Resonance Effect in Dynamic of the Mathematical Model for Baroreceptor

A. Ali, R. Hussain, F. Kappel

***Abstract** The best known nervous mechanism for control of arterial pressure is the baroreceptor loop. To simulate some fundamental regulation processes mathematical model is used which approximate the short-term behavior of the baroreceptor. The most important short term properties of baroreceptor in a clear mathematical model is presented in [2]. This model is applies in the dynamic features of the human system. The goal of our work is to see the resonance effect in the dynamic of the baroreceptor model presented in [2].*

Keywords: - baroreceptor, mechanism, fundamental, mathematical model.

I. INTRODUCTION

The central task of baroreceptor is to measure and transfer information of current blood pressure. This information is coded as an impulse frequency which will pass to circulatory center via afferent action potential. The relation between momentary pressure at the receptor and the resulting impulse frequency is time dependent and highly nonlinear.

II. BAROREFLEX MECHANISM

The baroreflex mechanism plays a central role in short-term control. This mechanism provides rapid negative feedback control of arterial blood pressure. A sudden drop in arterial pressure is sensed by the baroreceptors, starting a chain of events leading to an increase in heart rate and cardiac contractility. This pressure drop also stimulates the contraction of the vascular vessels. These responses tend to alter the arterial blood pressure toward its normal value hence producing negative feedback effect. This is short term control regulatory mechanism.

III. STATIONARY PROPERTY OF MODEL

The stationary signal frequency to pressure relation is obtained neglecting time dependent influences, in particular the variation of blood pressure with time. The corresponding response curve is obtained experimentally by including a stepwise increase of non pulsatile pressure and measurements of the resulting activity of the afferent nerve in an equilibrium situation, i.e., after the transient processes have decayed.

This response curve includes the existence of a stationary threshold pressure, below which no afferent signal can leave, an approximately linear mid-range segment, which in general has the maximal slope occurring in the graph, and a maximal frequency which represent saturation. The average threshold pressure for the carotid sinus preparation is approximately 60 mmHg, the steepest segment of the curve occur between 100 - 120 mmHg and the stationary maximal frequency of approximately 60-80 impulses/sec is reached at pressures higher than 160 mmHg. At the lower pressure threshold, the afferent activity can begin suddenly with a frequency of about 20 impulses /sec. The behavior of the receptors in the aorta is similar to that for receptor in the carotid sinus with pressure values shifted by about 30 mmHg in the direction of higher pressure. For the mathematical description of stationary pressure to signal frequency relation of the baroreceptors the following function is used in order to express the signal frequency $f(p)$ corresponding to arterial respectively aortic blood pressure p:

$$f(p) = \left(1 - \frac{(K-p_s)^u}{(p-p_0)^u + (K-p_0)^u}\right) f_{max}\, s(p-p_0) \qquad \text{(A)}$$

Here p_s denotes the threshold value for the blood pressure below of which the baroreceptors are insensitive, i.e $f(p) = 0$ for $p \le p_0$. The constant $K \ge p_0$ and the positive integer u determine the shape of the function $f(p)$ which is strictly increasing for $p \ge p_0$. It is easy to see that $\lim_{p\to\infty} f(p) = f_{max}$, where f_{max} denotes the stationary maximal signal frequency. Finally, s(.) denotes the unit step function.

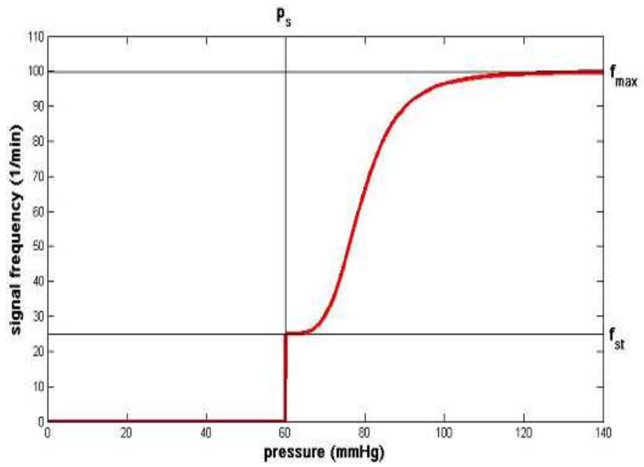

Figure1: Graph of $f(p)$given by (B)

In order to allow a sudden start of activity of the baroreceptors with a frequency $f_{st} > 0$ when the pressure increases through the threshold value we have to modify p_s equation (A) as follows (note that in order to start with a frequency f_{st} at p_s

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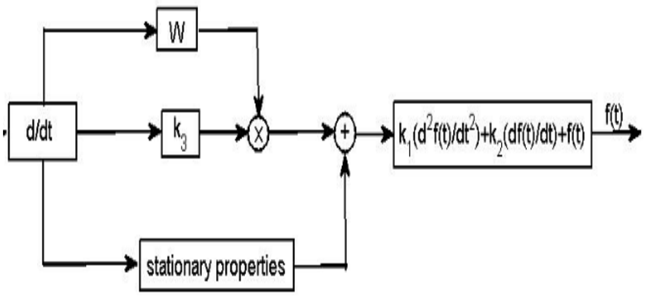
one must have $p_0 = p_s$)

$$f(p) = \left(\left(1 - \frac{(K-p_0)^u}{(p-p_0)^u + (K-p_0)^u}\right)(f_{max} - f_{st}) + f_{st}\right) s(p-p_s) \tag{B}$$

In Figure above depict the graph of the function $f(p)$ defined in equation (B).

IV. DYNAMICAL PROPERTY OF MODEL

The temporal behavior of the signal frequency, which is stimulated by pressure variations, depends on the absolute pressure level, which enters via the stationary relation described in the previous subsection, as well as on the speed of pressure changes, i.e., the derivative of pressure with respect to time. A rapidly increasing pressure signal can cause signal frequencies which exceed the stationary maximal frequency. If the pressure remains constant after the increase, the frequency response decays to the maximal stationary signal frequency [4]. Below the stationary pressure threshold of the receptor only fast pressure variation can cause a nervous reaction. If the pressure remains constant afterwards the signal frequency decreases to zero [3]. The consequences of a pressure increase are different from the consequences of a pressure decrease. This asymmetry varies in dependence on the species of animal used for the experiments. Since the signal frequency cannot become negative, a large or rapid pressure decrease can lead to a suspension of the afferent impulses during a period called the silent period. Afterwards the frequency reaches gradually the corresponding stationary value [3]. The above discussed dynamical aspects together with the stationary relation are represented in the following equations.



$$f_{aff}(t) = \begin{cases} f(t) & for\ f(t) \geq 0 \\ 0 & otherwise \end{cases}$$

$$k_1 \frac{d^2 f(t)}{dt^2} + k_2 \frac{df(t)}{dt} + f(t) = Z$$

$$Z = \left(\left(1 - \frac{(K-p_0)^u}{(p-p_0)^u + (K-p_0)^u}\right)(f_{max} - f_{st}) + f_{st}\right) s(p-p_s) + k_3 W\left(\frac{dp}{dt}\right)\frac{dp(t)}{dt} \tag{C}$$

and the function $W(x)$ is defined as

$$W(x) = \begin{cases} 1 & for\ x \geq 0 \\ w & for\ x < 0 \end{cases}$$

The function $W(x)$ represents the different weighting of positive and negative pressure slopes. Equation (C) cuts off negative frequencies which resulting a silent period. The parameters k_1, $k_2 > 0$ are time constants which determine the basic signal frequency, whereas k_3 represents a weight on the differential quotient of the pressure with respect to time.

We convert equation (C) into a system of 1st order equations by introducing

$$m(t) = \frac{df(t)}{dt}$$

$$\frac{df(t)}{dt} = m(t)$$

$$\frac{dm(t)}{dt} = -\frac{k_2}{k_1} m(t) - \frac{1}{k_1} f(t) + Z$$

f_{max}	k_1	f_{st}	k_2	p_0 & p_s	k_3	K	p	u	W
70	1	18	1.7	60	0.15	150	122	6	4

Table: Values of parameters used in simulations

V. SIMULATION RESULT

In the following we present some simulation result which shows some important properties of baroreceptors.

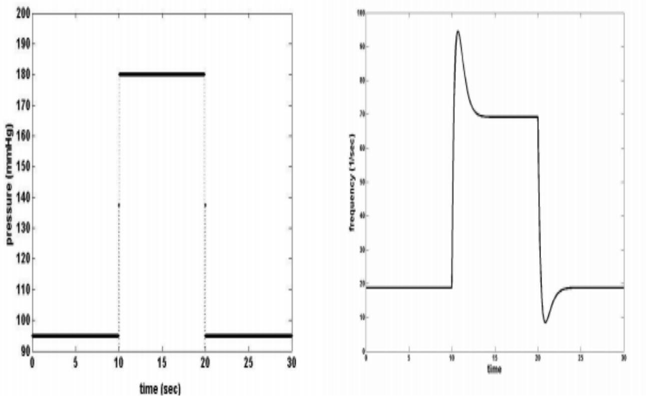


Figure-2 : Left panel is input (pressure), and right panel is response of model

We give input pressure in our model in the form of increasing and decreasing pressure jump with same magnitude i.e.,95-180 and 180-95 and response of our model is showing different behavior for increasing and decreasing pressure jump and shown in Fig-2.

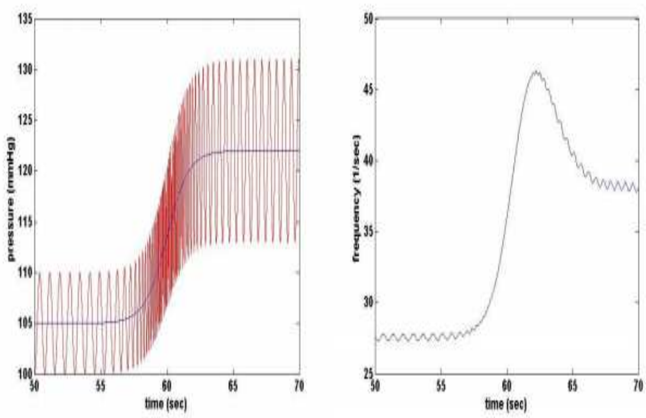


Figure-3: Input Left panel (pressure) and right panel is response of model

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Here we give input pressure and increases the pressure gradually, when pressure increases the amplitude is also increasing with increase in pressure. The response of model for different pressure level is different with variation in amplitude and is presented in Fig-3.

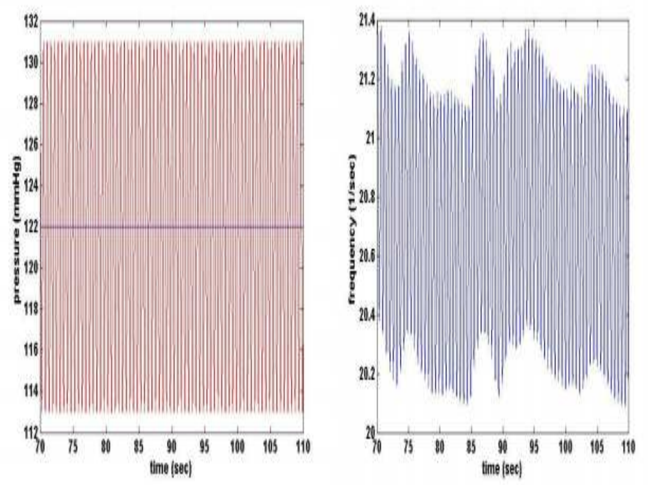

Figure-4: Left panel is input (pressure) and right panel is response of mode

Fig-4 here we give input pressure and maintain pressure at certain level, the response of model is quite interesting showing resonance effect

VI. CONCLUSION AND FUTURE WORK

Above presented model of the baroreceptor is working properly to the variation in blood pressure. Response of model is quite interesting showing resonance effect when input pressure is maintained at certain level i.e at constant pressure. In future our next step is to connect baroreceptor model with the global pulsatile model of the cardiovascular system.

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