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Static and Dynamic Analysis of Composite Laminated Plate

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Abstract- This work presents a static and dynamic analysis of Graphite /Epoxy composite plates. In the present work the behavior of laminated composite plates under transverse loading using an eight-node diso-parametric quadratic element based on First Order Shear Deformation Theory was studied, the element has six degrees of freedom at each node: translations in the nodal x, y, and z directions and rotations about the nodal x, y, and z axes . The static analysis includes the parametric studies on laminated plates to estimate the maximum deflection. The parametric study represented by variation in (aspect ratio, layer orientation, layer number, dimension of the plate and mesh size). The modeling of the plates was done by using ANSYS 12.0, and the results were compared with Finite Element Method code.The dynamic part of this study represented by evaluating the natural frequency of the plates. The boundary conditions considered in static and dynamic study, are simply supported and clamped boundary condition. In this study investigations were carried out on both square and rectangular composite laminated plates. The study start with isotropic plate of mild steel, and followed with orthotropic plate of Graphite/Epoxy composite .The results obtained from ANSYS program as well as Finite Element Method code, show a good agreement with the experimental results. The minimum deflection was found at an angle of 15 degree for clamped plate, and in case of simply supported plate the minimum deflection was found for angle 45 degree. It is also observed that the deflection for clamped boundary condition is less than in simply supported boundary condition for both isotropic and orthotropic plates. In isotropic plate the deflection in clamped plate is about 50% of simply supported. And for orthotropic plate the deflection for clamped is about (25 to 30)% of simply supported.

Keywords- Composite (Graphite/Epoxy) laminated plate, rectangular and square plate, Isotropic plate, Orthotropic plate, Free vibration (Natural frequency).

I. INTRODUCTION

The needs of the high rise building and aerospace industry led to the development and application of composite materials. Advances in the manufacturing process and technology of laminated composites have changed the use of the composites from secondary structural components to the primary ones. Practically laminated composites are commonly used as a part of building like sandwich penal, aeronautical and aerospace industries as the main part of the structure rather than aluminum or other metallic materials. Low weight, high strength and greater rigidity were of paramount interest.

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A variety of structural elements such as cylinders, beams, plates and shells could be potentially used for the analysis of laminated composites. The high stiffness-to-weight ratio coupled with the flexibility of the selection of the lamination scheme that can be tailored to match the design requirement makes the laminated plate an attractive structural component for many industries.Depending upon their applications, plates can be moderately thick. To use the laminated composite plates efficiently, it is necessary to develop appropriate analysis theories to predict accurately their structural and dynamical behavior. The increased utilization of composite materials in several engineering applications has led to intensive research activities in linear and non-linear, static and dynamic analysis of laminated composite plates. The majority of the investigations on laminated plates utilize either the classical lamination theory (CLT), or the first-order shear deformation theory (FSDT).

II. REVIEW OF LITERATURE

Sheikh et al.(2002) developed a high precision shear deformable element for the analysis of laminated composite plates of different shapes. **Jian et al. (2004)**.Studied the Vibration analysis of fully clamped arbitrary laminated plate.**Zhang and Yang(2009).** Presented the recent developments in finite element analysis for laminated composite platesbased on the various laminated plate theories for the free vibration and dynamics, buckling and post-buckling analysis, geometric nonlinearity and large deformation analysis, and failure and damage analysis of composite laminated plates .

Bhar, Phoenix, Satsangi(2010),studied the finite element analysis of laminated composite stiffened plates using FSDT. Dharma Raju and Suresh Kumar (2011) developed the analytical procedure to investigate the bending characteristics of anti-symmetric and cross ply laminated composite plates based on a higher order shear.

The objective of the present investigation is to study the behaviour of FSDT plates under transverse loading condition and to estimate the influence of stacking sequence, fiber orientation, layer thickness, aspect ratio and the number of layers in the laminated composite plates on the prediction of maximum strength of the plates.

III. MATERIAL & METHODOLOGYCOMPOSITE MATERIALS

The term composite is often used for a material that is made of two or more different components. Each of the components may have different mechanical and chemical properties. A composite composed of an assemblage of these different parts gives us a new material whose performance characteristic is superior to that of the individual parts taken separately.

The more common composites used are laminated composite plates which are typically made of different layers bonded together. Basically, each layer is generally orthotropic and has a different orientation of the fibers, as shown inFig.1.The properties which can be improved by forming a composite are stiffness, strength and weight reduction. The major advantage of composite material is ability of the controllability fiber alignment. By arranging layers and fiber direction, laminated material with required strength and stiffness properties to specific design conditions, can possibly be achieved. Composite are commonly formed in three different types' fibrous composites, particulate composites and laminated composites.

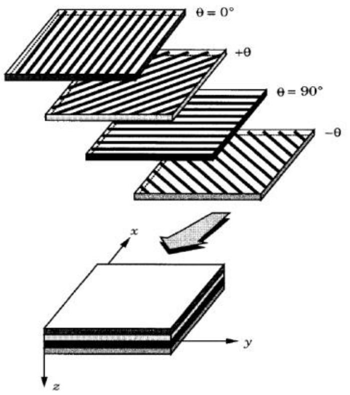

Fig.1: A laminate made up of lamina with different fiber orientations

CLASSICAL LAMINATED PLATE THEORY (CLPT)

The classical laminate plate theory is based on the Kirchhoff's assumptions, in which transverse normal and shear stresses through the thickness of the plates are neglected.

In formulating the theory following assumption and restriction are also considered:

1. The layers are perfectly bonded together (assumption).
2. The material of each layer is linearly elastic and has three planes of material
   symmetry (i.e., orthotropic) (restriction).Each layer is of uniform thickness (restriction).
3. The strains and displacements are small (restriction).
4. The transverse shear stresses on the top and bottom surfaces of the laminate are zero (restriction).

Considering the above assumption the displacement field of CLPT is of form

$$u(x,y,z) = u_0(x,y) - z\frac{\partial w_0}{\partial x}$$

$$v(x,y,z) = v_0(x,y) - z\frac{\partial w_0}{\partial y}$$

$$w(x,y,z) = w_0(x,y)$$

where(u_o, v_o, w_o)are the displacement components along the (x, y, z)coordinate directions, respectively, of a point on the mid-plane (i.e., $z = 0$). The displacement field implies that straight lines normal to the xy-plane before deformation remain straight and normal to the mid-surface after deformation.

FIRST-ORDER SHEAR DEFORMATION THEORY

The following assumptions are considered in formulating the theory:

1) The layers are perfectly bonded
2) The material of each layer is linearly elastic and has two planes of material symmetry
3) The strains and displacements are small
4) Deflection is wholly due to bending strains only
5) Plane sections originally perpendicular to the longitudinal plane of the plate remain plane, but not necessarily perpendicular to longitudinal plane
6) The transverse shearing strains (stresses) are assumed to be constant along the plate thickness

The displacement field of the first-order theory is of the form:

$$u(x,y,z) = u_0(x,y) - z\theta_x(x,y)$$

$$v(x,y,z) = v_0(x,y) - z\theta_y(x,y)$$

$$w(x,y,z) = w_0(x,y)$$

where (u_o, v_o, w_o, θ_x, θ_y) are unknown functions to be determined. As before,(u_o, v_o, w_o) denote the displacements of a point on the plane $z = 0$. θ_x and θ_y are the rotation of a transverse normal with respect to undeformed middle plane.

FREE VIBRATION

By free vibration we mean the motion of a structure without any dynamic external forces or support motion. The motion of the linear SDF systems without damping specializes to

$$m\frac{d^2u}{dt^2} + ku = 0$$

Free vibration is initiated by disturbing the system from its static equilibrium position by imparting the mass some displacement u(0) and velocity $\dot{u}$ (0) at time zero, defined as the instant the motion is initiated :

$\mathbf{u = u(0), \dot{u} = \dot{u}(0)}$

So, solution to the equation is obtained by standard methods**:**

$$u(t) = u(0)\cos\omega_n t + \frac{\dot{u}(0)}{\omega_n}\sin\omega_n t$$

Where natural circular frequency of vibration in unit radians per second=

$$\omega_n = \sqrt{\frac{k}{m}}$$

The time required for the undamped system to complete one cycle of free vibration is the natural period of vibration of the system.

$$T_n = \frac{2\pi}{\omega_n}$$

Natural cyclic frequency of vibration is denoted by $f_n = \frac{1}{\mathbf{Tn}}$, unit in Hz (cycles per second)

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FINITE ELEMENT ANALYSIS

The equations of equilibrium of a discredited elastic structure undergoing small deformations can be expressed as:

$$[M]\{\ddot{u}\} + [c]\{\dot{u}\} + [k]\{u\} = \{F(t)\}$$

For free undamped vibration, the equation reduces to:

$$[M]\{\ddot{u}\} + [k]\{u\} = \{0\}$$

If modal co-ordinates are employed the equation becomes

$$\{[k] - \omega^2 [M]\}\{\phi_n\} = \{0\}$$

There are various methods of finding the natural frequencies ω_i and modal vectors $\{\varphi_n\}_I$ once the system mass [M] and stiffness matrices [K] are formulated. Here an eight nodediso-parametric plate bending element has been chosen to discretise the plate. The necessary constitutive relationships have also been formed. The element is capable of incorporating transverse shear deformation through the implementation of first order shear deformation theory as applicable to composite.

The element stiffness matrix can be expressed as :

$$[K]_e = \int_{-1}^{+1}\int_{-1}^{+1} [B]^T [D][B]|J|\, d\xi\, d\eta$$

The Gaussian Quadrature formula is used for numerical integration. Reduced integration technique has been employed in order to avoid shear locking. Similarly the consistent element mass matrix is generated using,

$$[M]_e = \int_{-1}^{+1}\int_{-1}^{+1} [N]^T [m][N]|J|\, d\xi\, d\eta$$

Effect of rotary inertia is neglected.

FINITE ELEMENT MODELING

The static and dynamic analysis have been done using ANSYS 12.0 package. The composite laminated plates modeled using the element SHELL99 linear layer as shown in Fig.2may be used for layered applications of a structural shell model.It usually has a smaller element formulation time. SHELL99 allows up to 250 layers.The element has six degrees of freedom at each node: translations in the nodal x, y, and z directions and rotations about the nodal x, y, and z axes.

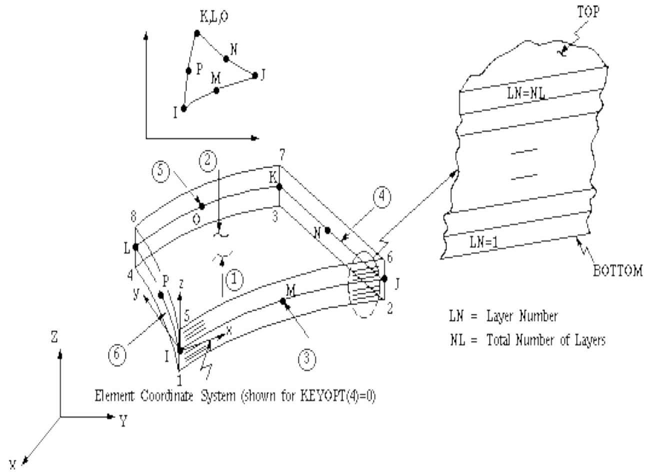


Fig.2: SHELL99 Linear Layered Structural Shell

MATERIAL AND DETAIL OF THE PLATE

An orthotropic plate subjected to transverse loading condition for both simply supported and clamped boundary conditions has been considered for the present study. The general stacking sequence considered for the laminates is [θ/θ], with gradual variation of θ. The composite plate of symmetric and anti-symmetric angle-ply laminates is modeled for different stacking sequence. The laminated plate consists of T300/5208 graphite/epoxy material. The details of geometric properties for orthotropic plate, Material properties of graphite/epoxy composite material and loading conditions are given in tables 1,2 and 3 respectively

Table 1: Geometric properties for orthotropic plate

Dimension	No. of layer	Stacking sequence	
0/0			
Length(b)= 1.0 m	2,4,6,8,10,12,16	15/-15	symmetric
Width(a)= 1.0 m		30/-30	un symmetric
Thickness = 0.01 m		45/-45	
		60/-60	
		75/-75	
		0/90	

Table 2: Material properties of graphite/epoxycomposite material

E_1= 175 e9 N/m^2
E_2= 7 e9 N/m^2
E_3= 7 e9 N/m^2
V_{12} = V_{13} = 0.25
V_{23} = 0.01
G_{12} = G_{13} = 3.5 e9 N/m^2
G_{23} = 1.4 e9 N/m^2
Density = 1550 kg/m^3

Table 3: Loading condition

Point load	Uniform distributed load	Boundary condition
P= 1000 N	q= 100 N/m^2	simply supported at four sides Clamped at four sides

IV. RESULTS AND DISCUSSION

Several cases were analyzed and grouped by the type of the sequence for the layers and the boundary condition of the plates (simply supported and clamped) existent for each case. One group consists of symmetric sequence for the fiber of layers in the plates. The other group consists of un-symmetric sequence of the fiber orientation plates. Central deflection of laminated composite plates are investigated for various stacking sequences, fibre orientations, layer thicknesses and the number of layers in the laminated composite plates. The problems studied during the present investigations are defined below.

PROBLEM 1:

An orthotropic plate subjected to transverse loading condition for both simply supported and clamped boundary conditions has been considered for the present study.The laminated plate consists of T300/5208 graphite/epoxy material.

In this problem various mesh size, have been used ranging from (4x4) to (24x24) to study the merging point of composite plate.No. of layer =4, with Stacking sequence of $[-15/15]_s$

For the same load condition and magnitude for both simply supported and clamped boundary condition of composite laminated plate, it is observed that deflection is
much restricted in clamed condition than in simplysupported boundary condition. It is observed that the merging point in case of simply supported boundary condition is (mesh size: 20X20 elements). In clamped boundary condition the merging point is (mesh size: 16X16 elements).

PROBLEM 2

A four layer rectangular plate with simply supported or clamped boundary condition, under uniformly distributed transverse load is taken for the investigation.The deflection at the center of a clamped symmetric angle-ply[-15/15] rectangular laminated composite plate by changing the length from (1 to 10)m and for fixed width(1)m through ANSYS and FEM code and it was found that the results are highly close agreement . This also showed that changing the length does not affect the deflection.

Next the length was fixed at (10) m,and the width was varied from (1 to 10)m. The maximum difference in deflection computed using ANSYS and FEM code is (3.567%) for width of (6.6667)m, and it was also noticed that by changing the width the deflection varied.

Deflection at the center of a simply supported anti-symmetric angle-ply$[-45/45]_s$rectangular laminated composite plate By changing a/b from (0.1 to 1) it was observed that the deflection changed, and maximum percentage of variation between ANSYS and FEM code was (4.9%) at (a/b =0.1).

PROBLEM 4:

Deflection at the center of a clamped symmetric cross-ply and angle-ply square laminated composite plate was studied by varying the angle from (0 to 90) in step of (15) and the best results are found for angle (15 and 75)

Next deflection at the center of a simply supported anti-symmetric cross-ply and angle-ply square laminated composite plate was studied for the same range of angle as in clamped plate and it was observed that the best results are obtained for angle (45)

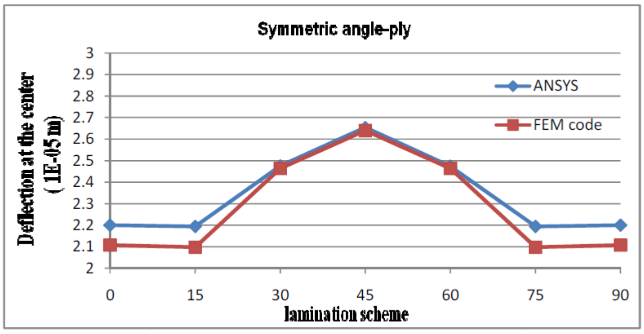


Fig.3.The deflection of symmetric composite plate with clamped bundary condion in different fibers orentation

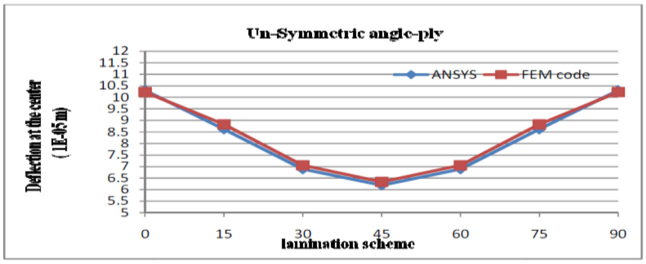


Fig.4.The deflection of anti-symmetric composite plate with simply supported boundary condion in different fibers orentation

PROBLEM 5:

An orthotropic plate with various number of layers is subjected to transverse loading condition for both simply supported and clamped boundary conditions has been considered for the present study, and the results obtained are given tabular form below in tables 4 and 5.

Table4. Deflection at the center of a simply supported symmetric angle-ply $[-45/45]_s$ square laminated composite plate

LAYER	ANSYS (1E-05 m)	FEM code (1E-05 m)	Percentage Variation Between Results %
2	9.949	14.72	32.412
4	7.200	6.890	4.499
6	6.091	5.793	5.014
8	5.882	5.601	5.017
10	5.770	5.500	4.901
12	5.762	5.450	5.724

Table5. Deflection at the center of a clamped symmetric angle-ply [-15/15]s square laminated composite plate

LAYER	ANSYS (1E-05 m)	FEM code (1E-05 m)	Percentage Variation Between Results %
4	2.105	2.097	0.381
6	2.052	2.037	0.736
8	2.042	2.035	0.344
10	2.033	2.029	0.197
12	2.033	2.029	0.197
16	2.032	2.022	0.495

Fig.5. The deflection of symmetric composite plate with simple supported boundary condion in different number of layer(mesh size:3200 element)

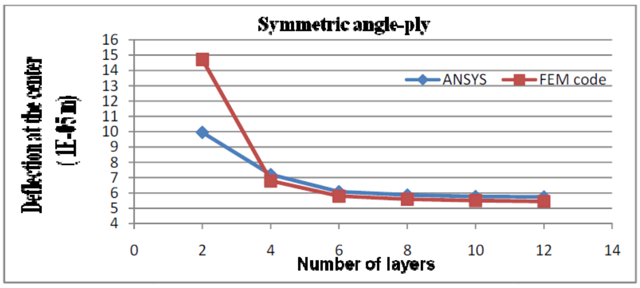
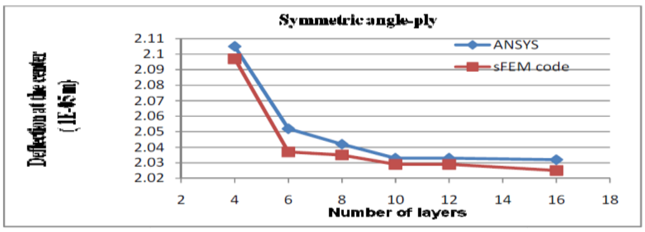


Fig.6. The deflection of symmetric composite plate with clamped boundary condion in different number of layer(mesh size:2048 element)



PROBLEM 6:

In this problem the natural frequency of composite laminated plate (graphite/epoxy) has been studied for both simply supported and clamped boundary condition. No. of layer =4

Table6. Natural Frequency for clamped laminated plate

Natural frequency	Mode.1	Mode.2	Mode.3	Mode.4	Mode.5
$[-15/15]_s$ No. of Plies= 4	108.36	134.49	184.10	256.04	282.04

Table7. Natural Frequency for simply supported laminated plate

Natural frequency	Mode.1	Mode.2	Mode.3	Mode.4	Mode.5
[-45/45/-45/45] No. of Plies= 4	64.692	114.62	144.62	249.21	263.29

V. CONCLUSIONS

The following conclusions are drawn from the present investigation:

1. It is observed that the deflection for clamped boundary condition is less than in simply supported boundary condition for both isotropic and orthotropic plates. In isotropic plate the deflection in clamped plate is about 50% of simply supported. And for orthotropic plate the deflection for clamped is about (25 to 30)% of simply supported.
2. The increase in mesh size gives results with more accuracy when it is compared with experimental work.
3. It shows that the ANSYS program gives a good agreement in results for composite plate when it compared with experimental work.
4. For simply supported and clamped cross-ply composite laminated plate, the deflection is less in case of symmetric arrangement of layers than in anti-symmetric arrangement.
5. For simply supported angle-ply composite laminated plate the deflection is less in case of anti-symmetric arrangement of layers than symmetric case.
6. For clamped angle-ply composite laminated plate the deflection is less in case of symmetric arrangement of layers than anti-symmetric.
7. Minimum deflection for angle-ply simply supported composite plate can be obtained using angle [-45/45] for fiber orientation, and using angle [-15/15]for angle-ply clamped composite plate.
8. The central deflection of composite laminated plate decrease by the increase of layer numbers for the same thickness, but this decreasing in deflection will be neglected after increasing the layer number above ten for both simply supported and clamped composite plate.
9. The deflection for simply supported composite plate depend on both short and long edge dimensions (width and length) of the plate, but in clamped plate depend on the short edge dimension (width) only.
10. The central deflection of clamped composite plate depends on the short edge dimension (width) of the plate.
11. The natural frequency for composite laminated plate in clamped boundary condition is more than in simply supported boundary condition.

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