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Large Sliding Frictional Contact Problems by a Penalty Based Approach

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Abstract— ***The finite element method is used to model large sliding frictional contact problems in which one body slides over another body and there is relative motion at the interface of the two bodies. The penalty approach has been used within finite element frame work to consider various constraints at the contact surface. The large sliding behavior between two bodies has been formulated by employing the node-to-segment (NTS) algorithm. The NTS technique searches for the active slave nodes and the corresponding master segments at the contact surface and then the contact stiffness matrix is evaluated to enforce the contact conditions. Finally, three problems are solved to demonstrate the applicability of the given technique in solving large sliding fictional contact problems. The given problems have been solved for both the frictionless contact as well as the frictional contact at the interface of the two bodies.***

Key words—FEM, Penalty Factor, Node-to-Segment Technique, Large Sliding.

I. INTRODUCTION

The modeling and simulation of solid mechanics problems, involving frictional contacts between two surfaces, is very important in computational solid mechanics. Large sliding contact problems include those problems in which one body slides over the surface of another body and there is relative motion at the interface of the bodies. The node-to-segment algorithm is the most widely used scheme for solving the contact problems involving large sliding at the interface [1, 2]. If the relative displacements between two contacting surfaces are negligibly small, the node-to-node technique is used [3, 4, 5]. However, such node-to-node algorithms cannot be employed to solve large sliding problems and we need to rely on node-to-segment (NTS) algorithms. Several other discretization techniques are available for modeling large sliding contact problems but the NTS technique is the most simple and flexible technique for solving this class of problems. The NTS technique determines the slave nodes and the master nodes located at the contact surface of the two mechanically contacting bodies.

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This technique searches for the active slave nodes and the corresponding master segments at the contact surface and then the contact stiffness matrix is evaluated to enforce the contact conditions. The two most commonly methods employed to enforce the contact constraints are the penalty approach [6] and the Lagrange multiplier method [7].

However, the penalty approach has been used here because it is simple and easy to understand. So far, several approaches have been used to model and simulate large sliding problems. Nistor *et.al* developed an efficient XFEM technique to model large sliding occurring at the interface [8]. Zavarise and Lorenzis developed the node-to-segment algorithm for 2D frictionless contact between two bodies [9]. A mortar based finite element formulation was employed by McDevitt and Laursen to solve the frictional contact between two solid bodies [10]. Chehel Amirani and Nemati coupled the conventional finite element method and the element free Galerkin method to simulate the contact behavior between two bodies [11]. Al-Dojayli and Meguid employed cubic splines to model the contact behavior between two bodies [12]. Khoei and Taheri Mousavi employed the node to segment algorithm within the XFEM framework to model frictionless large sliding occurring at the interface of the two contacting bodies [13].

The present paper aims at developing an efficient model to simulate large sliding between two solid bodies. The node-to-segment (NTS) algorithm has been used within the conventional FEM framework to formulate the large sliding behavior between the contacting bodies. The imposition of the contact constraints is based on the penalty approach. Finally, three 2D problems are solved to illustrate the applicability and efficiency of the given approach in modeling large sliding problems. The first problem models the large sliding behavior with horizontal contact surfaces between two bodies. The second problem investigates the large sliding behavior with inclined contact surfaces between two bodies. The last example models the frictional contact behavior during the sliding of a block on a frictional surface.

II. THE FINITE ELEMENT METHOD

The modeling and simulation of frictional contact problems in finite element method can be achieved by creating a mesh that conforms to the contact surface geometry, as shown in Fig. 1. Consider a contact surface between two bodies with $\boldsymbol{e_n}$ denoting the normal vector to the contact surface. The displacement and traction are then introduced on the contact surface by $\mathbf{u}$ and $\mathbf{t}$. In order to obtain an appropriate form that is suitable for the numerical treatment of contact behavior, the equilibrium equation can be written as

$$\int_{\Omega} \boldsymbol{\sigma}:\boldsymbol{\varepsilon}\, d\Omega - \int_{\Omega} \mathbf{b}:\mathbf{u}\, d\Omega - \int_{\Gamma_t} \mathbf{t}:\mathbf{u}\, d\Gamma_t = 0 \quad (1)$$

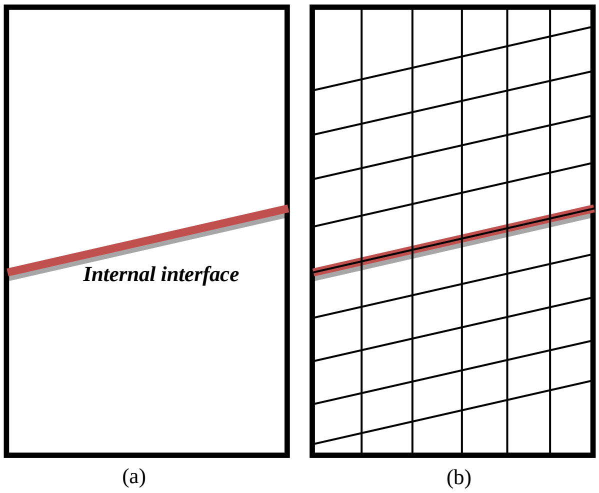


Fig. 1: Modeling of internal interface between two bodies (a) Problem definition (b) FEM mesh which conforms to the geometry of the interface

It is important that the displacement field of the domain and the displacement on the contact surface be kinematically admissible. The goal is to obtain the stress and displacement fields on the contact surface which satisfy the equilibrium and compatibility conditions. The displacement field is given by

$$\boldsymbol{u}^h(\boldsymbol{x}) = \sum_i N_i(\boldsymbol{x}) u_i \; for \;\; n_i \epsilon\, \boldsymbol{n} \quad (2)$$

In this equation, u_i represents the nodal displacement, $N(\boldsymbol{x})$ is the standard finite element shape function and $\boldsymbol{n}$ is the set of the nodal points of the domain. In matrix form, Eq. (2) can be written as

$$\begin{Bmatrix} u \\ v \end{Bmatrix} = \begin{bmatrix} N_i & 0 \\ 0 & N_i \end{bmatrix} \begin{Bmatrix} u_i \\ v_i \end{Bmatrix} = \boldsymbol{Nu} \quad (3)$$

where, $\mathbf{u} = [\mathrm{u_i} \;\; \mathrm{v_i}]^T$ is a vector of nodal degrees of freedom and $\boldsymbol{N} = \begin{bmatrix} N_i & 0 \\ 0 & N_i \end{bmatrix}$. The strain tensor can be written as

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_{xy} \end{Bmatrix} = \begin{Bmatrix} \frac{\partial u}{\partial x} \\ \frac{\partial v}{\partial y} \\ \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \end{Bmatrix} = \begin{bmatrix} \frac{\partial N_i}{\partial x} & 0 \\ 0 & \frac{\partial N_i}{\partial y} \\ \frac{\partial N_i}{\partial y} & \frac{\partial N_i}{\partial x} \end{bmatrix} \begin{Bmatrix} \mathrm{u}_i \\ \mathrm{v}_i \end{Bmatrix} = \mathbf{Bu} \quad (4)$$

where

$$\mathbf{B} = \begin{bmatrix} \frac{\partial N_i}{\partial x} & 0 \\ 0 & \frac{\partial N_i}{\partial y} \\ \frac{\partial N_i}{\partial y} & \frac{\partial N_i}{\partial x} \end{bmatrix} \quad (5)$$

Substitution of the trial function of Eq. (2) into the equilibrium equation (1) gives the discrete system of equations as

$$[\mathbf{K}]\{\mathbf{u}\} = \mathbf{f} \quad (6)$$

where

$$\mathbf{K} = \int_{\Omega} \mathbf{B}^T \mathbf{D} \mathbf{B}\, d\Omega \quad (7)$$

$$\mathbf{f} = \int_{\Gamma} \mathbf{N t}\, d\Gamma + \int_{\Omega} \mathbf{N b}\, d\Omega \quad (8)$$

where, $\mathbf{u}$ is the vector of unknown nodal displacements and $\mathbf{D}$ is the constitutive matrix relating stresses and strains.

III. FORMULATION OF LARGE SLIDING BY FEM

The large sliding problems can be modeled in FEM by creating a mesh that conforms to the contact surface geometry. The non-penetration condition has to be satisfied during the modeling of contact problems. The non-penetration constraint does not allow one contacting body to penetrate into the other body. We also need to enforce the constraints in the tangential direction of the contact surface. Various models have been developed from time to time for the proper enforcement of the contact constraints. Out of these approaches, the penalty method and the Lagrange multiplier technique are most commonly used techniques. However, the penalty approach has been used in this study for the imposition of contact constraints.

The node-to-segment (NTS) approach is a simple and most commonly used approach in modeling and simulating large sliding contact problems. Suppose any slave point $\boldsymbol{S}$ of a particular slave element with coordinate $\boldsymbol{x}^s$ makes a contact with the master segment $m_1 - m_2$ of any master element defined by the nodal coordinates $\boldsymbol{x}_1^m$ and $\boldsymbol{x}_2^m$, as shown in Fig. 2. If we take ξ as the coordinate along the master segment, the coordinates of the projection of the slave point $\boldsymbol{S}$ on the master segment are obtained as

$$\boldsymbol{x}^m(\xi) = \boldsymbol{x}_1^m + (\boldsymbol{x}_2^m - \boldsymbol{x}_1^m)\xi \quad (9)$$

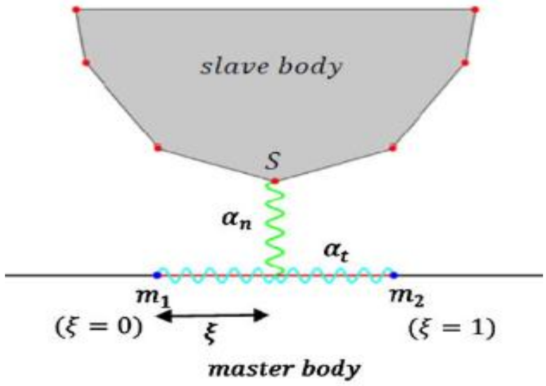


Fig. 2: Modeling of contact constraints by penalty method

The minimum distance (gap) between the slave node and the associated master segment is given as

$$\boldsymbol{g}_N = [\boldsymbol{x}^s - \boldsymbol{x}^m].\boldsymbol{e_n} \quad (10)$$

where $\boldsymbol{g}_N$ is the gap and $\boldsymbol{e_n}$ is the unit normal to the master surface. Using Eq. (10), we have

$$\boldsymbol{g}_N = [\boldsymbol{x}^s - (1-\xi)\boldsymbol{x}_1^m - \xi \boldsymbol{x}_2^m].\boldsymbol{e_n} \quad (11)$$

Thus, the shape functions for the slave-master can be written as

$$\boldsymbol{N}_{sm} = \begin{bmatrix} 1 & 0 & -(1-\xi) & 0 & -\xi & 0 \\ 0 & 1 & 0 & -(1-\xi) & 0 & -\xi \end{bmatrix} \quad (12)$$

The relative displacements at the interface of the two bodies can be obtained as $\boldsymbol{N}_{sm}\mathbf{u}$, with $\mathbf{u}$ denotes the displacements at the nodal points. Then, the relative displacements in the normal and tangential directions are given by

$$\mathbf{u}_N = (\boldsymbol{e_n} \otimes \boldsymbol{e_n}) \boldsymbol{N}_{sm} \mathbf{u} \quad (13)$$

$$\mathbf{u}_T = (\boldsymbol{e_t} \otimes \boldsymbol{e_t}) \boldsymbol{N}_{sm} \mathbf{u} \quad (14)$$

The normal and the tangential displacements can further be written as

$$\mathbf{u}_N = \boldsymbol{N_n}\mathbf{u} \quad (15)$$

$$\mathbf{u}_T = \boldsymbol{N_t}\mathbf{u} \quad (16)$$

where $\boldsymbol{N_n}$ and $\boldsymbol{N_t}$ are the normal and the tangential shape functions and are defined as

$$\boldsymbol{N_n} = (\boldsymbol{e_n} \otimes \boldsymbol{e_n})\boldsymbol{N}_{sm} \quad (17)$$

$$\boldsymbol{N_t} = (\boldsymbol{e_t} \otimes \boldsymbol{e_t})\,\boldsymbol{N}_{sm} \quad (18)$$

As mentioned earlier, the contact constraints need to be imposed at the contact interface of the two contacting bodies. The penalty method imposes the contact constraints by embedding springs in normal and tangential direction of the contact surface. This is shown schematically in Fig. 2.The contact constraints are applied by introducing the potential energy of the springs imposed in the normal direction and the tangential direction of the contact surface. Now, the total potential energy obtained at the contact surface can be expressed as

$$\pi = \frac{1}{2}\alpha_n \mathrm{u}_N^2 + \frac{1}{2}\alpha_t \mathrm{u}_T^2 \quad (19)$$

$$\pi = \frac{1}{2}\mathbf{u}_N^T \alpha_n \mathbf{u}_N + \frac{1}{2}\mathbf{u}_T^T \alpha_t \mathbf{u}_T \quad (20)$$

$$\pi = \frac{1}{2}\mathbf{u}^T \boldsymbol{N_n^T} \alpha_n \boldsymbol{N_n}\mathbf{u} + \frac{1}{2}\mathbf{u}^T \boldsymbol{N_t^T} \alpha_t \boldsymbol{N_t}\mathbf{u} \quad (21)$$

where α_n denotes the normal penalty parameter and α_t represents the tangential penalty parameter of the contact surface. The tangential penalty parameter α_t is zero for a frictionless contact problem. Eq. (21) can be further written as

$$\pi = \frac{1}{2}\mathbf{u}^T \boldsymbol{K_n^c}\mathbf{u} + \frac{1}{2}\mathbf{u}^T \boldsymbol{K_t^c}\mathbf{u} \quad (22)$$

where $\boldsymbol{K_n^c}$ represents the normal stiffness matrix and $\boldsymbol{K_t^c}$ denotes the tangential stiffness matrix. We have

$$\boldsymbol{K_n^c} = \boldsymbol{N_n^T}\alpha_n \boldsymbol{N_n} \quad (23)$$

$$\boldsymbol{K_t^c} = \boldsymbol{N_t^T}\alpha_t \boldsymbol{N_t} \quad (24)$$

The contact conditions are enforced by adding the normal stiffness matrix $\boldsymbol{K_n^c}$ and the tangential stiffness matrix $\boldsymbol{K_t^c}$ to the whole stiffness of the system. However, in case of frictionless sliding between the two contact surfaces, α_t is zero and hence only $\boldsymbol{K_n^c}$ is included. Thus, the finite element model for large sliding frictional contact problem can be written as

$$[\mathbf{K}]\{\mathbf{u}\} = \{\mathbf{f}\} \quad (25)$$

where

$$\boldsymbol{K} = \boldsymbol{K}_{mat} + \boldsymbol{K_n^c} + \boldsymbol{K_t^c} \quad (26)$$

$$\boldsymbol{K}_{mat} = \int_\Omega \boldsymbol{B}^T \boldsymbol{D}\,\boldsymbol{B}\, d\Omega \quad (27)$$

$$\boldsymbol{K_n^c} = \boldsymbol{N_n^T}\alpha_n \boldsymbol{N_n} \quad (28)$$

$$\boldsymbol{K_t^c} = \boldsymbol{N_t^T}\alpha_t \boldsymbol{N_t} \quad (29)$$

$$\boldsymbol{f} = \int_\Gamma \boldsymbol{N}\boldsymbol{t}\, d\Gamma + \int_\Omega \boldsymbol{N}\boldsymbol{b}\, d\Omega \quad (30)$$

In the above relation, the matrix $\boldsymbol{N}$ contains the finite element shape functions. The matrices $\boldsymbol{N}$ and $\mathbf{B}$ are defined as

$$\boldsymbol{N} = \begin{bmatrix} N_i & 0 \\ 0 & N_i \end{bmatrix} \quad (31)$$

$$\boldsymbol{B} = \begin{bmatrix} \frac{\partial N_i}{\partial x} & 0 \\ 0 & \frac{\partial N_i}{\partial y} \\ \frac{\partial N_i}{\partial y} & \frac{\partial N_i}{\partial x} \end{bmatrix} \quad (32)$$

However, it should be kept in mind that the tangential load P_T cannot be greater than the maximum frictional strength of the material. The maximum frictional strength is given by the Coulomb's law as $F_{max} = \mu_f \|\boldsymbol{P}_N\| + C_f$, where C_f represents the cohesion between the two bodies, $\boldsymbol{P}_N$ is the normal load and μ_f is the coefficient of friction. If the tangential load P_T is greater than the frictional strength F_{max}, then we have to update the values of the tangential load and the tangential penalty parameter as

$$\boldsymbol{P}_T = F_{max}\frac{\boldsymbol{u}_T}{\|\boldsymbol{u}_T\|} \quad \text{and} \quad \alpha_t = \frac{F_{max}}{\|\boldsymbol{u}_T\|} \quad (33)$$

IV. NUMERICAL RESULTS AND DISCUSSIONS

Now, we consider several numerical problems that were solved to check the applicability and accuracy of the given technique in solving large sliding contact problems in which one body slides over the surface of another body. Three 2D problems are modeled and simulated using FEM. The problems have been solved both by assuming frictionless contact at the interface as well as frictional contact at the interface of the two bodies. The first problem models and simulates large sliding of a rectangular block between two horizontal surfaces. The second problem models and simulates the large sliding of a rectangular block between two inclined surfaces. The last example investigates the behavior of a rectangular block sliding over a frictional surface.

A. Large Sliding with Horizontal Contact Surfaces

This problem models and simulates the contact behavior of a rectangular block sliding between two horizontal surfaces by employing FEM. A horizontal displacement of 2m is applied to the middle block, as shown in Fig. 3. A vertical displacement of 1mm is applied to the top and the bottom edges. The Young's modulus of $1 \times 10^8\ N/m^2$ has been assumed for the blocks. The Poisson ratio is assumed to be 0.3. The normal and tangential penalty parameters of the contact surface are assumed to be $1 \times 10^{10}\ N/m^2$ and $1 \times 10^5\ \mathrm{N/m^2}$, respectively. The coefficient of friction has been chosen to be 0.3. A uniform mesh of 144 elements has been considered in FEM simulation. The deformed configurations at various stages of the simulation are shown in Fig. 4. The stress distributions obtained by FEM for both frictionless as well as frictional sliding are shown in Fig. 5, Fig. 6, and Fig. 7.

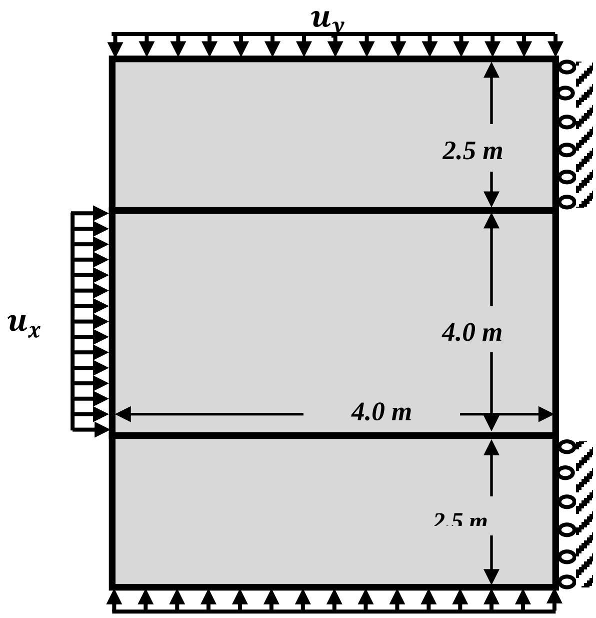
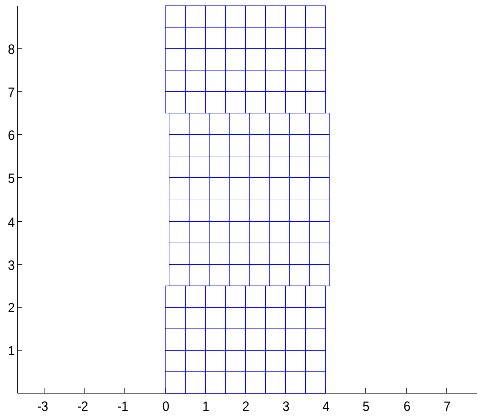


Fig. 3: Large Sliding with Horizontal Contact Surface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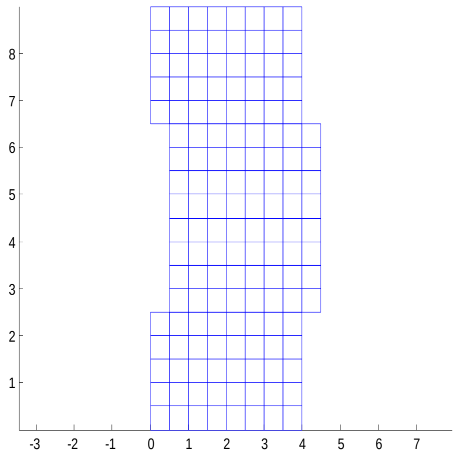
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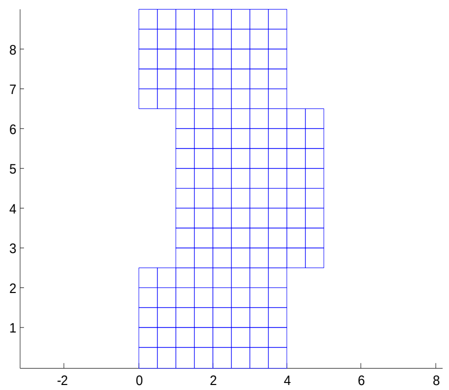
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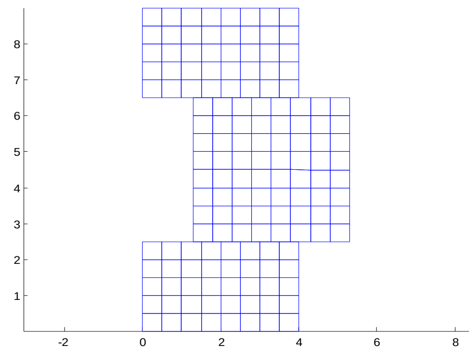
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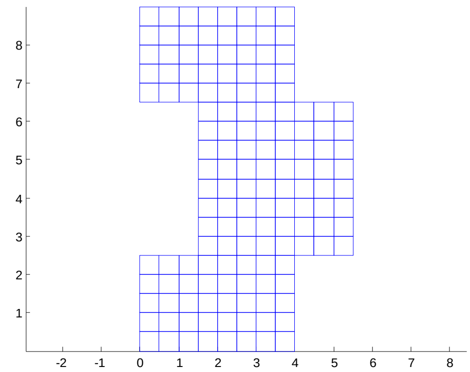
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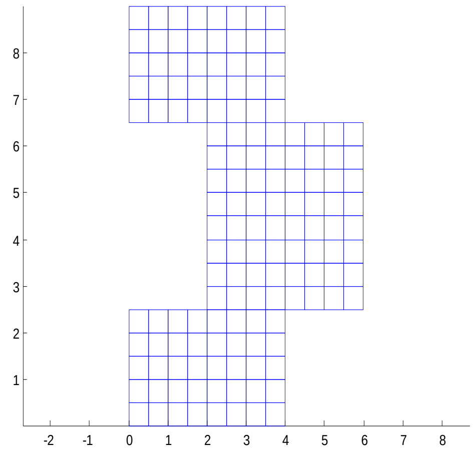
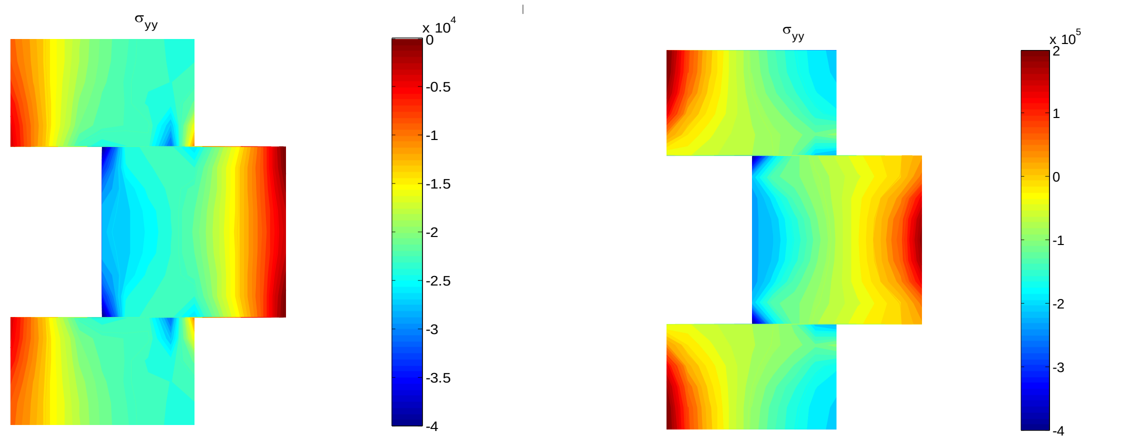
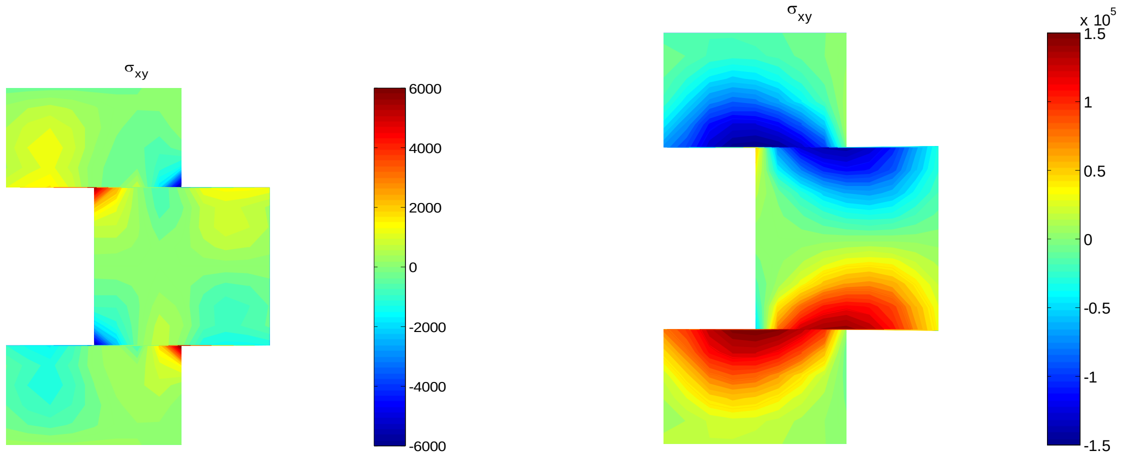

(f)

Fig. 4: Deformed configurations in large sliding with horizontal surface (a) $u_x = 0.1m$ (b) $u_x = 0.5m$ (c) $u_x = 1m$ (d) $u_x = 1.3m$ (e) $u_x = 1.5m$ (f) $u_x = 2m$



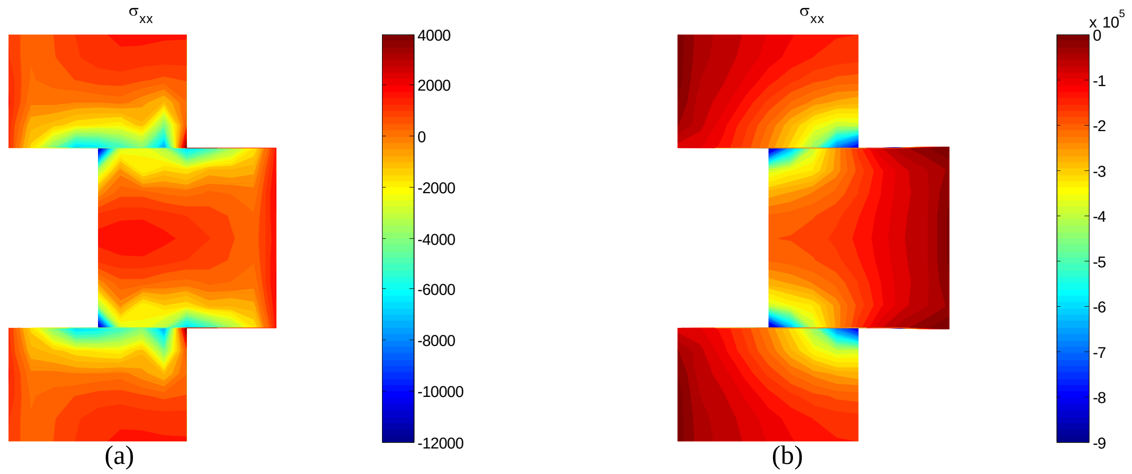
(a) (b)

Fig. 5: Normal stress σ_{yy} distributions (a) frictionless sliding (b) frictional sliding



(a) (b)

Fig. 6: Shear stress σ_{xy} distributions (a) frictionless sliding (b) frictional sliding



(a) (b)

Fig. 7: Normal stress σ_{xx} distributions (a) frictionless sliding (b) frictional sliding

B. Large Sliding with Inclined Contact Surfaces

This example models the contact behavior of a rectangular block sliding between two inclined surfaces, as shown in Fig. 8. FEM has been used for the purpose of simulation. A horizontal displacement of 2m is applied to the middle block. A vertical displacement of 1mm is applied to the top and the bottom edges. The material properties and the contact parameters are the same as in the previous problem. A uniform mesh of 144 elements has been considered in FEM simulation. The deformed configurations at various stages of the simulation are shown in Fig. 9. The stress contours obtained by FEM for both frictionless as well as frictional sliding are also shown in Fig. 10, Fig. 11, and Fig. 12.

(b)

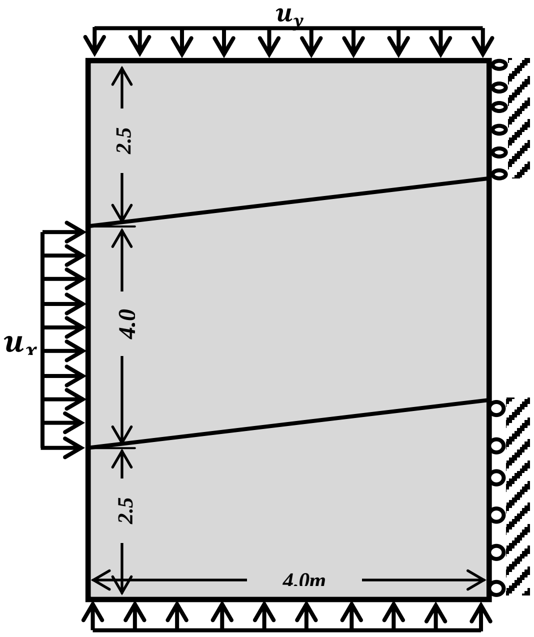
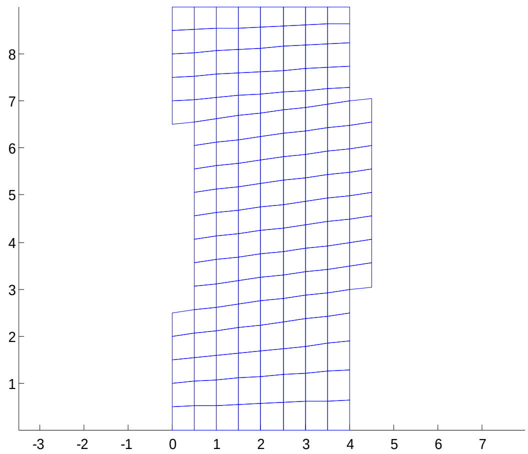
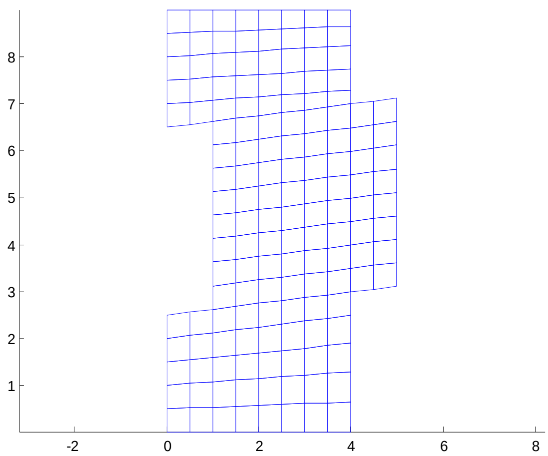


Fig. 8: Large Sliding with Inclined Contact Surfaces



(a)



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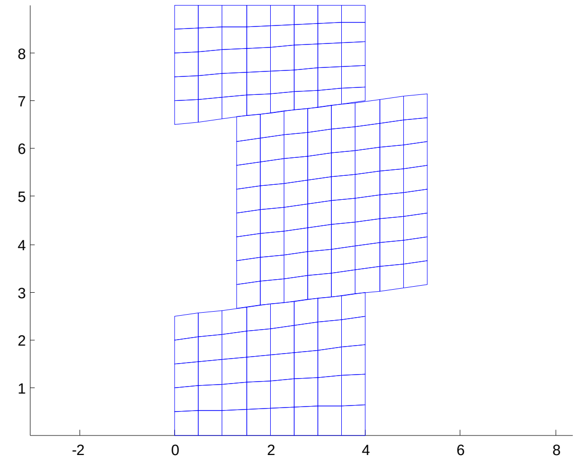

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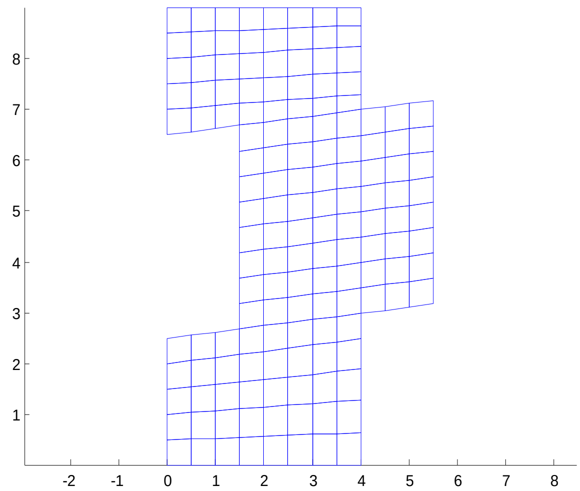

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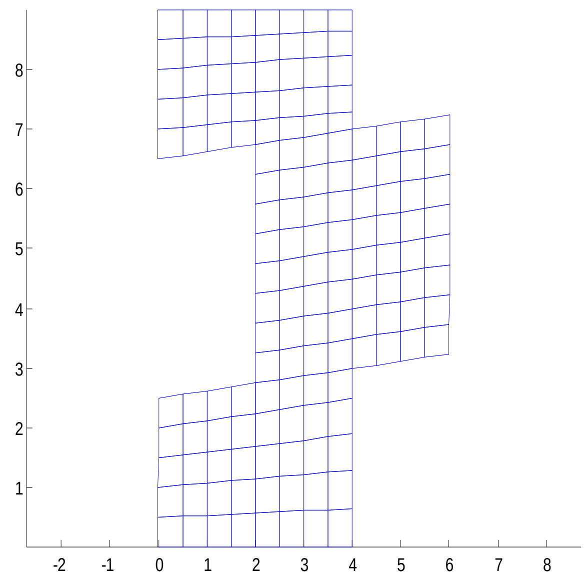
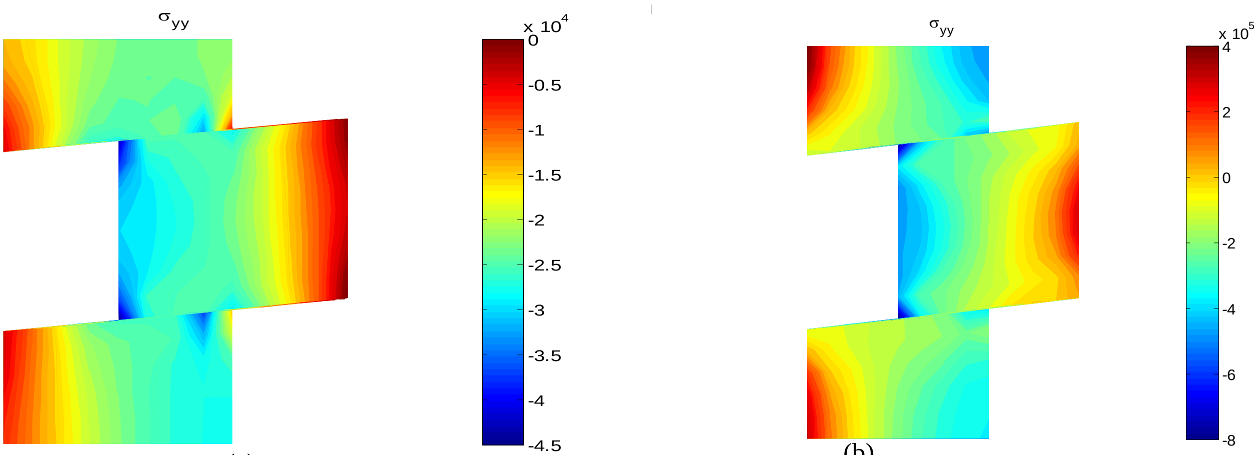


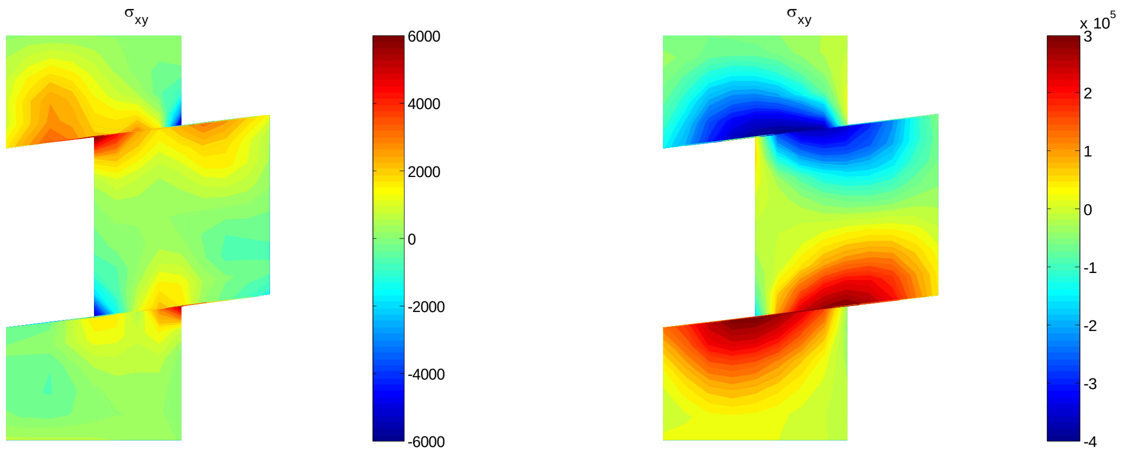
(e)

Fig. 9: Deformed configurations in large sliding with inclined surface (a) $u_x = 0.5m$ (b) $u_x = 1m$ (c) $u_x = 1.3m$ (d) $u_x = 1.5m$ (e) $u_x = 2m$



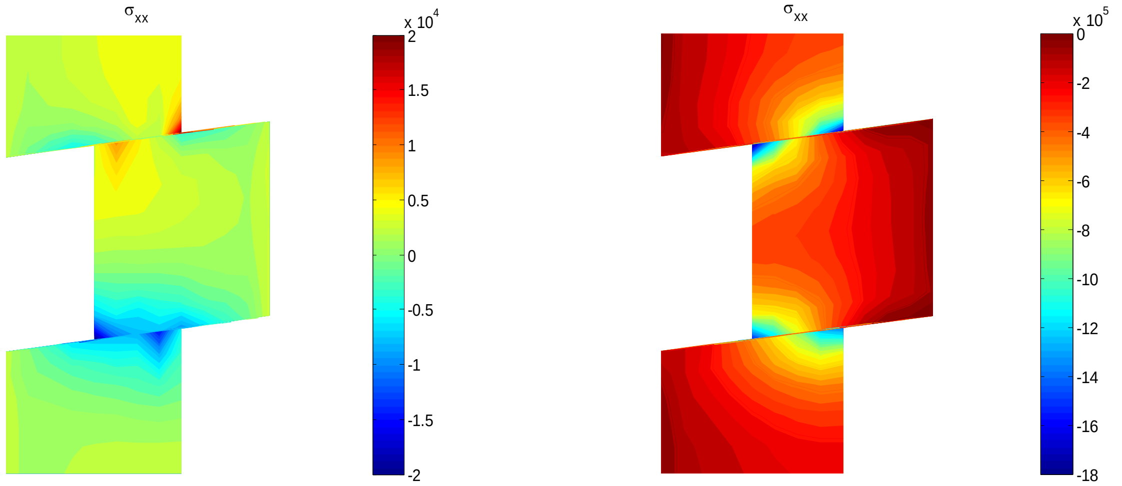
(a) (b)

Fig. 10: Normal stress σ_{yy} distributions (a) frictionless sliding (b) frictional sliding



(a) (b)

Fig. 11: Shear stress σ_{xy} distributions (a) frictionless sliding (b) frictional sliding



(a) (b)

Fig. 12: Normal stress σ_{xx} distributions (a) frictionless sliding (b) frictional sliding

C. Large Sliding of a Block on a Frictional Surface

The last example investigates the contact behavior for large sliding of a rectangular block over a frictional surface, as shown in Fig. 13. A horizontal displacement of 4m and a vertical displacement of 1mm are applied to the block. The simulation has been carried out by employing FEM. The material properties and the contact properties are the same as in the first example. The finite element method has been employed along with the NTS algorithm to model and simulate large sliding between the contact surfaces.

The deformed configurations at various stages of the simulation are shown in Fig. 14.

The normal stress σ_{xx} distribution obtained by FEM at various positions of the block on the frictional surface is shown in Fig. 15. The shear stress σ_{xy} and the normal stress σ_{yy} distributions obtained by FEM at various positions of the block on the frictional surface are shown in Fig. 16 and Fig. 17.

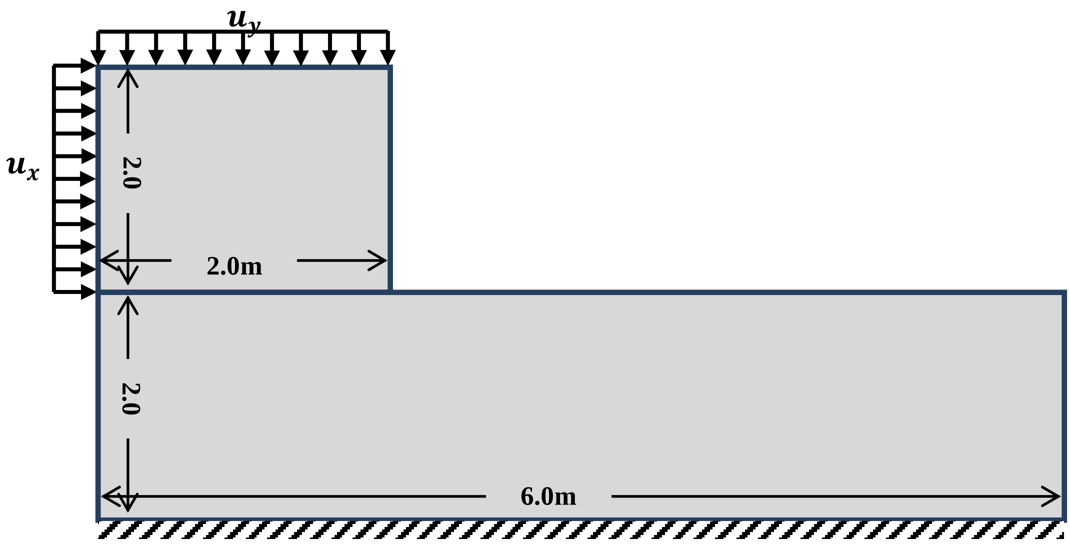


Fig. 13: Large sliding of a block on a frictional surface

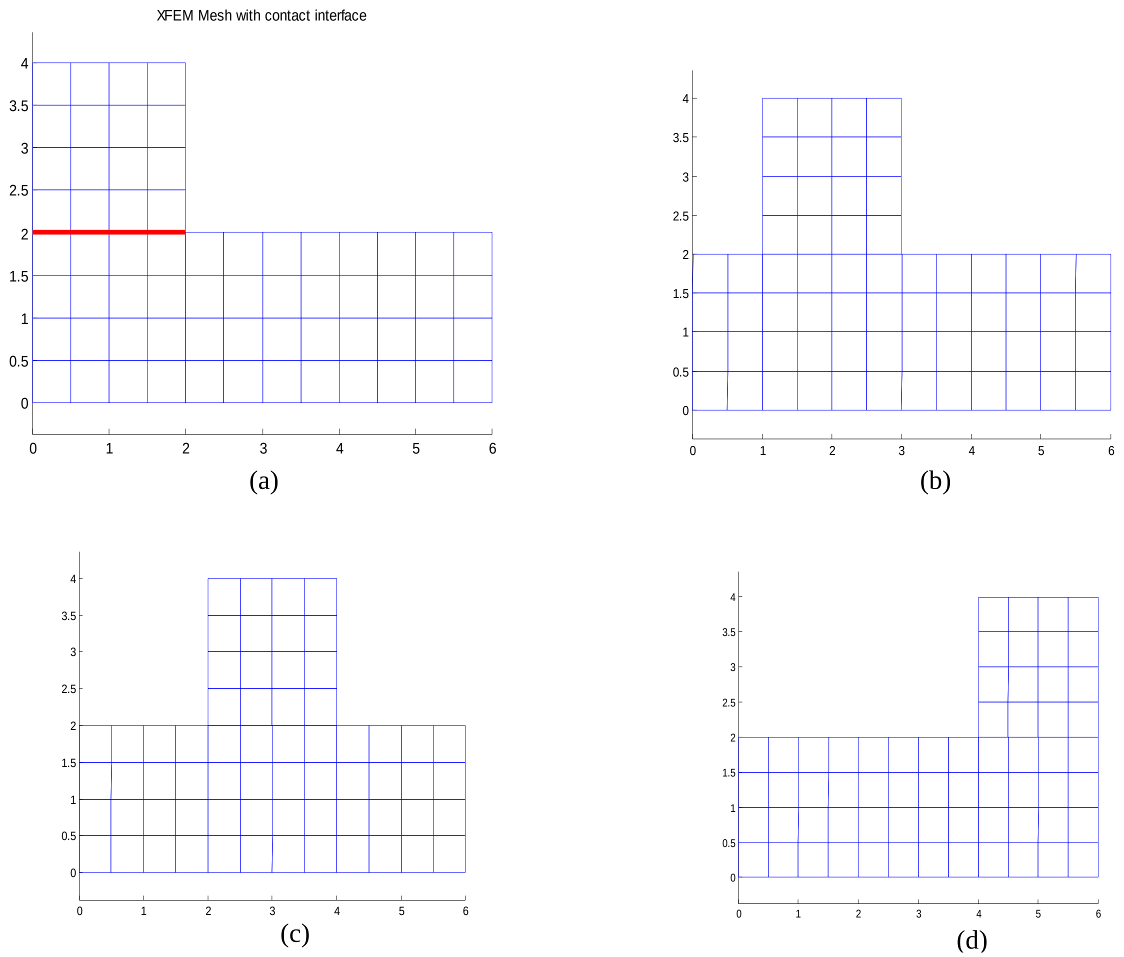
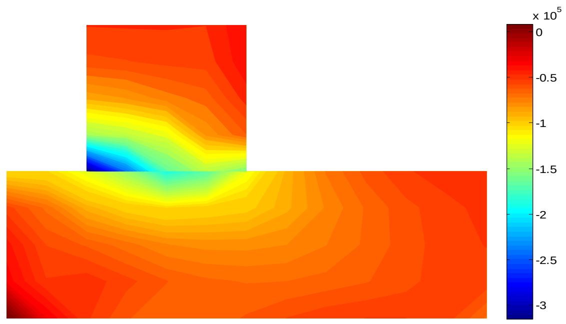
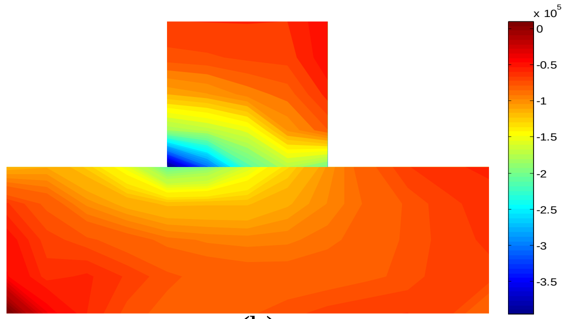


Fig. 14: Deformed configurations in large sliding of the block over a horizontal frictional surface (a) $u_x = 0m$ (a) $u_x = 1m$ (b) $u_x = 2m$ (c) $u_x = 4m$



(a)



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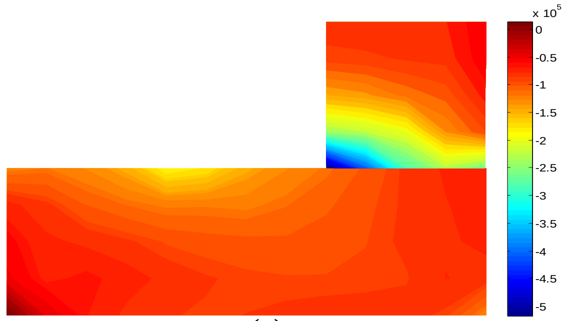
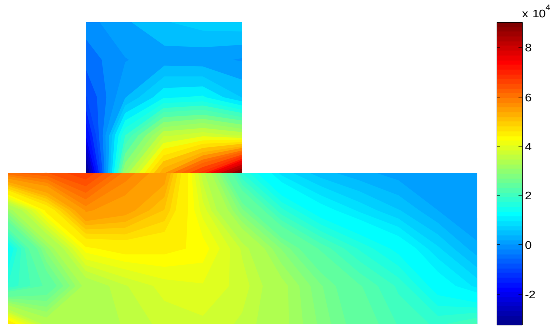
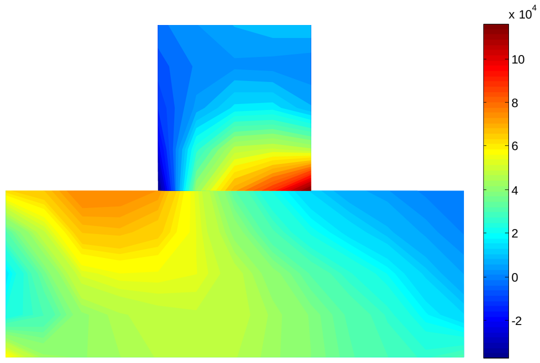


(c)

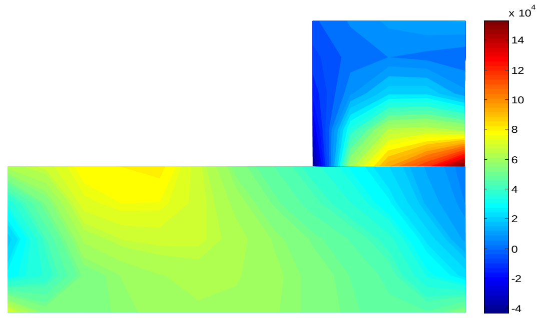
Fig. 15: Normal stress σ_{xx} distributions (a) $u_x = 1m$ (b) $u_x = 2m$ (c) $u_x = 4m$



(a)

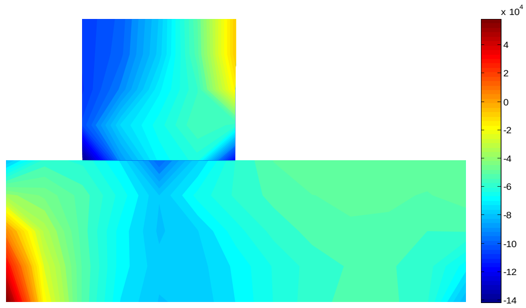


(b)

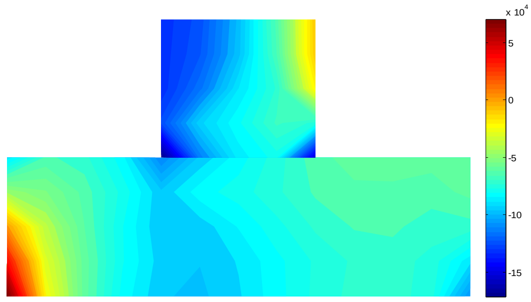


(c)

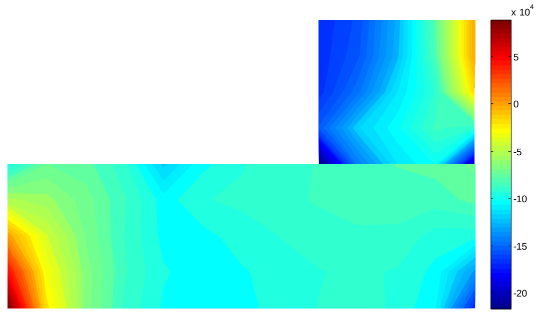
Fig. 16: Shear stress σ_{xy} distributions (a) $u_x = 1m$ (b) $u_x = 2m$ (c) $u_x = 4m$



(a)



(b)



(c)

Fig. 17: Normal stress σ_{yy} distributions (a) $u_x = 1m$ (b) $u_x = 2m$ (c) $u_x = 4m$

V. CONCLUSIONS

Contact problems are nonlinear in nature. There is a limited class of contact problems for which analytical solutions can be obtained. Therefore, various numerical techniques have been developed from time to time in order to solve these problems. Large sliding problems are those problems in which the relative displacements at the contact surface are sufficiently large. In this paper, node-to-segment approach has been used to model large sliding behavior between two solid bodies.

The contact constraints have been imposed by using the penalty approach. Three 2D large sliding problems have been solved by FEM to demonstrate the applicability of the node-to-segment approach in modeling and simulating large sliding contact problems and the results obtained in the study are found to be quite satisfactory. The results clearly demonstrate that the proposed technique can be efficiently used to model the large sliding problems, in which one body slides over the surface of another body. The penalty approach and the node-to-segment technique are powerful and accurate methods used to formulate the large sliding problems.

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