

# A Heuristic Algorithm for Resource Constrained Project Scheduling Problem with Discounted Cash Flows

Ritwik.A, Ginu Paul

**Abstract-** Project management is a complex decision making process involving the unrelenting pressures of time and cost. A project management problem typically consists of planning and scheduling decisions. The planning decision is essentially a strategic process wherein planning for requirements of several resource types in every time period of the planning horizon is carried out. Scheduling involves the allocation of the given resources to projects to determine the start and completion times of the detailed activities. Extensive research works have been carried out in Resource constrained project scheduling problems (RCPSP) and its variants. This paper mainly focuses on a resource constrained project scheduling problem with discounted cash flows (RCPSPDCF) as its variant. The study aims at providing fast heuristic solution for RCPSPDCF by utilizing the features of Particle Swarm Optimization (PSO).

**Keywords:** Project management; Scheduling; RCPSP; RCPSPDCF; PSO

## I. INTRODUCTION

Traditional project scheduling approaches such as critical path method (CPM) and project evaluation and review technique (PERT) focus on logical dependencies by assuming unlimited resource availability. However, the assumption of unlimited variety of resources may not be justified in many circumstances since only a fixed amount of resources are available or the cost of acquiring additional resources is very high. Therefore, many analytical or heuristic approaches have been proposed to solve such a resource constrained project scheduling problem (RCPSP), which consists of executing a group of activities limited by constraints.

Analytical methods adopted for solving RCPSP are integer programming (Talbot. F, 1982), dynamic programming and branch-and-bound or enumeration approaches to search for optimal solution. Heuristic methods for the RCPSP are aimed at searching for optimal solution in a more efficient way. Some of the existing techniques used are Tabu Search (TS), developed by Glover (1989), Simulated Annealing (SA) introduced by Kirkpatrick et al. (1983), Genetic Algorithm (GA) introduced in Holland (1975).

Particle Swarm Optimization is motivated by social behavior of organisms such as bird flocking and fish flocking. PSO algorithm, like GA, starts by initializing a population of random solutions and searches for optimal solution by updating generations. But PSO does not use any evolution operators. In PSO, the particles fly through the problem space by following its own experience and the best experience attained by the swarm as a whole. In contrast to other heuristic approaches, PSO can be easily implemented and it has a relatively fast searching process (Eberhart RC, shi Y, 1998). It is observed that project scheduling problem was studied with both regular as well as non regular objective functions. One of the prominent regular objective functions is makespan minimization, whereas the objective function such as the maximization of net present value and minimization of earliness-tardiness penalty cost fall among the non-regular objective functions.

However in the investigation, it was seen that literature on Resource constrained project scheduling problem with discounted cash flows is relatively scant. Though exact solutions and few heuristic solutions by Tabu search (Icmeli and Erenguc, 1994) were proposed for this problem, no attempts were made to find a heuristic solution using population based search algorithm like GA or PSO. Such algorithms can find solutions for even harder NP-hard problems with less computational effort.

### Nomenclature

N	Number of activities involved in a project
$f_i$	Finish time of activity i (1...N)
$d_i$	Duration of activity i
$P_i$	Set of activities that have already been scheduled (i.e., Predecessor)
$R_k$	Available amount of resources k (k=1...K)
K	Number of resource types
$r_{ik}$	Amount of resources k required by activity i
$A_t$	Set of ongoing activities at t
$f_n$	Finish time of n <sup>th</sup> activity
$V_i(t)$	The N-dimensional velocity for the i <sup>th</sup> particle in the t <sup>th</sup> iteration
$X_i(t)$	The N-dimensional position for the i <sup>th</sup> particle in the t <sup>th</sup> iteration
w(t)	Inertia weight used to control the impact of the previous velocity on the current velocity
$X_i^L$	Represents the local best (position or solution of the i <sup>th</sup> particle).
$X^G$	Represents the global best among all the population of particles achieved so far.
$c_1, c_2$	Positive constants
$r_1, r_2$	Random numbers between 0 and 1

### Greek symbols

$\delta_n$	Project Deadline
$\alpha$	Discount Rate

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It is seen that PSO gives superior performance in project scheduling problems (Hong et al., 2006). In this paper we have developed a heuristic (Particle Swarm Optimization) based solution methodology to find the best schedule which maximizes the net present value of the Resource Constrained Project Scheduling Problem with Discounted Cash Flows.

## II. DESCRIPTION AND FORMULATION OF RCPSPDF

A project scheduling problem can be characterized by the objective function, features of resources, and the pre-emption condition (Lee and Kim, 1996). Minimization of project duration is often used as an objective of a general project scheduling problem, while other objectives such as minimization of net present value of cash flows, and levelling of resource usage are also considered. Resources involved in a project can be single or multiple varieties, and can be renewable or non renewable. Pre-emption means that some activities can be interrupted during their execution, while non pre-emption means that some activities are not allowed to be interrupted once they are scheduled to start.

The Resource Constrained Project Scheduling Problem with Discounted Cash Flows (RCPSPDF) in this paper is based on the following assumptions: (1) the activities have certain known duration; (2) all predecessors must be finished before an activity can start (i.e., precedence constraints); (3) resources can be multiple varieties, available in limited amounts and renewable from period to period (multiple resource constraints); (4) activities are non pre-emptive, that is, cannot be interrupted when in progress; (5) managerial objective is to maximize the net present value; (6) all activities have known net cash flow. In addition, a project is considered to be represented by activity-on-node network topology, where two dummy activities with zero duration may be included for indicating the single start and end nodes of the project.

The RCPSPDF involves the scheduling of the project activities to maximize the net present value subject to precedence and resource constraints. The project is represented by an activity-on-the-node (AoN) network  $G=(N, A)$ , where the set of nodes,  $N$ , represents activities, and the set of arcs,  $A$ , represents finish-start precedence constraints with a time lag of zero. The project must finish before its imposed deadline  $\delta_n$ . The activities are numbered from dummy start activity 1 to the dummy end activity  $n$ . The duration of an activity is denoted by  $d_i (1 \leq i \leq n)$  and the performance of each activity involves a series of cash flow payments and receipts throughout the activity duration. Each activity is having randomly generated cash flows from a uniform distribution over the interval  $[-5000; 10,000]$ . It is assumed that  $cf_{it} (1 < i < n)$  denotes the known deterministic cash flow of activity  $i$  in period  $t$  of its execution. A terminal value of each activity on completion can be calculated by compounding the associated cash flow to the end of the activity as follows:

$$c_i = \sum_{t=1}^{d_i} (cf_{it}) e^{\alpha(d_i-t)},$$

Where  $\alpha$  represents the discount rate and  $c_i$  is the terminal value of cash flows of activity  $i$  at its completion. If the non-negative integer variable  $f_i (1 \leq i \leq n)$  denotes the completion time of activity  $i$ , its discounted value at the beginning of the project is  $c_i e^{-\alpha f_i}$ . The formulation of RCPSPDF (Vanhoucke et al., 2001) is given as follows:

$$\text{Maximize} \quad \sum_{i=2}^{n-1} c_i e^{-\alpha f_i} \quad (1)$$

Subject to:

$$f_j - f_i \geq d_i \quad \forall j \in P_i \quad (2)$$

$$i=1,2,\dots,N \quad (3)$$

$$f_n \leq \delta_n$$

$$\sum_{t=S_1, S_2, \dots, S_N} r_{ik} \leq R_k; \quad k = 1, 2, \dots, K; \quad (4)$$

Equation (1) represents the objective, while Eqs. (2), (3) and (4) represents the precedence constraint, project deadline constraint and resource constraints respectively.

## III. PARTICLE SWARM OPTIMIZATION (PSO)

PSO simulates a social behavior such as bird flocking to a promising position or region for food or other objectives in an area or space (Sylverin Kemmoe Tchomte et al., 2007). Like evolutionary algorithm, PSO conducts search using a population, which is called swarm, of individuals, which are called particles. Each particle represents a candidate position or solution to the problem at hand, to represent a potential solution. During searching for optima each PSO particle adjusts its trajectory towards its own previous best position, and towards the best previous position attained by any member of its neighborhood (i.e., the whole swarm). Thus, global sharing of experience or information takes place and particles profit from the discoveries of themselves (i.e., local best) and previous experience of all other companions (i.e., global best) during search process.

PSO is initialized with a population of  $M$  random particles and then searches for best position (solution or optimum) by updating generations until getting a relatively steady position or exceeding the limit of iteration number (i.e.,  $T$ ). In every iteration or generation, the local bests and global bests are determined through evaluating the performances, i.e., fitness values or objectives, of the current population of particles. Each particle is treated as a point in an  $N$ -dimensional space. Two factors characterize a particle status on the search space: its position and velocity (Kennedy J and Eberhart RC, 1995). The  $N$ -dimensional position for the  $i^{\text{th}}$  particle in the  $t^{\text{th}}$  generation (i.e., iteration) can be denoted as  $X_i(t) = \{x_{i1}(t), x_{i2}(t), \dots, x_{iN}(t)\}$ . Similarly, the velocity (i.e., distance change), also a  $N$ -dimensional vector, for the  $i^{\text{th}}$  particle in the  $t^{\text{th}}$  generation can be described as  $V_i(t) = \{v_{i1}(t), v_{i2}(t), \dots, v_{iN}(t)\}$ . The following equations (Kennedy J and Eberhart RC, 1995) can represent the updating mechanism of a population of particle's status from the ones of the last generation during search process:

$$V_i(t) = w(t)V_i(t-1) + c_1 r_1 (X_i^L - X_i(t-1)) + c_2 r_2 (X^G - X_i(t-1)) \quad (5)$$

$$X_i(t) = V_i(t) + X_i(t-1) \quad (6)$$

Where  $i = 1, 2, \dots, M$  and  $t = 1, 2, \dots, T$ ;  $X_i^L = \{x_{i1}^L, x_{i2}^L, \dots, x_{iN}^L\}$  represents the local best (position or solution) of the  $i^{\text{th}}$  particle associated with the best fitness encountered after  $t-1$  iterations,



while  $X^G = \{x_1^G, x_2^G, \dots, x_N^G\}$  represents the global best among all the population of particles achieved so far.  $c_1$  and  $c_2$  are positive constants (namely learning factors) and  $r_1$  and  $r_2$  are random number between 0 and 1;  $w(t)$  is the inertia weight used to control the impact of the previous velocities on the current velocity, influencing the tradeoff between the global and local exploration abilities during search.

Eq. (5) is used to calculate the particle's new velocity according to its previous velocity and the distances of its current position from its own best experience or position and the group's best experience or position. Then the particle flies toward a new position according to Eq. (6) (Shi Y and Eberhart RC, 1998).

### 3.1 PSO for RCSPDCF

In order to apply PSO, it is necessary to find a suitable mapping between the RCSPDCF at hand and the PSO particle. Priority-based scheduling (Lee and Kim, 1996) combined with a serial transformation scheme (Kolisch.R, 1996) is used to bridge the PSO particle with the RCSPDCF.

### 3.2 Particle representation of activities

The conflicts in scheduling multiple activities competing for limited resources can be resolved according to the priorities among schedulable activities, whose predecessors are all completed and which require no more resources than available amounts at time. The activities with higher priorities should be assigned the resources and scheduled prior to the ones with lower priorities. The priority of an activity is generally determined based on one or multiple factors such as the activity's critical index, duration, amounts of required resources, and the number of the directly successive activities, etc. Different computing models for the priorities by taking into account these factors will characterize the corresponding priority rules and may lead to different performances, because there is no systematic measure to select a heuristic rule or to decide which is better than others (Davis EW and Patterson JH, 1975). A multiple-pass heuristic method that considers different priority rules for each pass is proposed so as to select the best one among multiple solutions (Bell CE and Han J, 1991). The above multiple-pass heuristic actually reflects the concept of determining a better set of priorities that lead to a better schedule by adjusting the original priorities or their results. Such a concept is the basis on which priorities of activities are represented through PSO particles, so that the optimal schedule can be searched from a population of particle-represented priorities that are updated according to PSO mechanism. Instead of obtaining initial priorities from different heuristic rules, the initial priorities represented by PSO particles are randomly generated. As a point in a N-dimensional space, the N elements of a PSO particle can stand for the N activities in a project under study. Hence, the N parameters of a particle's position, i.e.,  $X_i(t) = \{x_{i1}(t), x_{i2}(t), \dots, x_{iN}(t)\}$ , can represent the priorities of the N activities, while the placements of the parameters in the N-dimensional particle reflectively correspond to indexes of the activities. The particle represented priorities must be transformed to feasible schedule based on precedence constraints and available resources from time to time. With known priorities of activities, there are two schemes, i.e., serial and parallel scheme (Kolish.R, 1996), to perform scheduling in

consideration of precedence constraints and resources constraints in a stage wise fashion, in determining sequences and start time of activities. Based on the particle representation of activity priorities and the serial scheme to transform the particle represented priorities to a feasible schedule, the framework of the PSO methodology for the RCSPDCF is developed.

### 3.3 Parameters Configuration For PSO

The PSO particle represents a series of priorities that range from 0 to 1, all parameters of the N dimensional particle positions, either initialized or updated during search, must be limited to [0,1]. The particle velocity, based on the current position should also be limited to [-1, 1] so as to prevent the updated position from oscillating too heavy.

The inertia weight  $w(t)$  can be constant or varying with iteration. Through experiments in our study, the constant inertia weight  $w(t) = 0.65$  is found best suitable. For the number of particles in the population or swarm, i.e., M or called P-size, more particles may increase success in searching for optima due to sampling state space more thoroughly. However, more particles require more evaluation runs, leading more optimization cost. Therefore, a medium number (e.g., the one equal to the number of activities except for dummy activities) of particles are generally selected for the PSO-based approach. The difference caused by different learning factors  $c_1$  and  $c_2$  is not obvious; hence, they are set to 2 as usual.

### 3.4 Procedure Of PSO Based Approach

Step 1 (Initialization). Set iteration counter as 0 and initialize M particles by randomly generating their positions,  $X_i(0) = \{x_{i1}(0), x_{i2}(0), \dots, x_{iN}(0), x_{i(N+1)}(0), x_{i(N+2)}(0), \dots, x_{i2N}(0)\}$ ,  $i = 1, \dots, M$ , within the search space [0, 1]. Meanwhile the velocities of the initialized particles, i.e.,  $V_i(0) = \{v_{i1}(0), v_{i2}(0), \dots, v_{iN}(0), v_{i(N+1)}(0), v_{i(N+2)}(0), \dots, v_{i2N}(0)\}$ ,  $i = 1, \dots, M$ , are also generated randomly and are limited to [-1, 1]. Then for each particle, the initialized positions is set as their local best positions, i.e.,  $X_i^L = \{x_{i1}^L, x_{i2}^L, x_{i3}^L, \dots, x_{iN}^L, x_{i(N+1)}^L, x_{i(N+2)}^L, \dots, x_{i2N}^L\}$ ,  $i = 1, \dots, M$ . Finally all particles should be evaluated according to the project duration corresponding to their schedules generated using the serial scheme, determining the global best  $X_i^G = \{x_{i1}^G, x_{i2}^G, x_{i3}^G, \dots, x_{iN}^G, x_{i(N+1)}^G, x_{i(N+2)}^G, \dots, x_{i2N}^G\}$  as step 6 does.

Step 2 (proceeding to the next iteration). After evaluating all initialized or updated particles in the population and finding out the local best and the global best, the iteration counter should be updated, i.e.,  $t = t + 1$ , so as to proceed to next iteration of the PSO search. Step 3 (Velocity updating). Based on the previous velocities and the distances of the current positions from the local best  $X_i^L = \{x_{i1}^L, x_{i2}^L, x_{i3}^L, \dots, x_{iN}^L, x_{i(N+1)}^L, x_{i(N+2)}^L, \dots, x_{i2N}^L\}$  and the global best  $\{x_{i1}^G, x_{i2}^G, x_{i3}^G, \dots, x_{iN}^G, x_{i(N+1)}^G, x_{i(N+2)}^G, \dots, x_{i2N}^G\}$ , all particle new velocities, i.e.,  $V_i(t) = \{v_{i1}(t), v_{i2}(t), \dots, v_{iN}(t), v_{i(N+1)}(t), v_{i(N+2)}(t), \dots, v_{i2N}(t)\}$ , are calculated using Eq. (5) and are subject to the velocity limit [-1, 1].

Step 4 (position updating). Based on the updated velocities, the new positions,  $X_i(t) = \{x_{i1}(t), x_{i2}(t), \dots, x_{iN}(t), x_{i(N+1)}(t), x_{i(N+2)}(t), \dots, x_{i2N}(t)\}$ , that each particle will fly towards should be calculated according to Eq. (6) and are subject to the position limit [0, 1].

Step 5 (Particle transformation). Each updated particle is transformed to the schedule using serial scheme based on the priorities and percentage deviation in start time, i.e., the current position  $X_i(t) = \{x_{i1}(t), x_{i2}(t), \dots, x_{iN}(t), x_{i(N+1)}(t), x_{i(N+2)}(t), \dots, x_{i2N}(t)\}$

Step 6 (Particle evaluation). For each particle, the project duration of the schedule transformed from the updated position  $X_i(t) = \{x_{i1}(t), x_{i2}(t), \dots, x_{iN}(t), x_{i(N+1)}(t), x_{i(N+2)}(t), \dots, x_{i2N}(t)\}$  is tested and if smaller than that of its previous local best (position), i.e.,  $X_i^L = \{x_{i1}^L, x_{i2}^L, x_{i3}^L, \dots, x_{iN}^L, x_{i(N+1)}^L, x_{i(N+2)}^L, \dots, x_{i2N}^L\}$ , then it will update the local best. Among all particles, search for the global best particle position corresponding to a schedule with maximum net present value. If this particular one is more than that of the previous global best, it should replace the previous one  $X_i^G = \{x_{i1}^G, x_{i2}^G, x_{i3}^G, \dots, x_{iN}^G, x_{i(N+1)}^G, x_{i(N+2)}^G, \dots, x_{i2N}^G\}$ .

Step 7 (Stopping criteria). The PSO will be terminated if the current iteration meets any one of the termination signals. The termination signals considered here include: (1) 20 consecutive iterations where the value of global best are the same, and (2) maximum total number of iterations. According to the procedure, the proposed PSO for the RCPSPDCF has been implemented using Java programming language with NetBeans IDE 7.0.1.

IV. COMPUTATIONAL ANALYSIS

To investigate the PSO based approach for RCPSPDCF a typical project network having 25 activities and two dummy activities with a complex precedence relationship is shown in Fig 1 (Hong et al, 2006). The project consists of a renewable type of resource. Each activity has certain duration which is indicated above the corresponding node. The cash flows and the resources are indicated below the node. The precedence constraints among activities are described with arrow lines. To this network individual cash flows are assigned to each activity (Vanhoucke, 2001). The assigned cash flows are as follows: 3745, -4349, 9357, -4801, 4686, -4117, 2952, 880, 5974, -4149, 4778, 468, 5679, -374, -288, 6356, -4129, 5749, -1025, -1782, 9229, 9005, 5473, 4163, 950. The discount rate is assumed to be 0.05. Because of the complex precedence relationship and the resource requirements, this is an even harder NP-hard problem. The optimum cash flow value is 13511, whereas the heuristic solution gave the cash flow value 0 as 13371.

V. CONCLUSION

PSO algorithm has been applied successfully on many of the scheduling problems. Here we have made a humble effort to adapt the PSO algorithm for solving the RCPSPDCF. The procedure has been coded in JAVA programming language using NetBeans IDE 7.0.1 version under Windows 7 environment and has been validated on a set of PSPLIB project networks. The results were obtained in a personal computer with Core i3 processor and 4GB RAM. For complex problems PSO may not give the optimum solution; but it can give good solutions in reasonable time. For solving problems with 40 or more activities, the use of PSO algorithm for finding near optimum solution can be highly justified as compared to finding the exact solution in regard to the time required for finding the solution.

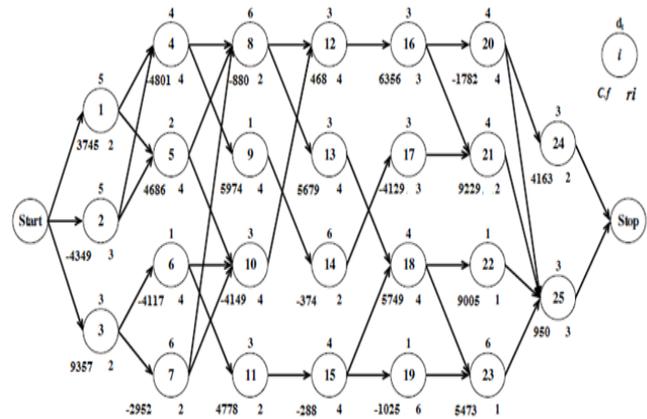


Fig. 1 A RCPSP network with associated cash flows

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