

Performance Comparison for the Design of Discrete Fourier-Invariant Signals

C Ilayarasu, K Boopathy Bagan, S Kanithan

Abstract— In this paper, the design methodology minimizes the difference between the signal and its spectrum using gradient based iterative method. The proposed method reduces the number of iterations and simulation time compared with the existing method. The novelty method of design includes discrete Fourier-invariant signals with minimum time-width (T) and bandwidth (B) product. These methods achieve theoretical Gabor lower bound on BT product. Finally, we show how the proposed discrete Fourier-invariant signals with minimum bandwidth time-width product are not affected by noise with the help of wavelet processing.

Index Terms— Eigenfunctions, Gabor uncertainty principle, shape invariant signals.

I. INTRODUCTION

Fourier transform is the tool which connects time and frequency domain of the signal [1]. For the signal $x(t)$ the Fourier transform $X(f)$ is given by

$$X(f) = \int_{-\infty}^{\infty} x(t) e^{-j2\pi ft} dt \quad (1)$$

The inverse Fourier transform is given by,

$$x(t) = \int_{-\infty}^{\infty} X(f) e^{j2\pi ft} dt \quad (2)$$

For the discrete Fourier transform the eigenvector of the transform matrix W is given by, $Wx = \lambda x$ where λ is the eigenvalue. The discrete Fourier transform has only four Eigenvalues: $\lambda = \pm 1$ and $\lambda = \pm j$. It is also verified that, transform matrix satisfying $W^4 = I$ form an equation $\lambda^4 = 1$ which gives four eigenvalues $\lambda = +1, -1, +j, -j$. The eigenvector for the eigenvalue $\lambda = 1$ will provide $Wx = x$. This equation tells the fact that, for any signal vector x and its discrete Fourier transform (Wx) will be same when $\lambda = 1$. An infinite number of eigenfunctions exist for each Eigenvalue. Eigen function plays a vital role in finding signal with the minimum BT product is found in [2]. A method for the generation of eigenfunctions and a set of all eigenvalues are discussed in

[3]. These results shows eigenfunctions have the same shape in both time and frequency domain. These results are obtained by adding any even signal and its Fourier transform.

$$X(f) = \lambda x(f) = \int_{-\infty}^{\infty} x(t) e^{-j2\pi ft} dt \quad (3)$$

Where the scalar λ is the eigenvalue and $x(f)$ is the Eigen function. Eigen function is invariant under the Fourier transform. In this paper eigenfunctions are called shape invariant signals. The objective of this paper is to design signals which are invariant in both time and frequency domains. In chapter two related works on eigenfunctions, eigenstructure of discrete Fourier transform are discussed. In chapter three the design methods of discrete Fourier-invariant signal are discussed. Experimental results and its explanations are given in chapter four. A summarization of experimental results and an outlook of further investigations are given in chapter five, finally accompanied with references.

II. RELATED WORKS

The existing method for the design of discrete Fourier-invariant signal is presented in [4]. It is given [5] that the Hermite-Gaussian functions are also eigenfunctions of the Fourier transform. In that paper it is shown that, Fourier eigenfunction achieve isoresolution in joint time-frequency analysis. Finally, other possible shape invariant functions, such as the Gaussian function; the hyperbolic secant or the Dirac series are discussed in [2]. The eigenstructure of the discrete Fourier transforms (DFT) and its applications has been broadly investigated in [6]. A method for computing an orthonormal set of eigen vectors for the discrete Fourier transform are given in [7]. Multiplicities eigenvalues of the generalized Discrete Fourier transform and method of generating eigenvectors are investigated in [8]. Fractional versions of Fourier transforms and its applications are also analyzed in that paper. The necessities of discrete fractional Fourier transform and its availability to the orthonormal eigenvectors are discussed in [9]. These orthonormal eigenvectors are basis for developing centered discrete fractional Fourier transform. Discrete Hermite-Gauss signals, which are a eigenvector of DFT are used for defining a discrete fractional Fourier transform, and the concepts of discrete fractional Fourier transform are discussed in [10]. The concepts on multi angle centered discrete Fourier transform are presented in [11].

Manuscript published on 30 August 2014.

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Gabor's basic principle on information and how the information can be transmitted from one place to other are discussed in [12]. Finally, the more information about definition and processing of signals are presented in [13].

III. DESIGN METHOD

The standard discrete Fourier transform of transforming size (N) map time samples {0, 1, 2, 3...N} into frequency sample points {0, 1/N... (N-1)/N}. The centered discrete Fourier transform (CDFT) [11] of the signal x(n) is defined by,

$$X(k) = \frac{1}{\sqrt{N}} \sum_{n=1}^N x(n) e^{-j2\pi \frac{(n-\frac{N+1}{2})(k-\frac{N+1}{2})}{N}} \quad (4)$$

Inverse CDFT for the spectrum X(k) is given by,

$$x(n) = \frac{1}{\sqrt{N}} \sum_{k=1}^N X(k) e^{-j2\pi \frac{(n-\frac{N+1}{2})(k-\frac{N+1}{2})}{N}} \quad (5)$$

Where x(n) is signal samples and X(k) are spectral samples. For CDFT, the time sampling points is centered around the zero time $\{-(N-1)/2, -(N-3)/2, \dots, (N-1)/2\}$ and the frequency sampling points are centered around the zero frequency $\{-(N-1)/2N, -(N-3)/2N, \dots, (N-1)/2N\}$. To make easier direct comparisons with frequency sampling points the time sampling points are additionally normalized to the transform size N. The normalized time sampling points and the frequency sampling points are given by,

$$\begin{aligned} -0.5 < t(n) = \frac{n - \frac{N+1}{2}}{N} < 0.5 \\ -0.5 < f(k) = \frac{k - \frac{N+1}{2}}{N} < 0.5 \end{aligned} \quad (6)$$

For k=1, 2...N and n=1, 2...N

The CDFT has the important property that, when N is a multiple of four, all four of its eigenvalues have equal multiplicities [6]. Important properties of discrete Fourier-invariant signals are symmetric, Identity between complex and real forms of the Fourier transform [1]. Symmetry property states that, any signal satisfying discrete Fourier-invariant condition must be even signal. So, the CDFT of the signal x(n) in matrix form is given by,

$$X = Wx, \quad W = \begin{pmatrix} W(1,1) & \dots & W(1,N) \\ \vdots & \ddots & \vdots \\ W(N,1) & \dots & W(N,N) \end{pmatrix} \quad (7)$$

Where

$$W(n,k) = \frac{1}{\sqrt{N}} \cos\left(\frac{2\pi \left(n - \frac{N+1}{2}\right) \left(k - \frac{N+1}{2}\right)}{N}\right) \quad (8)$$

For n=1, 2... N and k=1, 2...N

The definition for signal bandwidth (B), time-width (T) is found in Gabor's uncertainty principle [12].

$$B = \sqrt{\frac{\sum_{k=1}^N (f(k) - \bar{f}) \cdot |X(k)|^2}{\sum_{k=1}^N |X(k)|^2}} \quad (9)$$

$$\bar{f} = \frac{\sum_{k=1}^N f(k) \cdot |X(k)|^2}{\sum_{k=1}^N |X(k)|^2} \quad (10)$$

$$T = \sqrt{\frac{\sum_{n=1}^N (t(n) - \bar{t}) \cdot |x(n)|^2}{\sum_{n=1}^N |x(n)|^2}} \quad (11)$$

$$\bar{t} = \frac{\sum_{n=1}^N t(n) \cdot |x(n)|^2}{\sum_{n=1}^N |x(n)|^2} \quad (12)$$

Gabor's lower bound [12] on BT product of a signal is given by,

$$B.T \geq \frac{1}{4\pi N} \quad (13)$$

A. Existing Method

Existing method is an iterative design method based on minimization of maximum difference between signal vector and its CDFT spectrum [1].

The initial signal can be any symmetrical discrete signal given by,

$$x_n^{(0)} = x_{N+1-n}^{(0)} = s_n^{(0)} \text{ for } n = 1, 2, \dots, (N+1)/2 \quad (14)$$

In the k-th iteration the CDFT spectrum $S^{(k)}$ of the signal $s^{(k)}$ is given by,

$$S^{(k)} = Ws^{(k)} \quad (15)$$

Where



$$W = \begin{pmatrix} W(1,1) & \dots & W(1,P) \\ \vdots & \ddots & \vdots \\ W(P,1) & \dots & W(P,P) \end{pmatrix}, P = \frac{(N+1)}{2} \quad (16)$$

$$W = \begin{pmatrix} W(1,1) & \dots & W(1,P) \\ \vdots & \ddots & \vdots \\ W(P,1) & \dots & W(P,P) \end{pmatrix}, P = \frac{(N+1)}{2} \quad (24)$$

$$W(n, k) = \frac{2}{\sqrt{N}} \cos\left(\frac{2\pi\left(n - \frac{N+1}{2}\right)\left(k - \frac{N+1}{2}\right)}{N}\right) \quad (17)$$

$$W(n, k) = \frac{2}{\sqrt{N}} \cos\left(\frac{2\pi\left(n - \frac{N+1}{2}\right)\left(k - \frac{N+1}{2}\right)}{N}\right) \quad (25)$$

For n=1, 2... P and k=1, 2...P

For n=1, 2... N and k=1, 2...N

The difference vector $e^{(k)}$ and maximum difference between the signal and its spectrum are given by,

$$e^{(k)} = s^{(k)} - S^{(k)} \quad (18)$$

The difference vector $e^{(k)}$ between the signal and its spectrum are calculated by,

$$e_{max}^{(k)} = |e_{n_{max}}^{(k)}| = \max_{n=1, \dots, P} |e_n^{(k)}| \quad (19)$$

$$e^{(k)} = s^{(k)} - S^{(k)} \quad (26)$$

For each iteration maximum difference and its position are used for calculating the gradient of square maximum difference. The gradient of the square maximum difference between signal and its spectrum for n=1, 2 P is calculated by,

The gradient of the squared difference between signal and its spectrum for n=1, 2 N is calculated by,

$$g_n^{(k)} = \frac{\partial (e_{n_{max}}^{(k)})^2}{\partial s_n^{(k)}} = \begin{cases} -2e_{n_{max}}^{(k)} W_{n_{max},n} & n \neq n_{max} \\ 2e_{n_{max}}^{(k)} (1 - W_{n_{max},n}) & n = n_{max} \end{cases} \quad (20)$$

$$g_n^{(k)} = \frac{\partial (e_n^{(k)})^2}{\partial s_n^{(k)}} = 2e_n^{(k)} (1 - W(n, n)) \quad (27)$$

Finally a new signal with a minimum maximum difference between signal and its spectrum is given by,

Finally a new signal with a minimum difference between the signal and its spectrum is given by,

$$s^{k+1} = s^k - \Delta \cdot g_n^{(k)} \quad (21)$$

$$x^{k+1} = x^k - \Delta \cdot g_n^{(k)} \quad (28)$$

This obtained new signal is used as an initial signal for the next iteration. Iterations will be continued until the maximum difference reaches below the specified threshold. Extension of symmetrical part (s_{N+1-n}) will provide full version of the shape invariant signal.

This obtained new signal is used as an initial signal for the next iteration. Iterations will be continued until all the elements in difference vector reaches below the specified threshold. where Δ is the loop gain. Δ has to be made as large as possible for faster convergence. But, due to the stability problem and comparison with existing method Δ of 0.2 is applied in this paper .

Where Δ is the loop gain. Δ has to be made as large as possible for faster convergence. But, due to the stability problem Δ has a limiting value and based on evaluation loop gain of $\Delta=0.2$ applied.

C. Discrete Fourier-Invariant Signal with Minimum BT Product

B. Proposed Method

This is an iterative design method based on minimization of the difference between the signal and its spectrum instead of minimizing of maximum difference in the existing method. At the start of iteration initial signal can be any symmetrical discrete signal is given by,

In the existing approach, the obtained solution for lower size N are extended with zeros and used as the initial signal for following N. The existing iterative design procedure is repeated for each N to find a discrete Fourier-invariant signal. This will provide a discrete Fourier-invariant signal with minimum BT product. Let the initial signal for the design be $y(n)=[y(1) y(2)\dots y(N)]$. In the proposed method, the obtained solution for any value of N from the proposed iterative method is modified and it is used as the initial signal for the next iteration. Discrete Fourier-invariant signal is created with each iteration which has minimum BT compared to previous iterate. Iterations will be continued until the obtained BT product close to the Gabor's principle on the minimum BT product of a signal. Because of the full version of the signal is used for the design the transform matrix is given by,

$$x_n^{(0)} = x_{N+1-n}^{(0)} = s_n^{(0)} \text{ for } n = 1, 2, \dots, \frac{N+1}{2} \quad (22)$$

$$W = \begin{pmatrix} W(1,1) & \dots & W(1,N) \\ \vdots & \ddots & \vdots \\ W(N,1) & \dots & W(N,N) \end{pmatrix} \quad (29)$$

In the k-th iteration the CDFT spectrum $X^{(k)}$ of the signal $x^{(k)}$ is given by,

$$S^{(k)} = W S^{(k)} \quad (23)$$

Where

$$W(n, k) = \frac{1}{\sqrt{N}} \cos\left(\frac{2\pi\left(n - \frac{N+1}{2}\right)\left(k - \frac{N+1}{2}\right)}{N}\right) \quad (30)$$

The modifications done for obtaining initial signal of next iteration is given by,

$$x_i(n) = [x(1)/10, x(N)/10, x(2), x(3), \dots, x(N-1)]^T \quad (31)$$

Where, $x_i(n)$ is an initial signal for the next iteration. After the final iteration results without modifications will provide discrete Fourier-invariant signal with a minimum BT product which is close to the Gabor's lower bound. A signal with any BT product can be easily obtained by varying the number of iterations. This method also provides a better BT reduction compared to the existing method.

IV. EXPERIMENTAL RESULTS AND PERFORMANCE ANALYSIS

The results of the design of discrete Fourier-invariant signal and performance comparisons of proposed, existing method are summarized below. The results of the design of discrete Fourier-invariant signal with minimum BT product and its possible application are also summarized. The results are evaluated using Matlab simulation tool. Figure 1 illustrates proposed design method for a triangular shaped input signal. It also shows how the shape of initial signal is modified using gradient based iterative method to get discrete Fourier-invariant signal. Figure 1 shows an initial signal and its CDFT spectrum, discrete Fourier-invariant signal and its CDFT spectrum. In this approach all the elements in a difference vector reaches below the threshold value of 10^{-10} after 19 iterations.

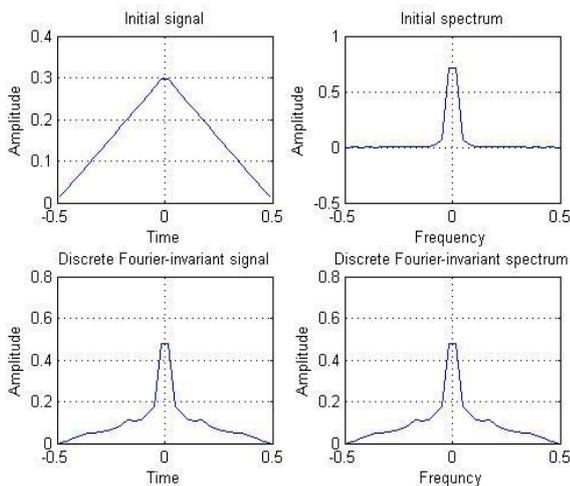


Fig 1: Proposed design of discrete Fourier-invariant signals for N=32; $|e_n| \leq 10^{-10}$ achieved after 19 iterations.

Figure 2 illustrates proposed design method for a rectangular pulse of Transform size N=1015. It shows the initial CDFT spectrum for a rectangular pulse as a sinc pulse. The discrete Fourier-invariant signal is obtained with 16 iterations. Even though the proposed method based on minimizing a difference vector for a comparison with the existing method maximum difference versus the number of iterations is illustrated in Figure 3. It shows the proposed design require less number of iterations compared to the existing method. The existing method focuses on reducing the maximum

difference between signal and spectrum in each iteration. But, the proposed method, in each iteration every element has minimum difference vector compared to the existing method. Thus, instead of reducing only one element in the difference vector in the existing method, the proposed method concentrates on reducing all the elements in difference vector. Because of every element in a difference vector is minimized, the every element of gradient vector decreases abruptly in every iteration. Due to abrupt decrease of elements in the gradient vector in each iteration the proposed method takes less number of iterations compared to the existing method. This comparison is made by taking the triangular input signal of size N=32 and same loop gain of 0.2 is applied in both existing and proposed method.

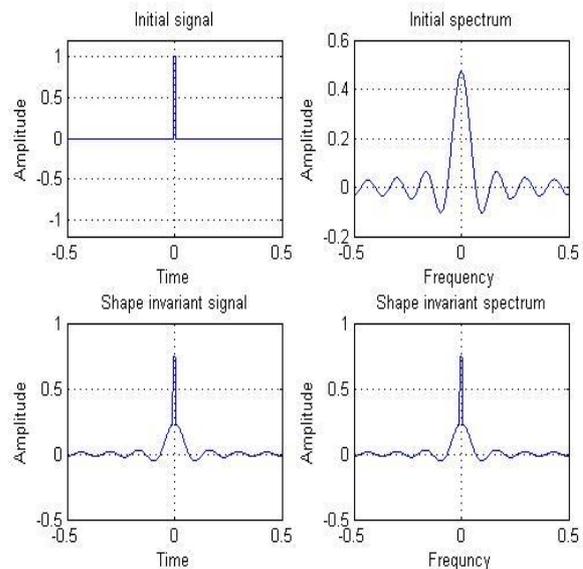


Figure 2: Proposed design method for the rectangular pulse of size N=1015

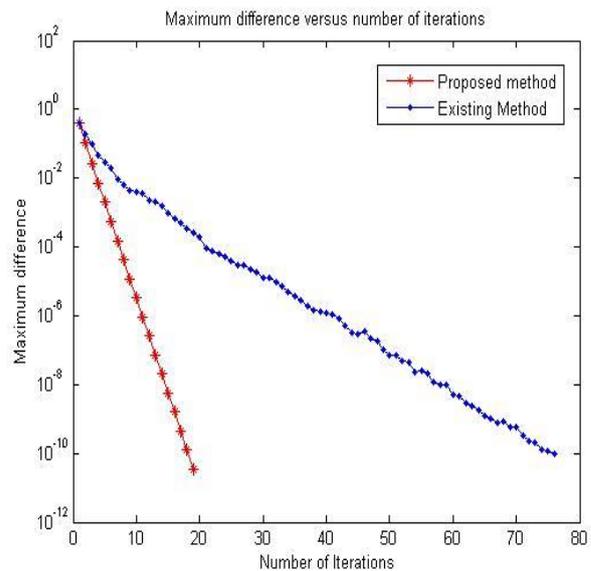


Figure 3: Maximum difference versus Number of iterations for N=32.

Figure 4 shows how the proposed method responds to the various values of loop gain. It can be revealed that, for a loop gain of 0.2 achieve a less number of iterations compared to the other loop gain. Results are obtained by taking the cosine input signal of size $N=32$. Figure 5 illustrates the number of iterations required by triangular shaped input signal of different transform size(N).In the existing method the rate of decrease of gradient vector in each iteration is slow to increase with transforming size (N).For example, gradient vector after 200 iterations is in the order of 10^{-9} of transform size $N=100$.But for a transform size $N=200$,the gradient vector after 200 iterations is in the order of 10^{-6} .In the proposed method the rate of decrease of gradient vector is same for any size of N. So the number of iterations required by the proposed method is almost constant for most value of N and it is increasing with N in the existing method. Results are obtained by taking triangular shaped input signal for different transform size.

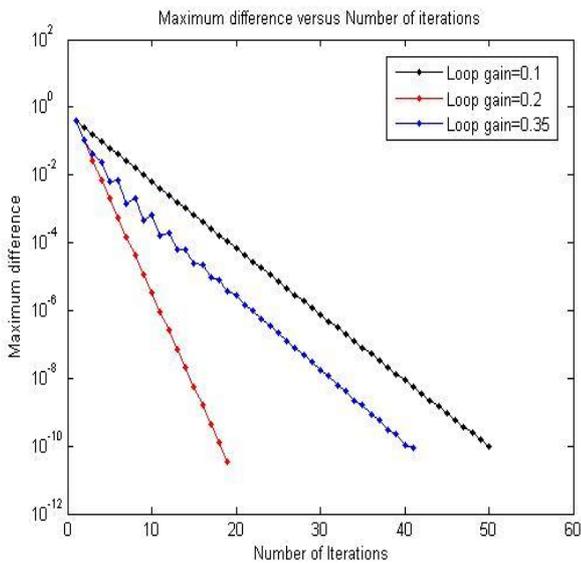


Figure 4: Maximum difference versus Number of iterations for different loop gain.

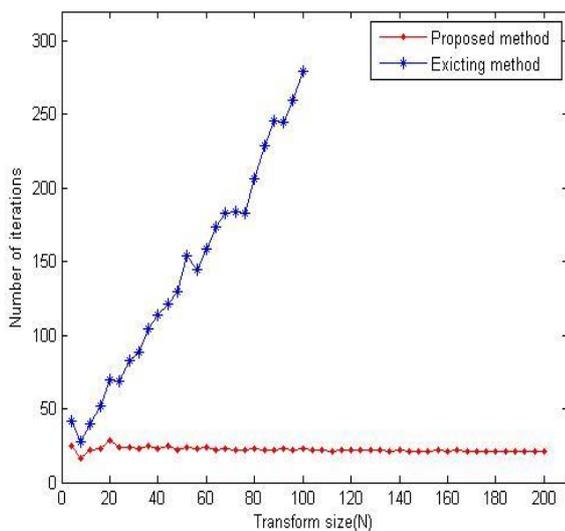


Figure 5: Transform size (N) versus Number of iterations

Figure 6 illustrates how the proposed design method requires less number of iterations compared with existing methods. Simulation time of an existing method increases exponentially as like number of iterations. Although the

proposed method takes the same number of iterations for all transform size N, it is not necessary that the proposed method to provide same simulation time to all the value of N. It is just because, if we increase transform size (N) there is an additional complexity of calculating CDFT spectrum which requires matrix multiplication and gradient for the extra points in each iteration. Results are obtained by taking triangular shaped input signal for the various transform sizes. An obtained solution for any value of N from the proposed iterative method is modified and it is used as the initial signal for the next iteration. After the final iteration, obtained solution will give the discrete Fourier-invariant signal with the minimum BT product is shown in Figure 7. Results are shown by taking a triangular shaped signal of size $N=250$ as an input. The obtained results will achieve a BT product which is close to Gabor's lower bound BT product and also achieve less BT product compared to the existing approach.

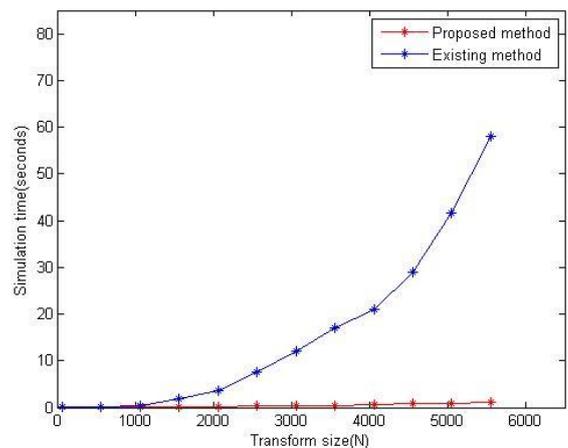


Figure 6: Transform size (N) versus Simulation time (seconds)

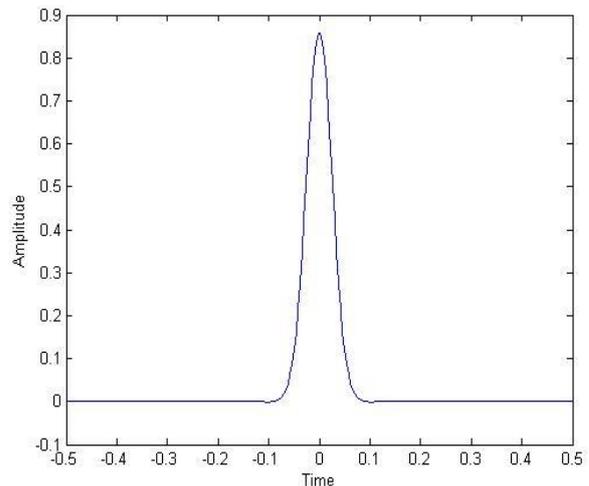


Figure 7: Proposed method for discrete Fourier-invariant signal with minimum BT product of transform size $N=250$

Figure 8 illustrates how the designed discrete Fourier-invariant signal for various sizes (N) approaches theoretical Gabor lower bound on the BT product of a signal.

For example, the proposed method for transforming size $N=250$ provide a BT product of 3.1834×10^{-4} . For the same transform size BT product of an existing method is 3.2372×10^{-4} . The theoretical minimum BT product for $N=250$ is 3.1831×10^{-4} . Figure 9(a) and 9(b) illustrates initial cosine signal and its CDFT spectrum. For the initial cosine signal, the discrete Fourier-invariant signal with the minimum BT product is obtained by proposed design and results are illustrated in figure 9(c). Figure 9(d) shows the spectrum of obtaining discrete Fourier-invariant signal with minimum BT product. The additive white Gaussian noise (AWGN) of signal to noise ratio -2 dB added with obtaining minimum BT signal and results are illustrated in figure 9(e). This noise added signal given as input to the discrete wavelet transform for noise reduction and results are shown in figure 9(f). This wavelet signal output is used as an initial input signal and again design procedure of signal with minimum BT is followed and the results are illustrated in figure 9(g) and 9(h). These results provide better reconstruction of the minimum BT signal. Thus, we can reconstruct the original cosine signal from finally recovered signal with minimum BT product.

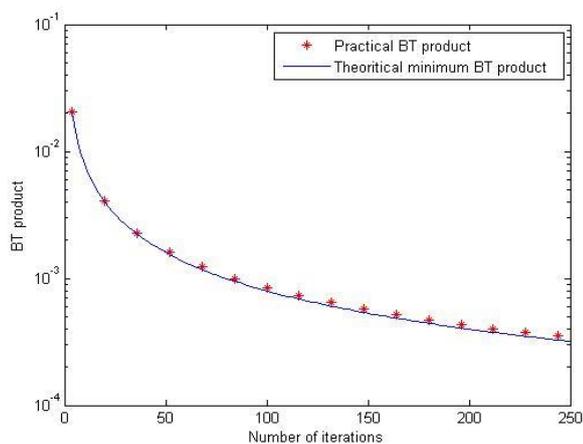


Figure 8: Transform size (N) versus BT product

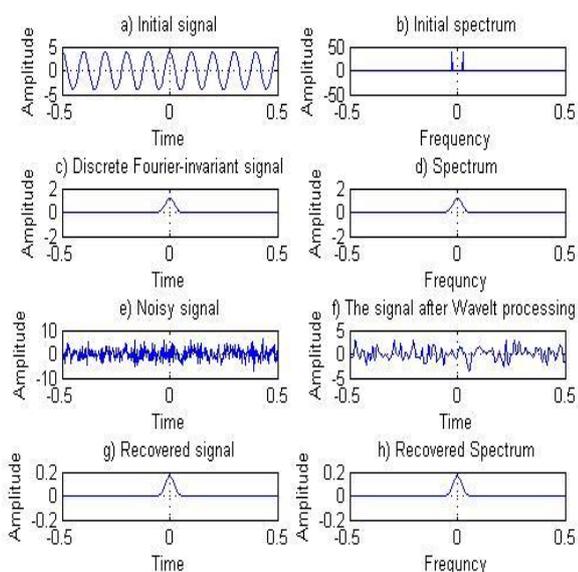


Figure 9: An example of noise reduction with discrete Fourier-invariant with minimum BT product

V. CONCLUSION

A method for the design of discrete Fourier-invariant signal is proposed. The proposed iterative design of discrete Fourier-invariant signals starts with any symmetrical initial signal and iteratively minimizes the difference between the signal and its CDFT spectrum using the gradient method. It is shown that the proposed method takes less simulation time and least number of iterations compared with the previous design method. A method for the design of discrete Fourier-invariant signal with the minimum BT product is proposed. The signal obtained from the proposed design is very close to Gabor's lower limit on minimum BT product of any signal is shown. Finally, possible applications of discrete Fourier-invariant signal with the minimum BT are shown. Challenges are how to implement the proposed method for multidimensional signal and to find various applications of the discrete Fourier-invariant signal. The other possible application is to apply this signal in digital filter and combine them under perfect reconstruction condition for compression application.

ACKNOWLEDGMENT

I would like to thank to my institute for providing me a wonderful peaceful environment and proper guide. Also, I would like to say special thanks to my guide and other co-author for the motivation and needful help.

REFERENCES

1. Lathi, B P signal processing and linear systems, Berkeley-Cambridge press, Carmichael, CA 1998.
2. L. R. Soares, H. M. de Oliveira, R. J. S. Cintra, and R. C. de Souza, "Fourier eigenfunctions, uncertainty gabor principle, and isoresolution wavelets," in Symp. Brasileiro de Telecomun., Rio de Janeiro, 2003.
3. P. P. Vaidyanathan, "Eigenfunctions of the Fourier transform," IETE J. Educ., vol. 49, pp. 51–58.
4. Discrete Fourier-Invariant Signals: Design and Application, Maja Temerinac-Ott, Member, IEEE, and Miodrag Temerinac, Senior Member, IEEE. IEEE TRANSACTIONS ON SIGNAL PROCESSING, VOL. 60, NO. 3, MARCH 2012
5. B. Santhanam and T. Santhanam, "Discrete Gauss-Hermite functions and eigenvectors of the centered discrete Fourier transform," in Proc. IEEE Int. Conf. Acoust., Speech, Signal Process. (ICASSP 2007), Apr. 2007, vol. 3, pp. III-1385–III-1388.
6. S.-C. Pei and K.-W. Chang, "Generating matrix of discrete Fourier transform eigenvectors," in Proc. IEEE Int. Conf. Acoust., Speech, Signal Process. (ICASSP 2009), Apr. 2009, pp. 3333–3336.
7. M. T. Hanna, N. P. A. Seif, and W. Ahmed, "Discrete fractional Fourier transform based on the eigenvectors of tridiagonal and nearly tridiagonal matrices," Digit. Signal Process., vol. 18, pp. 709–727, 2008.
8. B. Dickinson and K. Steiglitz, "Eigenvectors and functions of the discrete Fourier transform," IEEE Trans. Acoust., Speech, Signal Process., vol. 30, no. 1, pp. 25–31, Feb. 1982.
9. S.-C. Pei, W.-L. Hsue, and J.-J. Ding, "Discrete fractional Fourier transform based on new nearly tridiagonal commuting matrices," IEEE Trans. Signal Process., vol. 54, no. 10, pp. 3815–3828, Oct. 2006.
10. C. Candan, "On higher order approximations for Hermite-Gaussian functions and discrete fractional Fourier transforms," IEEE Signal Process. Lett. vol. 14, no. 10, pp. 699–702, Oct. 2007.
11. J. Vargas-Rubio and B. Santhanam, "On the multiangle centered discrete fractional Fourier transform," IEEE Signal Process. Lett., vol. 12, no. 4, pp. 273–276, Apr. 2005.

12. E. Anderson, Z. Bai, C. Bischof, S. Blackford, J. Demmel, J. Dongarra, J.D. Croz, A. Greenbalm, S. Hammarling, A. McKenny, and D. Sorensen, LAPACK User's Guide, 3rd ed. Philadelphia, PA:SIAM, 1999.
13. GD. Gabor, "Theory of communication. Part 1: The analysis of information," *Elect. Eng.—Part III: Radio Commun. Eng. J. Inst.*, vol. 93, no. 26, pp. 429–441, Nov. 1946.

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