots p_r,$
where $|p_1| < |p_2| < |p_3| \ldots\ldots\ldots\ldots < |p_r|,$ and then modified cluster centre p_{ei} can be obtained by using the algorithm of modified pole clustering suggested in this paper.
Let m pair of complex conjugate poles in the j^{th} cluster be $[(\alpha_1 \pm j\beta_1), (\alpha_2 \pm j\beta_2) \ldots\ldots\ldots\ldots (\alpha_m \pm j\beta_m)]$ where

$|\alpha_1| < |\alpha_2| < |\alpha_3| \ldots\ldots\ldots\ldots < |\alpha_m|,$

Now using the same algorithm separately for real and imaginary parts of the complex conjugate poles, the modified cluster centre is obtained, which is written as

$\Phi_{ej} = A_{ej} \pm jB_{ej}$
Where
$\acute{\Phi}_{ej} = A_{ej} + jB_{ej}$ and $\grave{\Phi}_{ej} = A_{ej} - jB_{ej}$

An interactive computer oriented algorithm [13] has been developed, which automatically finds the modified cluster centre and is give as follows:

Step-1 Let r real poles in a cluster be

$|p_1| < |p_2| < |p_3| \ldots\ldots\ldots\ldots < |p_r|,$

Step-2 Set $j = 1$.
Step-3 Find pole cluster centre

$$C_j = \left[\sum_{i=1}^{r}\left(\frac{-1}{|p_i|}\right) \div r\right]^{-1}$$

Step-4 Set $j = j+1$.
Step-5 Now find a modified cluster centre from

$$C_j = \left[\left(\frac{-1}{|p_i|} + \frac{-1}{|c_{j-1}|}\right) \div 2\right]^{-1}$$

Step-6 Is $r = j$? , if No, and then go to *step-4* otherwise go to *step-7*.
Step-7 Take a modified cluster centre of the k[th] –cluster as $p_{ek} = c_j$
For synthesizing the k[th] -order denominator polynomial, one of the following cases may occur
Case-1
If all modified cluster centres are real, then denominator polynomial of the k[th] -order reduced model can be obtained as

$$D_k(s) = (s - p_{e1})(s - p_{e2}) \ldots\ldots\ldots (s - p_{ek}) \quad (3)$$

Where $p_{e1}, p_{e2} \ldots\ldots\ldots\ldots\ldots\ldots p_{ek}$ are 1[st].2[nd],......k[th] modified cluster centre respectively.
Case-2:
If all modified cluster centres are complex conjugate then denominator polynomial of the k[th] -order reduced model can be obtained as

$$D_k(s) = (s - \acute{\Phi}_{e1})(s - \grave{\Phi}_{e1}) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (s - \acute{\Phi}_{e\frac{k}{2}})(s - \grave{\Phi}_{e\frac{k}{2}}) \quad (4)$$

Case-3
If some cluster centre are real and some are complex conjugate. For example (k-2) cluster centres are real and one pair of cluster centre is complex conjugate, then k[th] –order denominator can be obtained as

$$D_k(s) = (s - p_{e1})(s - p_{e2}) \ldots\ldots (s - p_{e(k-2)})(s - \acute{\Phi}_{e1})(s - \grave{\Phi}_{e1}) \quad (5)$$

hence, the denominator polynomial $D_k(s)$ is obtained as

$$D_k(s) = f_0 + f_1 s + \ldots\ldots + f_k s^k \quad (6)$$

Step-2
Determination of the numerator of k[th] order reduced model using Factor Division algorithm [14]
After obtaining the reduced denominator, the numerator of the reduced model is determined as follows

$$\overline{N(s)} = \frac{N(s)}{D(s)} \times D_k(s) = \frac{N(s)}{D(s)/D_k(s)}$$

Where $D_k(s)$ is reduced order denominator

There are two approaches for determining of numerator of reduced order model.

(i) By performing the product of N8(s) and Dk(s) as the first row of factor division algorithm and D8(s) as the second row up to sk-1 terms are needed in both rows.
(ii) By expressing N8(s)Dk(s)/D8(s) as N8(s)/[D8(s)/Dk(s)] and using factor division algorithm twice; the first time to find the term up to sk-1 in the expansion of D8(s)/Dk(s)(i.e. put D8(s) in the first row and $D_k(s)$ in the second row, using only terms up to s^{k-1}), and second time with $N_8(s)$ in the first row and the expansion $[D_8(s)/D_k(s)]$ in the second row.

Therefore the numerator $N_k(s)$ of the reduced order model ($R_k(s)$) in eq.(2) will be the series expansion of

$$\frac{N_8(s)}{\dfrac{D_8(s)}{D_k(s)}} = \frac{\sum_{i=0}^{k-1} c_i s^i}{\sum_{i=0}^{k-1} d_i s^i}$$

About s=0 up to term of order s^{k-1}.
This is easily obtained by modifing the moment generating[14].which uses the familiar routh recurrence formulae to generate the third, fifth, and seventh etc rows as,

$$\begin{array}{lcllll}
\alpha_0 = \dfrac{d_0}{f_0} & < & \dfrac{d_0}{f_0} & \dfrac{d_1}{f_1} & \dfrac{d_2}{f_2} \;.\; \ldots\ldots\ldots\ldots & \dfrac{d_{k-1}}{f_{k-1}} \\
\alpha_1 = \dfrac{g_0}{f_0} & < & \dfrac{g_0}{f_0} & \dfrac{g_1}{f_1} & \dfrac{g_2}{f_2} \;\ldots\ldots\ldots\ldots & \dfrac{g_{k-2}}{f_{k-2}} \\
\alpha_2 = \dfrac{l_0}{f_0} & < & \dfrac{l_0}{f_0} & \dfrac{l_1}{f_1} & \dfrac{l_2}{f_2} \;\ldots\ldots\ldots\ldots & \dfrac{l_{k-3}}{f_{k-3}} \\
\ldots\ldots\ldots\ldots & & & & & \\
\ldots\ldots\ldots\ldots & & & & & \\
\ldots\ldots\ldots\ldots & & & & & \\
\alpha_{k-2} = \dfrac{p_0}{f_0} & < & \dfrac{p_0}{f_0} & \dfrac{p_1}{f_1} & & \\
\alpha_{k-1} = \dfrac{q_0}{f_0} & < & \dfrac{q_0}{f_0} & & &
\end{array}$$

Where
$g_i = d_{i+1} - \alpha_0 * f_{i+1}$ i=0,1,2,....................
$l_i = g_{i+1} - \alpha_1 * f_{i+1}$ i=0,1,2......................
....
....
$q_0 = p_1 - \alpha_{k-2} * f_1$

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Therefore, the numerator $N_m(s)$ of eq.(2) is given by $N_k(s)=\sum_{i=0}^{k-1} \alpha_i s^i$

IV. METHOD FOR COMPARISON

In order to check the accuracy of the proposed method the relative integral square error ISE index in between the transient parts of the reduced models and the original system is calculated using Matlab/Simulink. The relative integral square error RISE is defined as

$$ISE=\int_0^{\infty}[y(t)-y_k(t)]^2\,dt$$

V. NUMERICAL EXAMPLES

The proposed method explains by considering numerical example, taken from the literature. The goodness of the proposed method is measured by calculating integral square error (ISE) between the transient parts of the original and reduced model using MATLAB. The ISE should be minimum for better approximation i.e close the $R_k(s)$ to $G_n(s)$, which is given by

$ISE=\int_0^{\infty}|y(t)-y_k(t)|^2dt$

Where, y(t) and $y_k(t)$ are the unit step responses of original and reduced system respectively.

Example :- Consider an eight-order system from the literature

$$G_8(s)=\frac{N_8(s)}{D_8(s)}$$

Where

$N_8(s)=40320+185760s+222088s^2+122664s^3+36380s^4+5982s^5+514s^6+18s^7$

And

$D_8(s)=40320+109584s+118124s^2+67284s^3+22449s^4+4536s^5+546s^6+36s^7+s^8$

The poles are: -1,-2,-3,-4,-5,-6,-7,-8

Let the 2nd -order reduced model is required to be realized, for this purpose only two real clusters are required.

Let the first and second cluster consists the poles (-1, -2, -3, -4) and (-5, -6, -7, -8) respectively. The modified cluster centres are computed as

$P_{e1}=-1.06371$

$P_{e2}=-5.13271$

Using eqn (3), the denominator polynomial $D_2(s)$ is obtained as

$D_2(s)=5.45971+6.19642s+s^2$

Using the eqns (8) to (10), the following coefficients of numerator $N_2(s)$ of reduced order model are calculated

Consider $D_8(s)/D_2(s)$ gives

$\alpha_0=7385.0076 <$ $\begin{matrix} 40320 & 109584 \\ 5.45971 & 6.19642 \end{matrix}$

$\alpha_1=11689.8867 <$ $\begin{matrix} 63823.39 \\ 5.45971 \end{matrix}$

Now Considering $N_8(s)/D_8(s)/D_2(s)$

$\alpha_0=5.45971 <$ $\begin{matrix} 40320 & 185760 \\ 7385.0076 & 11689.8867 \end{matrix}$

$\alpha_1=16.51137 <$ $\begin{matrix} 121936.6086 \\ 7385.0076 \end{matrix}$

Thus Reduced Numerator is given as

$N_2(s)=5.45971+16.51137s$

Thus the Reduced model is given as

$$R_2(s)=\frac{5.45971+16.51137s}{5.45971+6.19642s+s^2}$$

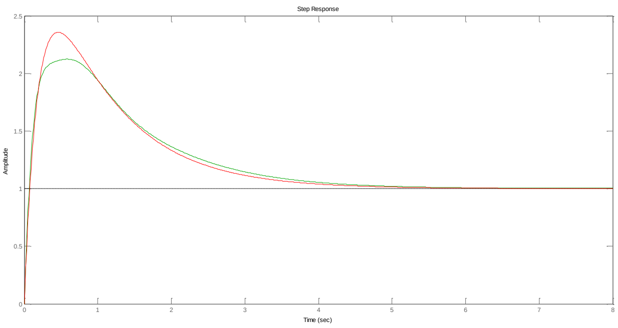

Figure1. Step Response Comparison between original system and reduced system

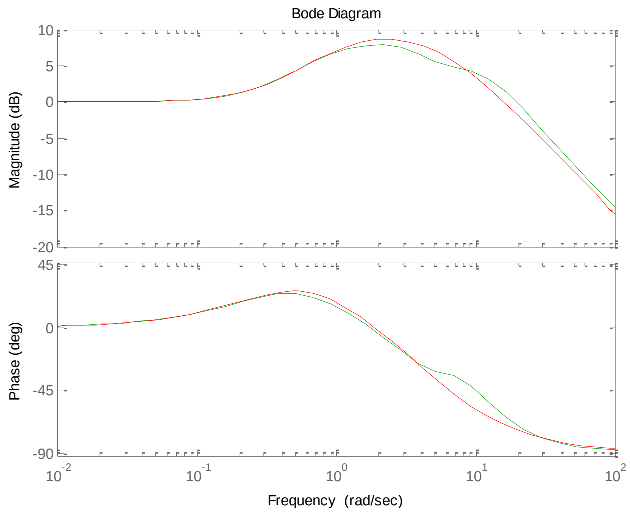

Figure 2. Bode Plots of original system and reduced system

COMPARISON OF THE REDUCTION METHODS

Original System

RiseTime: 0.0597
SettlingTime: 4.3580
SettlingMin: 0.9619
SettlingMax: 2.3563
Overshoot: 135.6268
Undershoot: 0
Peak: 2.3563
PeakTime: 0.4599

Reduced System

RiseTime: 0.0528
SettlingTime: 4.8905
SettlingMin: 0.9354
SettlingMax: 2.1236
Overshoot: 112.3589
Undershoot: 0
Peak: 2.1236
PeakTime: 0.5858

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VI. CONCLUSIONS

The authors proposed an order reduction method for the linear single-input-single-output high order systems. The determination of denominator polynomial of the reduced model is done by using the modified cluster method while the numerator coefficients are computed by factor division method. The merits of proposed method are stable, simplicity, efficient and computer oriented. The proposed algorithm has been explained with an example taken from the literature. The step responses and Bode plots of the original and reduced system of second order are shown in the Figure-1 and Figure-2 respectively. A comparison of proposed method with the other well known order reduction methods in the literature is shown in the Table-I from which we can conclude that proposed method is comparable in quality.

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