Development of Mathematical Model for Coupled Tank System using System Identification (SI)

S. Y. S. Hussien, H. I. Jaafar, N. A. Selamat, E. F. Shair, A. F. Z. Abidin

Abstract*—*This paper presents* the *development of mathematical model for Coupled Tank System (CTS) using System Identification (SI) method. In this research, real model of CTS-001 is used as a medium to generate transfer function experimentally. By 1058 input and output data from the execution process are recorded and analyzed to develop a model. Implementation of SI toolbox is applied and explained clearly in order to generate a model of CTS. Then, the model of CTS is tested via open and closed loop method to observe the stability and transient responses of the system. This output response from the modeling function can be taken as the benchmark to achieve a good response for future implementation of CTS. All the execution and performances can easily be monitored in MATLAB environment.

Index Terms*—*Coupled Tank System (CTS), Mathematical Model, System Identification (SI), Transfer Function.

I. INTRODUCTION

The Coupled Tank System (CTS) apparatus model CT-001 consists of two small tower tanks (Tank 1 and Tank 2). At the bottom of the CTS consists of a reservoir, which functions as storage for the water. It is applied in various fields such as liquid storage tank, feeding tank, product tank, the intermediate buffer containers and water tanks [1-2]. Water is pumped into the top of each tank by two independent pumps [3]. The level of water in each may visually read on the attached scale at the front of the tanks. It is illustrated in Figure 1 [4-5]. Each tank is fitted with an outlet, at the side near the base, and this outlet is connected by a plastic hose, which returns the water to the reservoir. The amount of water which returns to the reservoir is approximately proportional to the level of water in the tank since the plastic water-return tube at the base of the tank functions as a pseudo-linear hydraulic resistance. A screw clamp valve is provided on the tubing to increase this resistance. The level of water in each tank is monitored by a capacitive-type probe, which in conjunction with electronic circuits in the box at the rear of the unit, provided an output signal proportional to the water level. For normal operation, this DC output voltage is in the range 0 to ±5 Volts, for which, the zero level represents the rest point of the water when the tank is empty (approximately 20 mm), and the full state is when the water level begins to overflow down the rear plastic standpipes, this occurring at approximately 300 mm.

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More complex tank arrangements can be easily simulated by using the internal baffle which controls leakage between the two tanks. By turning the wing-nut on the top of the tank assembly, the baffle will be raised a little bit sufficient to provide useful ranges of inter-tank resistance. A spring returns the baffle to the closed position when the wing-nut is released [6].

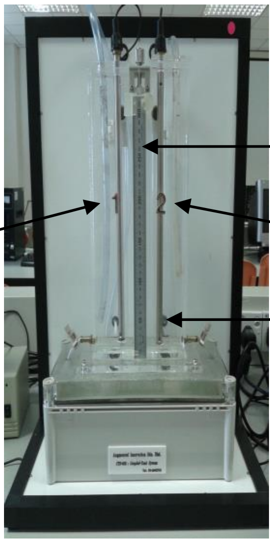

Figure 1: Coupled Tank CTS-001

II. PROCEDURE FOR SYSTEM IDENTIFICATION

A. Calibration Process

In order to obtain the transfer function, data from the experimental are collected. Thus, some calibration needs to be done in order to make sure the experimental process is accurate. The calibration data are shown in Table 1, Table 2 and Table 3. These data are captured by DAQ card.

Table 1: Calibration readings for Tank 1 level sensor with pump voltage of 1.6 V

Region 1		Region 2		Region 3	
Water Level (cm)	Voltage (V)	Water Level (cm)	Voltage (V)	Water Level (cm)	Voltage (V)
1.70	0.00	1.70	0.00	1.70	0.00
5.00	0.02	5.00	0.02	5.00	0.01
10.00	1.10	10.00	1.11	10.00	1.12
15.00	2.14	15.00	2.14	15.00	2.12
20.00	3.05	20.00	3.07	20.00	3.04
25.00	3.95	25.00	3.97	25.00	3.95
30.00	5.00	30.00	5.00	30.00	5.00

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Table 2: Calibration readings for Tank 2 level sensor with pump voltage of 1.8 V

Region 1		Region 2		Region 3	
Water Level (cm)	Voltage (V)	Water Level (cm)	Voltage (V)	Water Level (cm)	Voltage (V)
1.7	0.00	1.7	0.00	1.7	0.00
5.00	0.58	5.00	0.59	5.00	0.53
10.00	1.70	10.00	1.70	10.00	1.68
15.00	2.48	15.00	2.48	15.00	2.41
20.00	3.23	20.00	3.21	20.00	3.25
25.00	4.04	25.00	4.04	25.00	4.01
30.00	4.91	30.00	4.95	30.00	4.92

Table 3: Calibration of the controlled inflow (pump)

Input voltage to pump, U	Initial height, H_i (cm)	Final height, H_f (cm)	Volume, V (cm^3)	Time taken to fill up V, T	Rate of change of volume $Q=\frac{V}{T}$
0.0	1.7	1.7	54.4	0.00	0.00
2.2	1.7	15.0	480.0	13.84	34.67
2.4	1.7	15.0	480.0	11.31	42.44
2.6	1.7	15.0	480.0	9.77	49.13
2.8	1.7	15.0	480.0	9.51	50.45
3.0	1.7	15.0	480.0	9.35	51.31
3.2	1.7	15.0	480.0	9.18	52.28
3.4	1.7	15.0	480.0	8.14	58.99
3.6	1.7	15.0	480.0	7.89	60.80
3.8	1.7	15.0	480.0	7.84	61.25
4.0	1.7	15.0	480.0	7.29	65.87
4.2	1.7	15.0	480.0	7.53	63.75
4.4	1.7	15.0	480.0	7.19	66.77
4.6	1.7	15.0	480.0	7.48	64.16
4.8	1.7	15.0	480.0	7.84	61.11
5.0	1.7	15.0	480.0	7.47	64.23

B. DAQ Card

DAQ card is used to communicate data between controller and plant. The required signal is sent from the controller to control the pump at the tank through DAQ card. The signal cannot send directly to the tank because there is no USB connection at the tank which is why DAQ card is used. After the signal is sent, the sensor and actuator in the tank react in the feedback loop and send the information signal to the controller for the next iteration. The process is still the same even though it is in the opposite direction as shown in Figure 2.

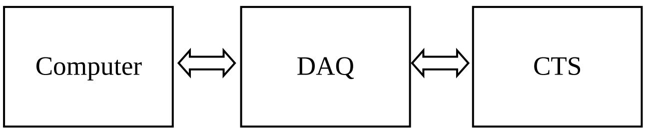

Figure 2: Communication path between computer and CTS

C. System Identification

System Identification (SI) is the process of developing or improving the mathematical representation of a physical system using experimental data. The SI is useful to generate a transfer function that is represented as a system model before designing any suitable controller for control purpose [7]. The identification process is more useful, especially in industrial production processes [8-9]. Efficient and effective control of these processes have immense economical advantages and its success depends on the type of control strategy such as the liquid is required to be maintained at a specific height or a certain range [10-11]. The SI tool can be opened by using MATLAB with the coding of "*ident*". This tool is used to compare two types of data and develop the transfer function. Figure 3 shows the SI tool to model the system.

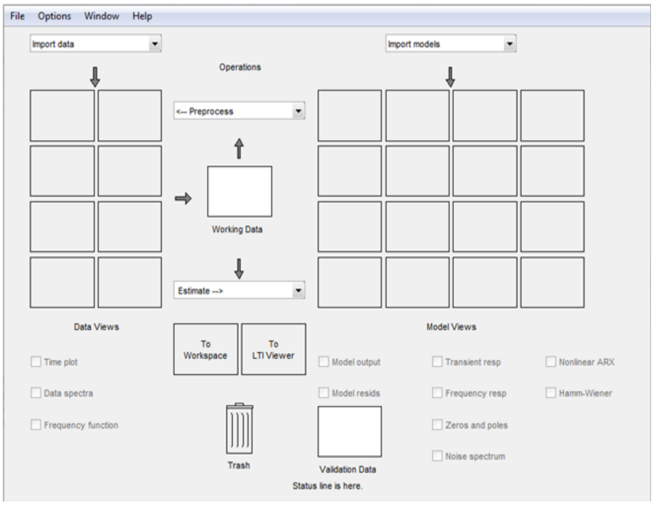

Figure 3: SI tool

Figure 4 shows the two types of data as the inputs to model the system. The "*mydata*" is selected as the first input and "*mydata1*" as the second input. The sampling time or interval for the data is 0.7.

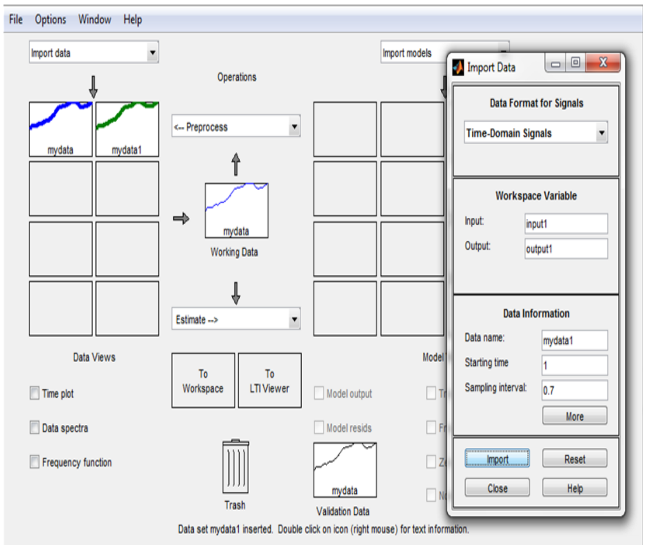

Figure 4: Selection data of first and second input

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Figure 5 shows the first input, "*mydata*" is selected as the working data and "*mydata1*" is selected as the validation data. Then, this tool compared the data between working data and validation data based on the model selected (*transfer function model*) for this system as illustrated in Figure 6.

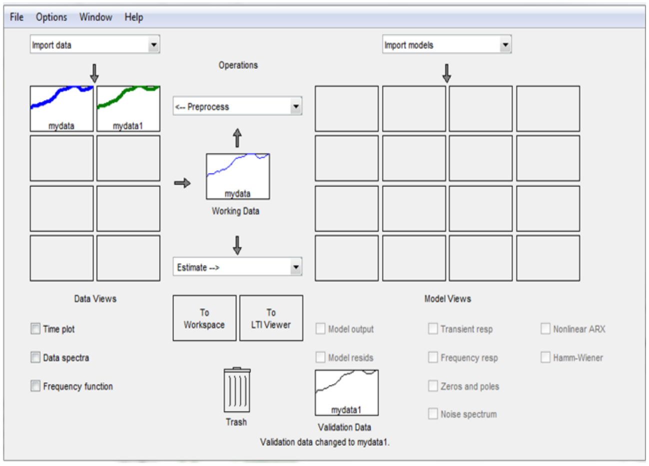

Figure 5: Data selection in working data and validation data

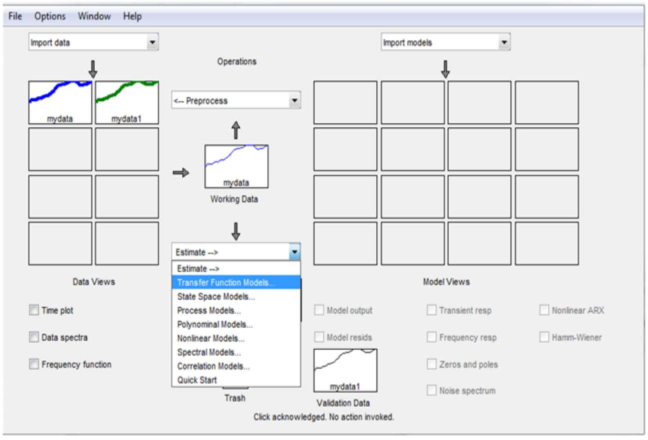

Figure 6: Model selection (Transfer Function Model)

Based on the model, number of poles and number of zeros were determined according to the characteristic of the system. Then, the SI tool will estimate the transfer function for the CTS model. Based on the characteristics of CTS, the number of poles is 2 and number of zero is none as shown in Figure 7.

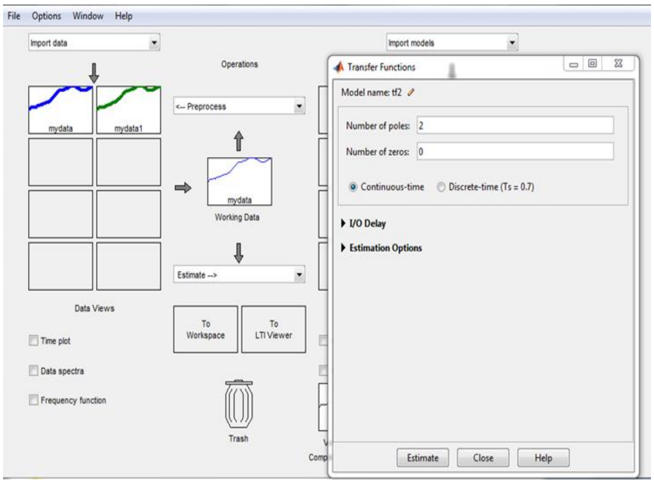

Figure 7: Characteristics selection based on poles and zeros number

Once the number of poles and zeros have been determined, measured and simulated model output can be observed. According to Figure 8, the output model for CTS shows best fit as 83.25%. Based on [7], it is considered accepted because it is more than 80%. Then, the complete transfer function model can be represented as shown in Figure 9.

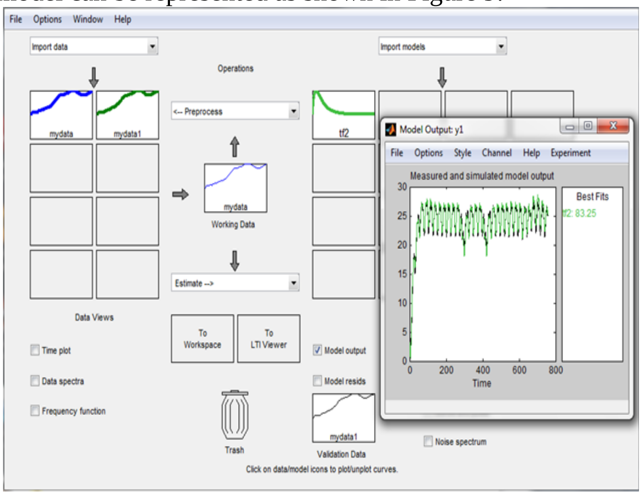

Figure 8: Model output of CTS in the SI

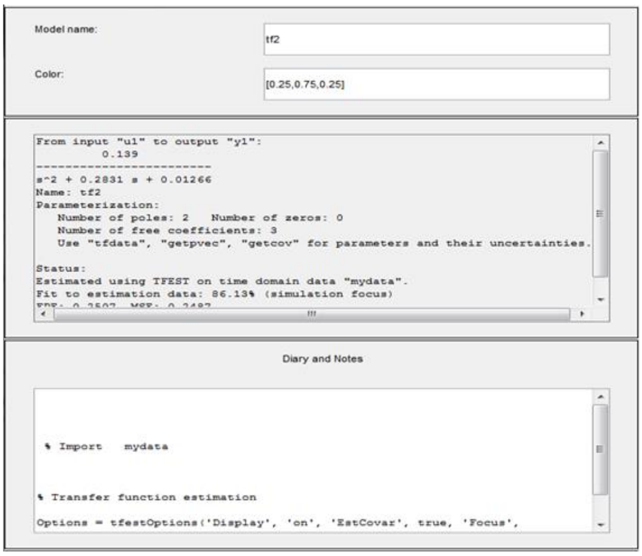

Figure 9: Transfer function of CTS

Thus, the actual transfer function of the CTS with the complete mathematical model can be presented as:

$$G_p(s) = \frac{0.139}{s^2 + 0.2831s + 0.01266} \tag{1}$$

III. STABILITY OF THE COUPLED TANK SYSTEM

The stability of the system can be determined by using Routh-Hurwitz stability criterion. It is a mathematical test that is necessary and sufficient condition for the stability of a Linear Time Invariant (LTI) control system. Table 4 shows the Routh-Hurwitz table for the transfer function of the CTS as in (1) based on the characteristic equation. The system is considered stable only if the coefficients in the first columns are all positives.

Characteristic equation:

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$$1 + KG_p(s)H(s) = 0 \tag{2}$$

$$1 + \frac{K(0.139)}{s^2 + 0.2831s + 0.01266} = 0 \tag{3}$$

$$s^2 + 0.2831\,s + 0.01266\ + 0.139\,K\ = 0 \tag{4}$$

Table 4: Routh-Hurwitz table for CTS

s^2	1	0.01266+ 0.139K	0
s^1	0.2831	0	0
s^0	$-\dfrac{\begin{vmatrix} 1 & 0.01266+0.139K \\ 0.2831 & 0 \end{vmatrix}}{0.2831}$ $= -\dfrac{[0-0.2831\,(0.01266+0.139K)]}{0.2831}$ $= 0.01266+0.139K$	0	0

According to Table 5, CTS is stable. It can be proved by the coefficients sign in the first column that is all positives sign.

Table 5: Summarize of Routh-Hurwitz table

s^2	(+) 1
s^1	(+) 0.2831
s^0	(+) [0.01266+0.139K]

IV. Result and Discussion

The open loop system is a system without a controller. The plant of the CTS is obtained from the SI in the previous section. The input voltage injected in the system is 1 Volt and the level converter will convert the input voltage to the water level which for this case the level is 1 cm. The block diagram and the response of the system are shown in the Figure 10 and Figure 11. Table 6 shows the performance of the transient responses for the system in terms of Settling Time (Ts), Rise Time (Tr), Overshoot (OS) and Steady State Error (SSE).

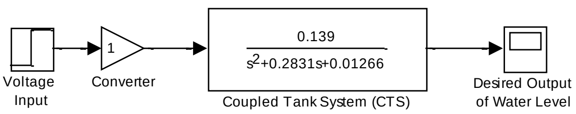

Figure 10: Block diagram of open loop system for CTS

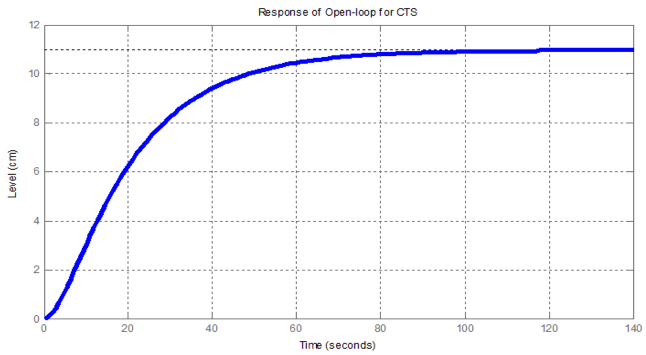

Figure 11: Response of open loop system for CTS

Table 6: Transient responses of the open loop CTS

Method	Ts (s)	Tr (s)	OS (%)	SSE (cm)
Open loop	75.3274	41.4668	0.0000	9.9721

As expected, the value of SSE is high (9.9721 cm) and cannot be controlled due to no implementation of any controller or closed loop process. Even though, the percentage of OS is zero, but it needs a long time to stabilize. From the open loop analysis, the transient responses can be improved. Thus, the open loop system is converted into a closed loop system due to safety or production restriction reasons [12]. The unity feedback is added which makes the system now in the closed loop system as shown in Figure 12. Based on the closed loop response in Figure 13, the percentage of OS is increased to 29.3131%. However the Ts and Tr is reduced at 27.8203 s and 3.6258 s compared to open loop. Even though the output of the water level still does not achieve the desired water level, the SSE can be reduced as minimum as 0.1852 cm. It is difficult to reach the desired control response with short transition time and small overshoot without any controller implementation [3]. This is one of the reasons why the appropriate controller with suitable tuning method is useful to balance all the transient response to a minimum value [13]. Table 7 shows the closed loop response for CTS.

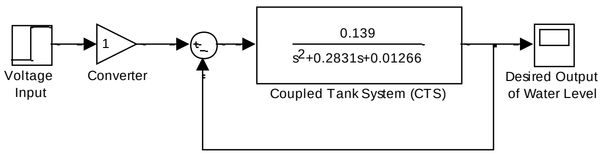

Figure 12: Block diagram of closed loop system

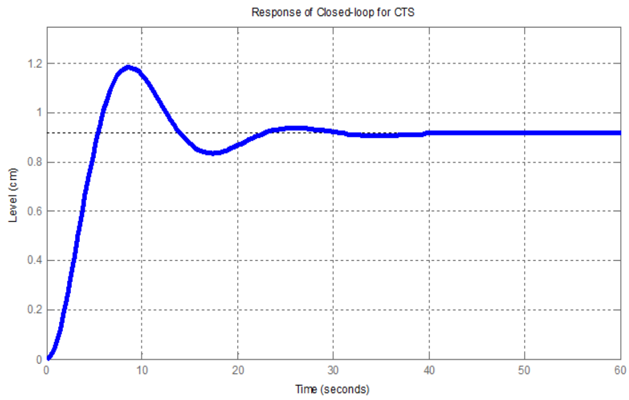

Figure 13: Response of closed loop system

Table 7: Transient responses of the closed loop CTS

Method	Ts (s)	Tr (s)	OS (%)	SSE (cm)
Closed-loop	27.8203	3.6258	29.3131	0.1852

V. Conclusion

In this project, development of mathematical model of Coupled Tank System (CTS) using System Identification (SI) is successfully obtained. The overall process is started from calibration and experimental data collection based on real model CTS-001. All the 1085 data are recorded and used in the MATLAB SI toolbox to infer a model in the transfer function form.

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It can be concluded that the transfer function of CTS obtained is a good model since the best fit of the simulated and simulated model output is higher than 80%. Then, the transfer function is successfully tested via open and closed loop method to observe the stability and transient responses of the system. Therefore, this model of transfer function can be used for other experiment for example level control of CTS by using a certain controller with the appropriate tuning method. By using this approach, a good transient response and better performance can be achieved.

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