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Direction of Arrival Estimation on the Performance of WCMSR Technique

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Abstract*— This paper presents direction-of-arrival (DOA) estimation of wideband signals, and wideband covariance matrix sparse representation (W-CMSR) method is proposed. In W-CMSR, covariance matrix is taken such that the lower left triangular elements are aligned to form a new measurement vector. In W-CMSR technique we use constraint of sparsity, sparse representations are those representations that account for most or all information of a signal with a linear combination of a small number of elementary signals called atoms. Often the atoms are chosen from a so called over-complete dictionary. It means that given a signal firstly we form the dictionary which contains the atoms that represent the signal and then after that we find the smallest set of atoms from the dictionary to represent the signal. No prior information of the incident signal is required in W-CMSR, and no decomposition is done. Half-wavelength spacing restriction can be changed from the highest to the lowest frequency of the incident wideband signals.

Index Terms*— Direction-of-arrival (DOA) estimation, over complete representation, sparse representation, wideband signal, source localization.

I. INTRODUCTION

Wideband signals have many applications but widely used in sonar and radar systems, and many methods have been proposed to estimate directions-of-arrival (DOA), such as JLZA. Most of those methods split the incident wideband signals into narrowband components, and then realize wideband DOA estimation with incoherent or coherent techniques. However, there are two significant disadvantages within these kinds of methods. First, DOA pre-estimates of the incident signals are required for spectral focusing, and the precision of those pre-estimates largely influences the performance of DOA estimation. Second, they need a priori information of the incident signal number which may not be available, especially in the non- cooperative scenarios. In W-CMSR technique we use constraint of sparsity, sparse representations are those representations that account for most or all information of a signal with a linear combination of a small number of elementary signals called atoms. Often the atoms are chosen from a so called over-complete dictionary. It means that given a signal, firstly we form the dictionary which contains the atoms that represent the signal and then after that we find the smallest set of atoms from the dictionary to represent the signal. In this paper, we examine the problem of wideband DOA estimation using array output covariance matrix representation under sparsity constraint.

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We put forward a method called wideband covariance matrix sparse representation (W-CMSR). In W-CMSR, the lower left triangular elements of the array output covariance matrix are arranged to form a new measurement vector, and then this vector is reorganized on an over-complete dictionary to find wideband DOA estimation. Due to the representation model used, the a priori information of source number is not necessary in W-CMSR. Meanwhile, W-CMSR deal with the incident wideband signals holistically and do not split them in the frequency domain, thus it does not get into the problem of combining narrowband signal components to obtain the final wideband DOA estimates. Further study shows that W-CMSR owns an essential superiority in making better use of the array geometry to enhance its proficiency of separating multiple simultaneous signals, which may bring the inspiring watershed of more signals than sensors in array geometries. Besides, since the signal information within the signal band is exploited holistically, the half-wavelength restriction can be relaxed for ambiguity avoidance.

This paper explains the area of wideband DOA estimation in the following five aspects.

1) W-CMSR depends less on the a priori information of the incident signal number rather than the ordinary subspace-based methods.

2) No spectral decomposition is required, thus W-CMSR is resistant to imperfect DOA pre-estimates, and will not move into the tremendous efforts of concentrating matrix selection or non-identical narrowband DOA estimate fusion.

3) The a priori information of signal spectrum can be used in W-CMSR to improve the performance of DOA estimation.

4) Well-built array geometries help W-CMSR to obtain increased ability in separating more simultaneous signals than sensors.

5) The half-wavelength spacing condition in avoiding ambiguity is relaxed from the highest to the lowest signal frequency like JLZA-DOA has achieved.

II. PROBLEM FORMULATION

Suppose that ‘K’ wideband signals strike onto an M-element array from different directions $\theta_1, \theta_2, \ldots\ldots\ldots\ldots, \theta_k$ respectively and ‘N’ snapshots are assembled, the snapshot at time is given by:

$$x(t) = \left[\sum_{k=1}^{K} S_K\left(t+\tau_{1,k}\right) + v_1(t), \ldots\ldots\ldots, \sum_{k=1}^{K} S_K\left(t+\tau_{M,k}\right) + v_M(t)\right]$$

Where,

Sk(t) – Waveform of the Kth signal.

Tm,k - Propagation delay of Kth signal from Mth sensor.

Vm - Additive noise at Mth sensor.

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The incident signals and the additive noise are considered to be all Gaussian distributed and mutually independent. Thus the waveforms of the 'K' independent signals satisfy:

$$E[S_{k1}(t)S_{k2}(t)] = 0$$

$$E[S_{k1}(t)S_{k2}^{*}(t)] = \eta_{k1}\eta_{k1}(t_1 - t_2)\delta(k_1 - k_2)$$

Where

$\eta_{k1} \rightarrow$ Power of k^{th} signal

$$\eta_k(\tau) \rightarrow Unified\ correlation = \frac{E[S_k(t+\tau)S_k^{*}(t)]}{\eta_k}$$

It satisfy $\eta_k(\tau) = 0$ (we unify it to make this correlation function immune to signal power.

III. DIRECTION OF ARRIVAL ESTIMATION VIA W-CMSR

The wideband DOA estimation method presented in this section is based on the correlation functions of the wideband signals, which can be obtained from the array output covariance matrix. The flutter-free covariance matrix of the wideband array output is given below:

$$R = \begin{bmatrix} \sum_{k=1}^{K}\eta_k + \sigma_v^2 & \sum_{k=1}^{K}\eta_k r^{*}(\tau_{2,k} - \tau_{1,k}) & \cdots & \sum_{k=1}^{K}\eta_k r^{*}(\tau_{M,k} - \tau_{1,k}) \\ \sum_{k=1}^{K}\eta_k r(\tau_{2,k} - \tau_{1,k}) & \sum_{k=1}^{K}\eta_k + \sigma_v^2 & \cdots & \sum_{k=1}^{K}\eta_k r^{*}(\tau_{M,k} - \tau_{2,k}) \\ \vdots & \vdots & \ddots & \vdots \\ \sum_{k=1}^{K}\eta_k r(\tau_{M,k} - \tau_{1,k}) & \sum_{k=1}^{K}\eta_k r(\tau_{M,k} - \tau_{2,k}) & \cdots & \sum_{k=1}^{K}\eta_k + \sigma_v^2 \end{bmatrix}$$

Where, σ_v^2 is the variance of noise. The directions of the incident signals can be obtained from the correlation family of $\{r(\tau_{M,k} - \tau_{1,k})\}$m1,m2=1,,,,,,M . Many array geometries can be employed to the sensors and direction of arrival can be obtained for every array geometry.

A. *General W-CMSR (GLA)*

The covariance matrix of the array output is conjugate symmetric, so its upper right triangular elements can be replaced by the lower left triangular ones. Because the main diagonal elements are added with unknown noise variance we slide over them and arrange the lower left triangular elements column-by-column to acquire the following measurement vector

$$Y = [R_{2,1}, \dots, R_{M,1}, R_{3,2}, \dots, R_{M,2}, R_{M-1,M-2}, R_{M,M-2}, R_{M,M-1}]^T$$

Where, Rm1,m2 signify the (m1,m2)th element of R. This vector can be split into 'K' components as

$$Y = \sum_{k=1}^{K}\eta_k y_k$$

Where,

$$Y_K = [r(\tau_{2,k} - \tau_{1,k}), \dots, r(\tau_{M,k} - \tau_{1,k}), \dots, r(\tau_{M,k} - \tau_{M-1,k})]^T$$

These signal components depend on the correlation function of the incident signals and their propagation delays along with the array, which are directly linked to the signal directions. Thus, the signal directions can be predicted from if the signal components can be separated. Represent the propagation delay of a signal at direction θ thus

$$Y^{(\theta)} = [r(\tau_2^{(\theta)} - \tau_1^{(\theta)}), \dots, r(\tau_M^{(\theta)} - \tau_1^{(\theta)}), \dots, r(\tau_M^{(\theta)} - \tau_{M-1}^{(\theta)})]^T$$

If one discretizes the [-900 900] scope $\Delta\theta$ then Φ ={-900, -900 +$\Delta\theta$,….900} with interval of $\Delta\theta$, then thus Y can be rewritten in an over-complete condition as

$$Y = Y(\phi)n$$

Where 'n' is a sparse vector and $\{Y^{\phi} = y^{\theta} | \theta \in \phi\}$, Based on the this model, the following restricted sparsity- enforcing objective function can be represented for wideband DOA estimation-

$$\hat{n} = \operatorname{argmin}\|n\|_0, \quad \text{subject to } Y = Y(\phi)n$$

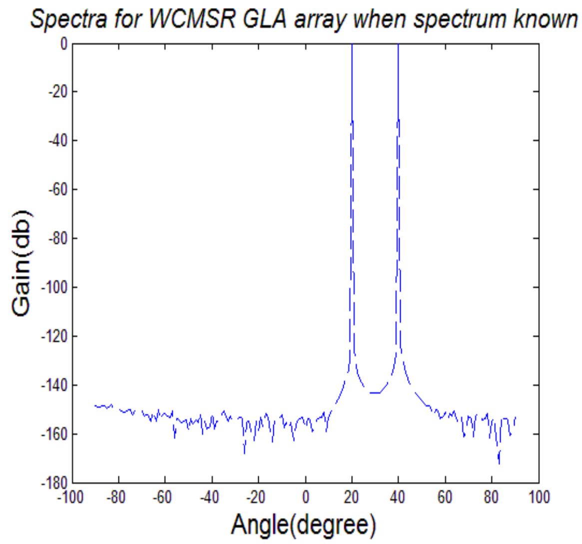
Where $\|.\|_0$→L0-norm, It corresponds to number of non-zero elements.

Argmin → Reduces the number of non-zero elements to get the direction of signals.

The calculation of the over-complete dictionary $Y(\phi)$plays a significant role during the DOA estimation process. Further two techniques are used to find the correlation function:

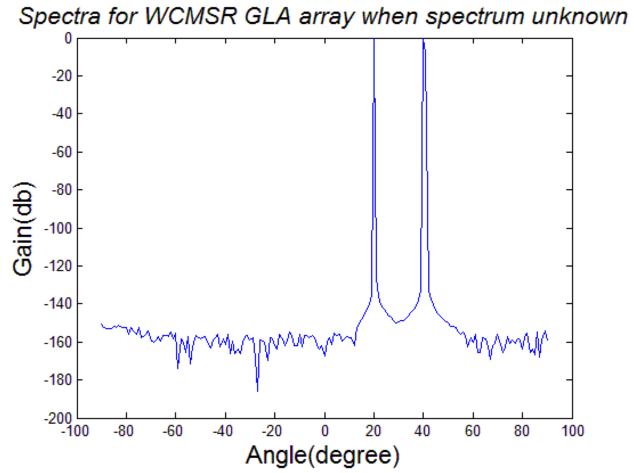
1) It makes use of the a priori information of the signal spectrum- In some cases, the correlation function can be calculated from the prior information of signal modulation such as the amplitude of the unified correlation functions of phase- (PSK) signals own a triangular shape approximately, with its three vertexes located at $(\pm\frac{1}{B}, 0)$ and (0,1) thus if the code rate is given or estimated with other methods, the unified correlation function can be computed straightforwardly as follows:

$$r(\tau) = \begin{cases} (1 - |B\tau| e^{j2\pi f_c t} & |\tau| \le \frac{1}{B} \\ 0 & |\tau| > \frac{1}{B} \end{cases}$$



2) If no prior information is available with us than correlation is found out using array output only in following manner-

$$r(\tau) = \frac{1}{P_1}\int_{\Omega} P(\omega)\, e^{j\omega\tau} d\omega$$



B. *Uniform Linear Array (ULA)*

When one or more pair of the array sensors are equally spaced and repeated elements exist. These elements increase the problem scenario without providing any useful information. Hence, we suggest combining them to simplify W-CMSR and save computational load. To make notation, we take the uniform linear array (ULA) for example, but the subsection analysis can be extended to other array geometries alike with some useful modifications. The covariance matrix of the ULA has a conjugate symmetric Toeplitz structure, so can be represented by its elements along the first column.

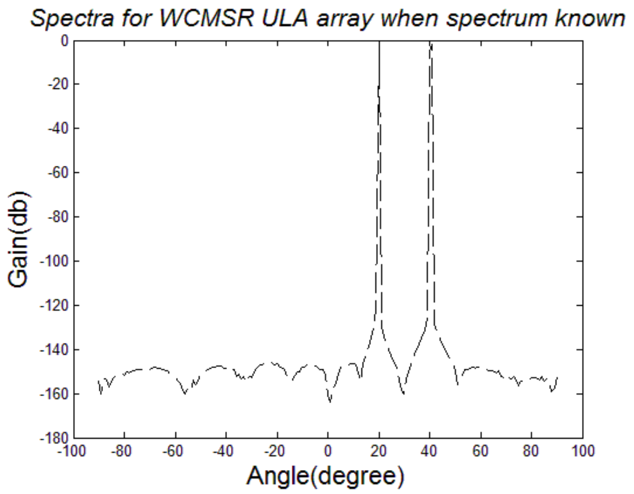
$$A = \begin{bmatrix} a_0 & a_1 & a_2 & \cdot & a_{n-1} \\ a_{-1} & a_0 & a_1 & \cdot & a_{n-2} \\ a_{-2} & a_{-1} & a_0 & \cdot & a_{-1} \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & a_2 \\ \cdot & \cdot & \cdot & \cdot & a_1 \\ a_{-n+1} & \cdot & \cdot & \cdot & a_0 \end{bmatrix} \rightarrow \text{Toeplitz structure}$$

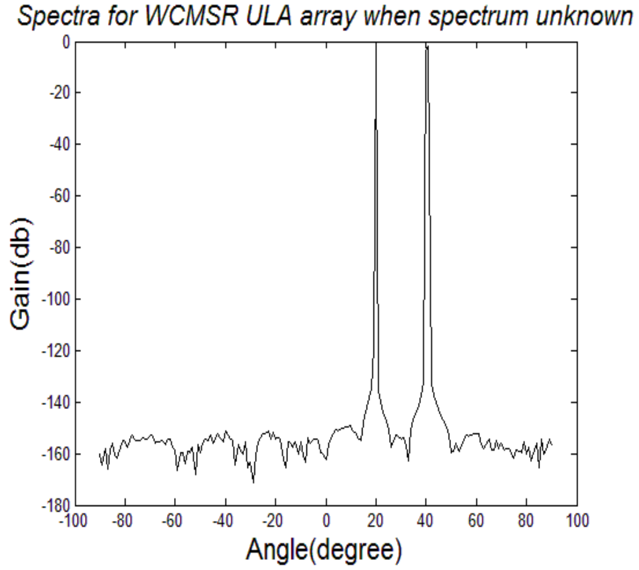
After averaging the diagonal lower left covariance elements, we obtain the following measurement vector:

$$\hat{Z} = [\widetilde{Z}_1, \dots \dots \dots, \widetilde{Z}_{M-1}]$$

Then the spatial distribution of the incident signals can be realised by solving the following constrained L1-norm-based optimization problem-

$$\hat{n} = \operatorname{argmin} \|n\|_1, \ \text{subject to} \ \|\hat{Z} - Z^{\phi} n\|_2 < \beta'$$





IV. RESULT ANALYSIS

Simulations are carried out to see the performance of W-CMSR in wideband DOA estimation, and then compare it with previous methods, like CSSM. Aperture of six half-wavelengths are being used in the simulations i.e., the seven-element ULA and GLA are been simulated. If they are not stated specifically, the half-wavelength is set with respect to the highest signal frequency to avoid ambiguity in all the methods. The optimization tool of SEDUMI is introduced to simplify the L1-norm-based optimization. In other methods, the spatial scope [-900,900] is sampled with an interval of 10. The snapshots are divided into sections of 64 snapshots in CSSM. W-CMSR is being used for correlation computation when the incident signals are PSK ones with code-rate known. And when calculating the correlations we estimate the signal power spectrum $P(\omega)$ from the array output first with the function periodogram in Matlab, and then IFFT is used to compute the correlation at certain time delays. In the simulations, the array output is used at the first sensor to interpret $P(\omega)$ to save computational load. Inter-spacing of the array is relaxed from half-wavelength with respect to the highest signal frequency to that with respect of the lowest signal frequency, in order to demonstrate the half-wavelength relaxation property of W-CMSR. Suppose two 0-dB BPSK signals with central frequency of 70 MHz and bandwidth of 20% onto a 7-ULA from the directions of 20 and 40 degrees is impinged, 256 snapshots are collected. The method of CSSM and W-CMSR (ULA and GLA) with μ'=1.5, are used to estimate the directions of those signals, the signal spectrum for W-CMSR is assumed to be unknown. First, inter-spacing of the array at half-wavelength with respect to the highest signal frequency is fixed then the inter-spacing of the array is enlarged to half-wavelength with respect to the lowest signal frequency, which is one and a half times of the above simulation, and fix the other settings unchanged.

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V. CONCLUSION AND FUTURE WORK

By studying the simulation results we can conclude that W-CMSR is the best technique for direction of arrival estimation so far because of the following points:

i. Angle of arrival has sharp peaks.
ii. Spectral decomposition of wideband signals is not required, hence DOA is more accurate.
iii. Resolution is high.
iv. No prior information about the unknown sources is needed to obtain their angle of arrival.
v. Half-wavelength spacing in avoiding ambiguity is relaxed from the highest to the lowest signal frequency.

Thus we can see that W-CMSR technique several advantages and the most important ones are that no prior information of number of incident signal is required and also the wideband signal is treated holistically that is no splitting into narrow bands is required and also half wavelength spacing condition in avoiding ambiguity is been relaxed from the highest to the lowest signal frequency. From the work presented in this paper it is clear that there can be some improvements and potential for continued work. Although the proposed method overcome problems of few items that could be addressed more deeply which include the physical locations of array elements which may be determined by enabling optimization algorithms to gain more reliable estimates of the parameters linked to the DOA problem. This way, estimating precise DOAs at low SNR values of the source signals having small angle resolutions will be possible.

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