International Journal of Innovative Technology and Exploring Engineering (IJITEE)
ISSN: 2278-3075 (Online), Volume-4 Issue-8, January 2015

Stability Analysis of Mathematical Model Comparing Solute Kinetics in Low & High Hemodialysis Patients

R. Hussain, A. Ali, N. Arif

Abstract. This paper is about the stability analysis of model, which we have taken from "The mathematical model comparing solute kinetics in low- and high-BMI hemodialysis patients" [2]. The purpose of this study is to check the stability of three types of patients i.e small medium and large during dialysis and in between the dialysis treatment. In all cases we get the stable solution for above model presented in [2].

Index Terms—Stability analysis, hemodialysis, mathematical modeling.

I. INTRODUCTION

The model is a four compartmental model taken from [2]. The block diagram of model is given below:

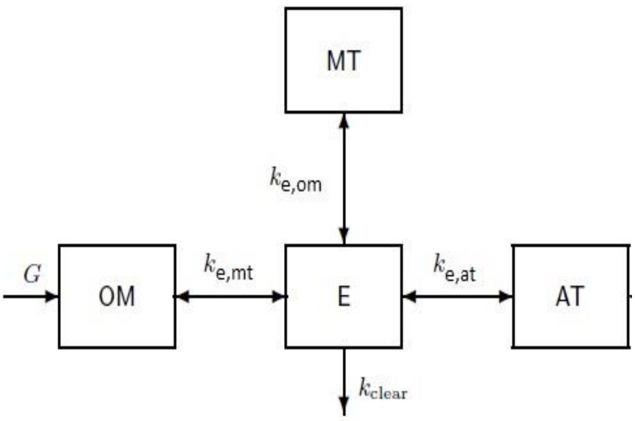

Figure: Block diagram

$$\frac{dx_{om}}{dt} = G - \frac{k_{e,om}}{v_{om}} x_{om} + \frac{k_{e,om}}{v_{om}} x_e \quad (1)$$

$$\frac{dx_{mt}}{dt} = \frac{k_{e,mt}}{v_{mt}} x_e - \frac{k_{e,mt}}{v_{mt}} x_{mt} \quad (2)$$

$$\frac{dx_{at}}{dt} = \frac{k_{e,at}}{v_{at}} x_e - \frac{k_{e,at}}{v_{at}} x_{at} \quad (3)$$

$$\frac{dx_e}{dt} = \frac{k_{e,om}}{v_e} x_{om} + \frac{k_{e,mt}}{v_e} x_{mt} + \frac{k_{e,at}}{v_e} x_{at} - \frac{1}{v_e}(k_{e,om} + k_{e,mt} + k_{e,at}) x_e - k_{clearance} \quad (4)$$

Manuscript published on 30 January 2015.
*Correspondence Author(s)
Dr. Rashida Hussain, Mirpur, University of Science &Technology (MUST) AJK, Mirpur, AJK, Pakistan.
Dr. Ashar Ali, Mirpur, University of Science &Technology (MUST) AJK, Mirpur, AJK, Pakistan.
Nida Arif, Mirpur University of Science &Technology (MUST) AJK, Mirpur, AJK, Pakistan.

The above set of equations can be written as:

$$\frac{dc_{om}}{dt} = \frac{G}{v_{om}} - \frac{k_{e,om}}{v_{om}} c_{om} + \frac{k_{e,om}}{v_{om}} c_e \quad (5)$$

$$\frac{dc_{mt}}{dt} = \frac{k_{e,mt}}{v_{mt}} c_e - \frac{k_{e,mt}}{v_{mt}} c_{mt} \quad (6)$$

$$\frac{dc_{at}}{dt} = \frac{k_{e,at}}{v_{at}} c_e - \frac{k_{e,at}}{v_{at}} c_{at} \quad (7)$$

$$\frac{dc_e}{dt} = \frac{k_{e,om}}{v_{om}} c_{om} + \frac{k_{e,mt}}{v_{mt}} c_{mt} + \frac{k_{e,at}}{v_{at}} c_{at} - \frac{1}{v_e}(k_{e,om} + k_{e,mt} + k_{e,at}) c_e - k_{clearance} \quad (8)$$

Where

$$k_{clearance} = \begin{cases} k_{clear} c_e & \text{during dialysis} \\ 0 & \text{in between dialysis} \end{cases}$$

c_{om}	c_{mt}	c_{at}	c_e
35	35	35	35

Table 1: Initial Condition

$k_{e,om}$	$k_{e,mt}$	$k_{e,at}$	k_{clear}
0.045	0.03177	0.0193	0.1

Table 2: Rate Constants

Size	v_e	v_{om}	v_{mt}	v_{at}
Large	58	21	6.54	4.46
Medium	40.6	17.22	2.93	2.25
Small	23.2	11.22	1.486	0.034

Table of Parameter

For stability analysis of model, we divide it into two sections. Section 1 is for the inter-dialytic interval. And in second section we determine the Stability analysis for dialytic interval.

1. Stability of Model in between dialysis ($\delta = 0$)

1.1 LARGE PATIENTS

First we choose equation (5) and (8)

$$\frac{dc_{om}}{dt} = \frac{G}{v_{om}} - \frac{k_{e,om}}{v_{om}} c_{om} + \frac{k_{e,om}}{v_{om}} c_e \quad (9)$$

$$\frac{dc_e}{dt} = \frac{k_{e,om}}{v_{om}} c_{om} + \frac{k_{e,mt}}{v_{mt}} c_{mt} + \frac{k_{e,at}}{v_{at}} c_{at} - \frac{1}{v_e}(k_{e,om} + k_{e,mt} + k_{e,at}) c_e - k_{clearance} \quad (10)$$

Published By:
Blue Eyes Intelligence Engineering and Sciences Publication (BEIESP)

After putting values from table in equations (9) and (10), we get Critical points (c_{om}, c_e) = (16.098,9.420)

Eigenvalue (-0.0032,-0,0006)

As both eigenvalues are –ve so the solution is stable and is presented in Fig 1.1

Secondly we choose equations (6) and (8)

$$\frac{dc_{mt}}{dt} = \frac{k_{e,mt}}{v_{mt}} c_e - \frac{k_{e,mt}}{v_{mt}} c_{mt} \quad (11)$$

$$\frac{dc_e}{dt} = \frac{k_{e,om}}{v_{om}} c_{om} + \frac{k_{e,mt}}{v_{mt}} c_{mt} + \frac{k_{e,at}}{v_{at}} c_{at} - \frac{1}{v_e}(k_{e,om} + k_{e,mt} + k_{e,at}) c_e - k_{clearance} \quad (12)$$

After putting values from table in equations (11) and (12), we get Critical points (c_{mt}, c_e) = (34.964, 34.964)

Eigenvalue (-0.0055, -0.0010)

Both eigenvalues are –ve so solution is stable and is presented in Fig 1.2

Third we choose equations (7) and (8)

$$\frac{dc_{at}}{dt} = \frac{k_{e,at}}{v_{at}} c_e - \frac{k_{e,at}}{v_{at}} c_{at} \quad (13)$$

$$\frac{dc_e}{dt} = \frac{k_{e,om}}{v_{om}} c_{om} + \frac{k_{e,mt}}{v_{mt}} c_{mt} + \frac{k_{e,at}}{v_{at}} c_{at} - \frac{1}{v_e}(k_{e,om} + k_{e,mt} + k_{e,at}) c_e - k_{clearance} \quad (14)$$

After putting values from table in equations (13) and (14), we get Critical points (c_{at}, c_e) = (20.421, 20.421)

Eigenvalue (-0.0048, -0.0012)

Again both eigenvalues are –ve so solution is stable and is presented in Fig 1.3

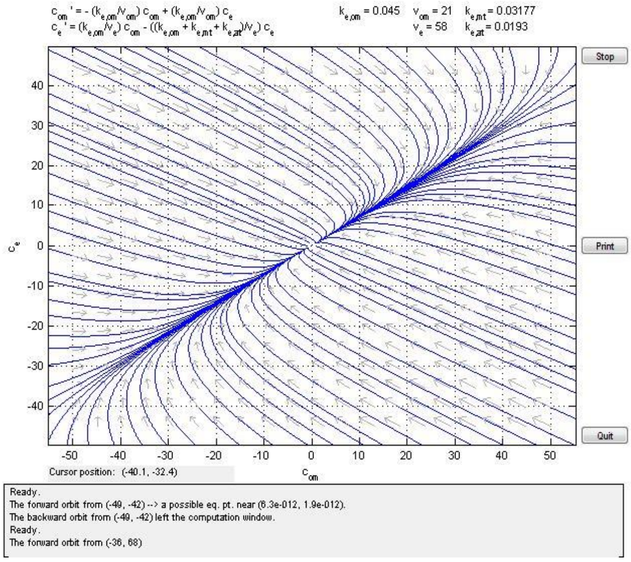

Figure 1.1: Stability of c_{om} vs c_e for large patients

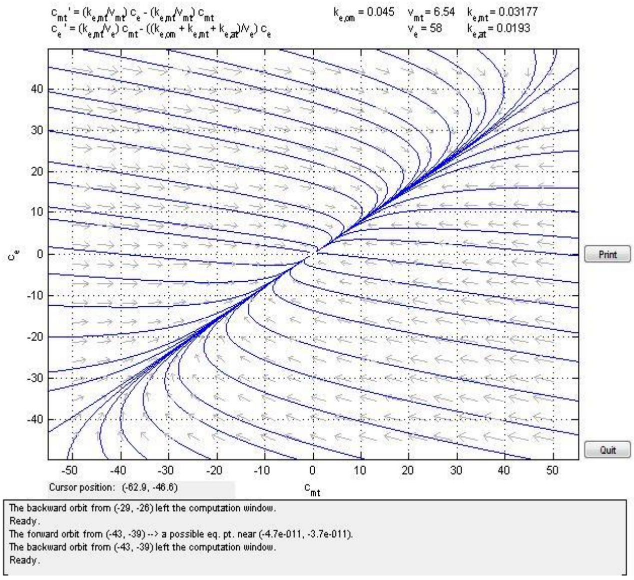

Figure 1.2: Stability of c_{mt} vs c_e for large patients

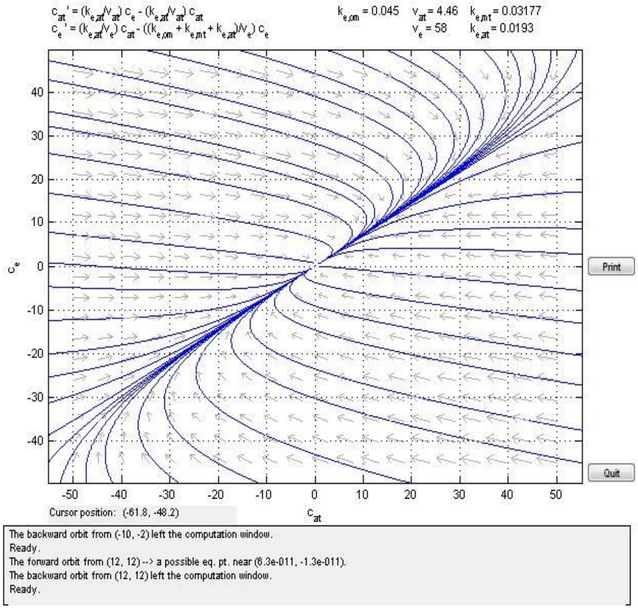

Figure 1.3: Stability of c_{at} vs c_e for large patients

The same pair of equations have been used for medium and small patients.

1.2 MEDIUM PATIENTS

After putting values from table in equations (9) and (10), we get Critical points (c_{om}, c_e) = (47.500, 40.825)

Eigenvalue (-0.0042, -0.0008)

Both eigenvalues are –ve so the solution is stable and is presented in Fig 1.4

After putting values from table in equations (11) and (12), we get Critical points

(c_{mt}, c_e) =(34.862, 34.862)

Eigenvalue (-0.0117, -0.0015)

As Both eigenvalues are –ve so the solution is stable and is presented in Fig 1.5

Published By:
Blue Eyes Intelligence Engineering and Sciences Publication (BEIESP)

After putting values from table in equations (13) and (14), it gives Critical points (c_{at}, c_e) = (35.016, 35.016)

Eigenvalue (-0.0092, -0.0018)

Both eigenvalues are –ve so the solution is stable and is presented in Fig 1.6

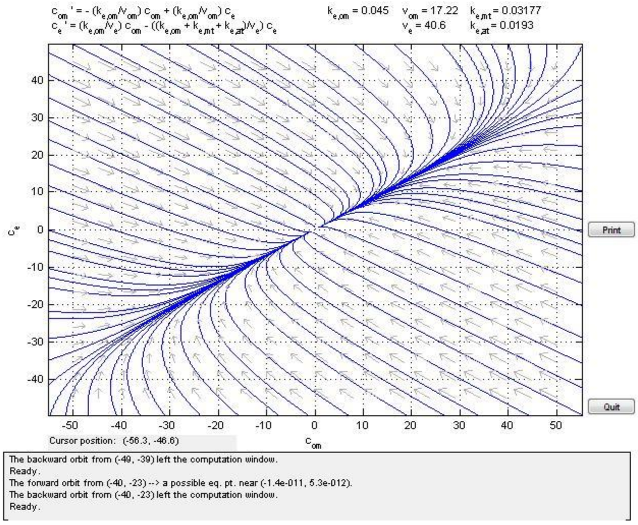

Figure 1.4: Stability of c_{om} vs c_e for medium patients

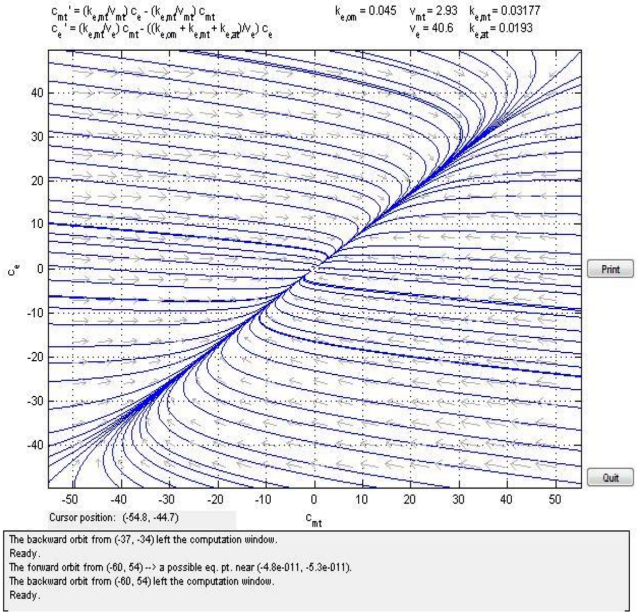

Figure 1.5: Stability of c_{mt} vs c_e for medium patients

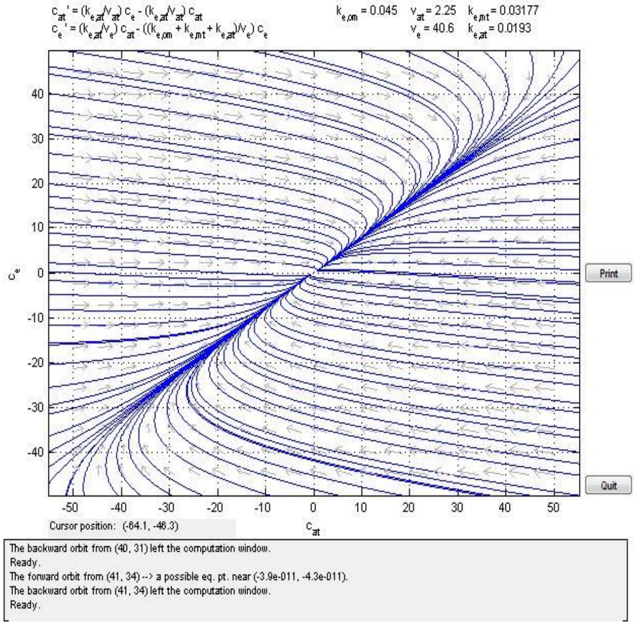

Figure 1.6: Stability of c_{at} vs c_e for medium patients

1.3 SMALL PATIENTS

After putting values from table in equations (9) and (10), it gives Critical points

(c_{om}, c_e) = (47.567, 40.9)

Eigenvalue (-0.0013, -0.0068)

Again all eigenvalues are –ve so the solution is stable and is presented in Fig 1.7

After putting values from table in equations (11) and (12), it gives Critical points (c_{mt}, c_e) = (35.022, 35.022)

Eigenvalue (-0.0229 , -0.0026)

As both eigenvalues are –ve so the solution is stable and is presented in Fig 1.8

After putting values from table in equations (13) and (14), it gives Critical points

(c_{at}, c_e) = (34.991, 34.991)

Eigenvalue (-0.5685, -0.0033)

Again all eigenvalues are –ve so the solution is stable and is presented in Fig 1.9

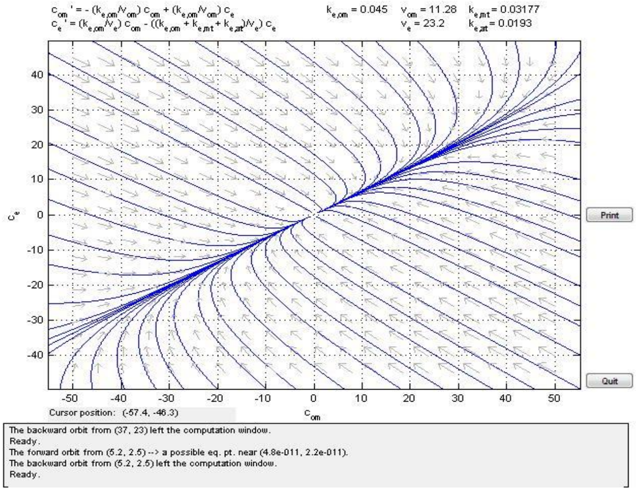

Figure 1.7: Stability of c_{om} vs c_e for small patients

Published By:
Blue Eyes Intelligence Engineering
and Sciences Publication (BEIESP)

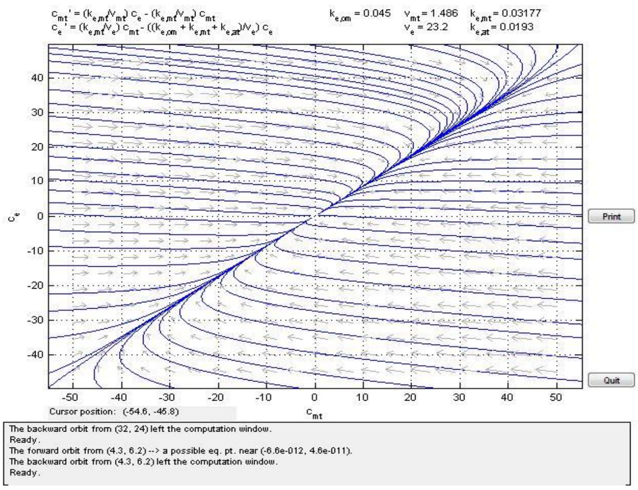

Figure 1.8: Stability of c_{mt} vs c_e for small patients

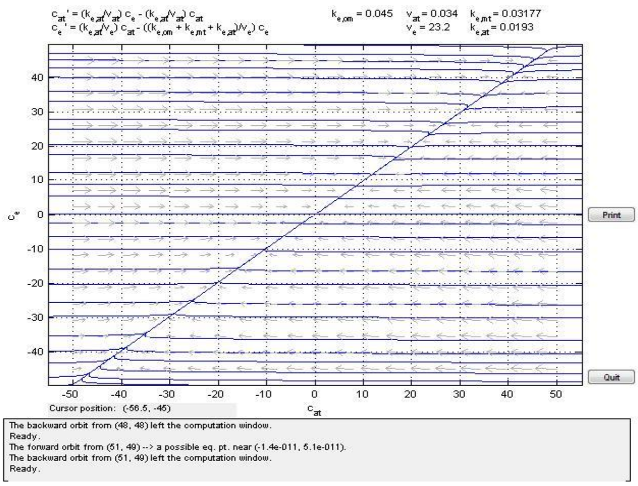

Figure 1.9: Stability of c_{at} vs c_e for small patients

II. STABILITY OF MODEL DURING DIALYSIS ($\delta = 1$)

Using the same pair of equations which we have already used for in between dialysis for three cases (large, medium and small).

2.1 LARGE PATIENTS

After putting values from table in equations (9) and (10), it gives Critical points (c_{om}, c_e) = (20.535, 13.858)

Eigenvalue (-0.0013, -0.0042)

Solution of above equations are stable as eigenvalues are –ve and is presented in Fig 2.1

After putting values from table in equations (11) and (12), it gives Critical points (c_{mt}, c_e) = (13.714, 13.714)

Eigenvalue (-0.0059, -0.0023)

Solution of above pair of equations is again stable due to –ve eigenvalues and is presented in Fig 2.2

After putting values from table in equations (13) and (14), it gives Critical points (c_{at}, c_e) = (8.905, 8.905)

Eigenvalue (-0.0051, -0.0026)

Again stable solution due to –ve eigenvalues and is presented in Fig 2.3

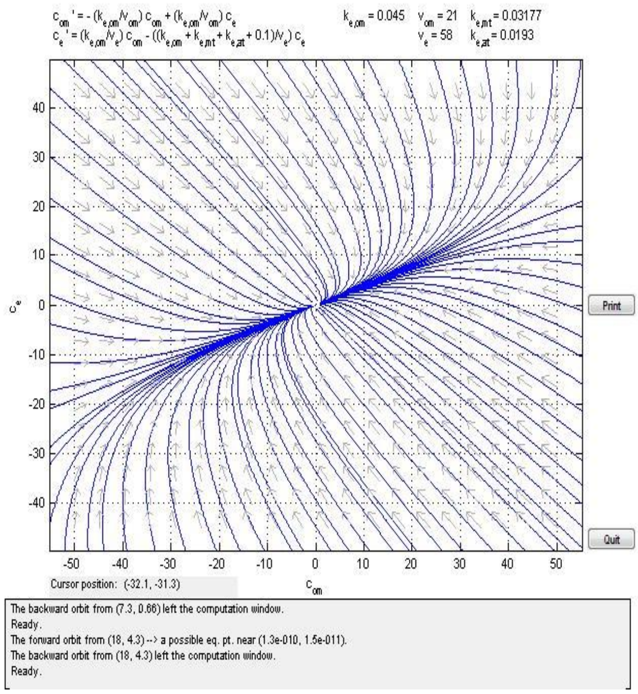

Figure 2.1: Stability of c_{om} vs c_e for large patients

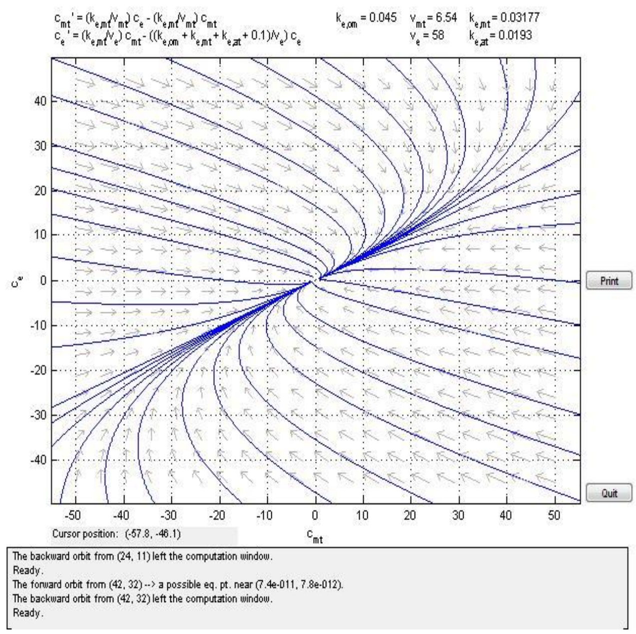

Figure 2.2: Stability of c_{mt} vs c_e for large patients

Published By:
Blue Eyes Intelligence Engineering
and Sciences Publication (BEIESP)

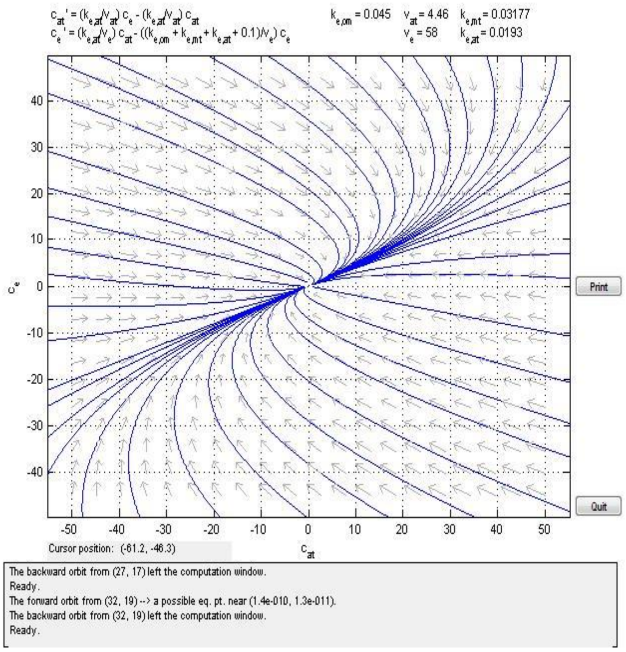

Figure 2.3: Stability of c_{at} vs c_e for large patients

2.2 MEDIUM PATIENTS

After putting values from table in equations (9) and (10), it gives Critical points

$$(c_{om}, c_e) = (20.502, 13.828)$$

Eigenvalue (-0.0017, -0.0058)

Due to –ve eigenvalues solution of above model is stable and is shown in Fig 2.4

After putting values from table in equations (11) and (12), it gives Critical points

$$(c_{mt}, c_e) = (13.686, 13.686)$$

Eigenvalue (-0.0120, -0.0037)

Again solution presented in Fig 2.5 is stable due to –ve eigenvalues

After putting values from table in equations (13) and (14), it gives Critical points $(c_{at}, c_e) = (15.214, 15.214)$

Eigenvalue (-0.0095, -0.0039)

Stable solution presented in Fig 2.6

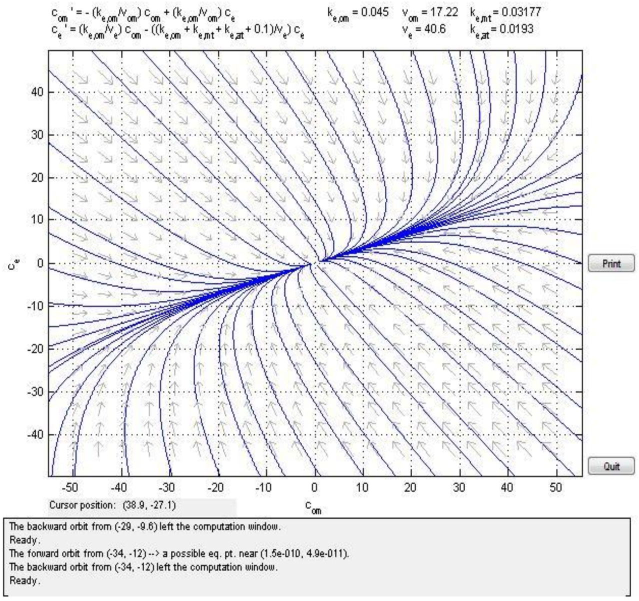

Figure 2.4: Stability of c_{om} vs c_e for medium patients

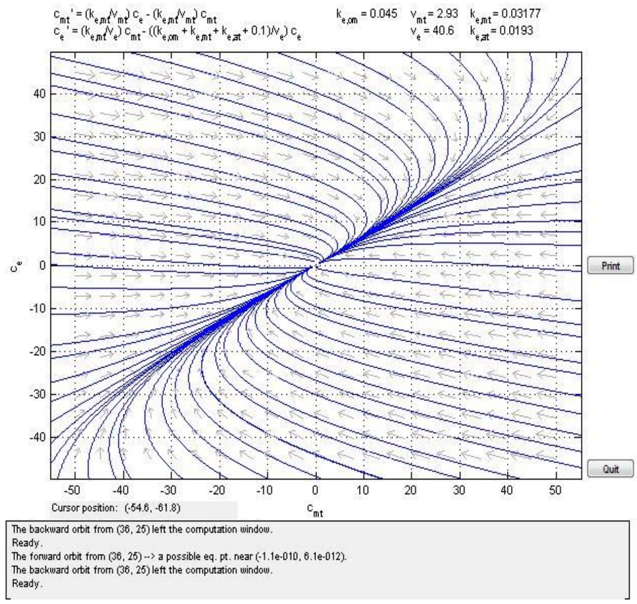

Figure 2.5: Stability of c_{mt} vs c_e for medium patients

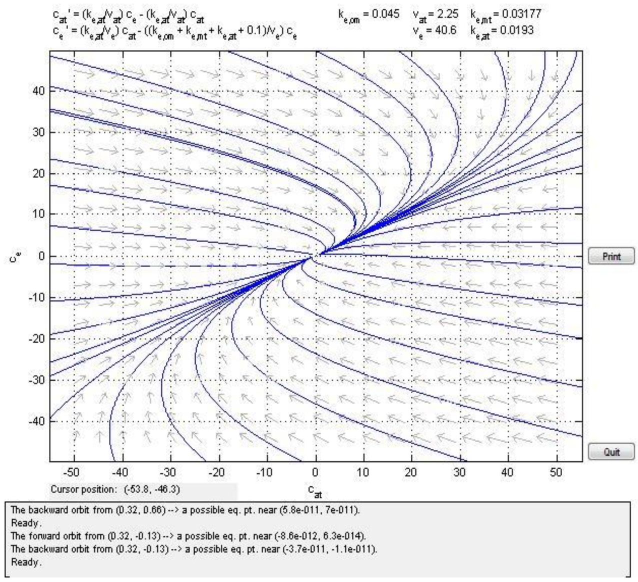

Figure 2.6: Stability of c_{at} vs c_e for medium patients

2.3 SMALL PATIENTS

After putting values from table in equations (9) and (10), it gives Critical points

$$(c_{om}, c_e) = (20.488, 13.822)$$

Eigenvalue (-0.0027, -0.0098)

Stable solution presented in Fig 2.7 for small patients.

After putting values from table in equations (11) and (12), it gives Critical points

$$(c_{mt}, c_e) = (13.702, 13.702)$$

Eigenvalue (-0.0233, -0.0065)

Stable solution for small patients is presented in Fig 2.8

After putting values from table in equations (13) and (14), it gives Critical points

Published By:
Blue Eyes Intelligence Engineering
and Sciences Publication (BEIESP)

$(c_{at}, c_e) = (15.199, 15.199)$

Eigenvalue (-0.5685, -0.0076)

Stable solution for small patients is presented in Fig 2.9

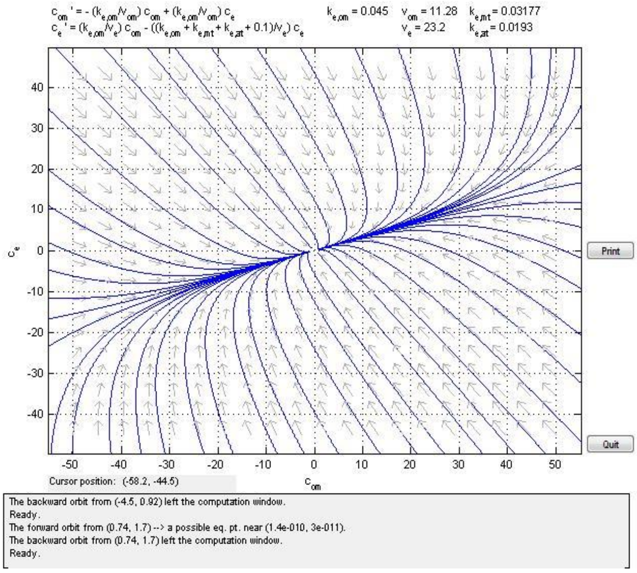

Figure 2.7: Stability of c_{om} vs c_e for small patients

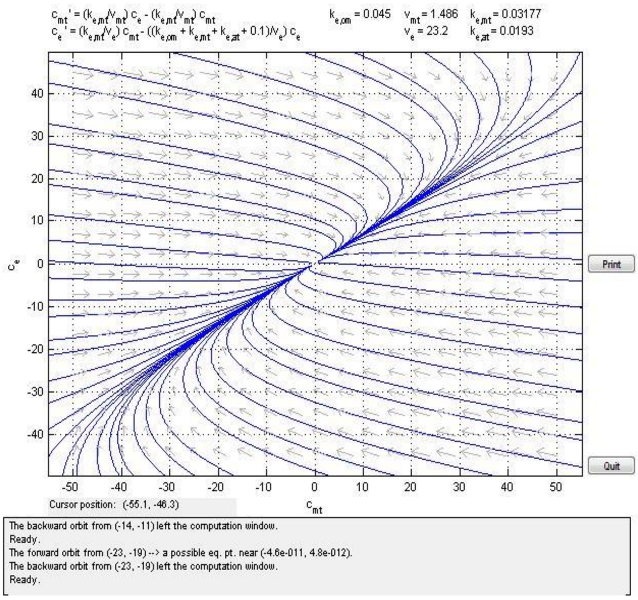

Figure 2.8: Stability of c_{mt} vs c_e for small patients

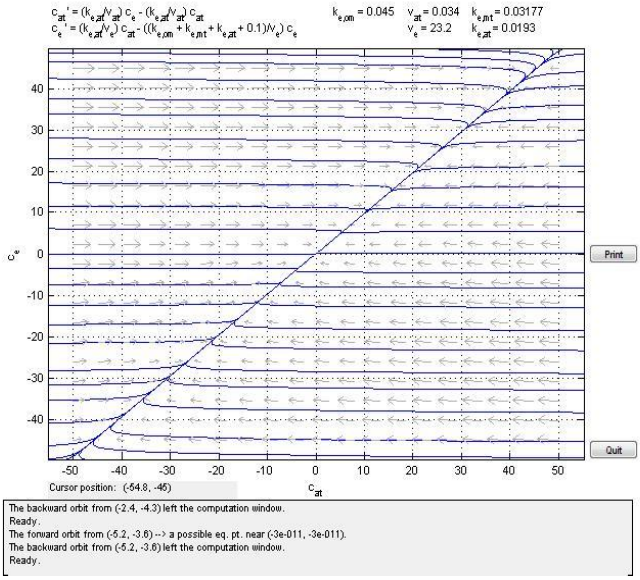

Figure 2.9: Stability of c_{at} vs c_e for small patients

III. CONCLUSION

Stability analysis of the model presented in [2] was done for different types of patients (large, medium, and small) during dialysis and in between the dialysis. We conclude that in both cases the model solution was stable.

REFERENCES

1. D. Cronin-Fine, F. Gotch, N. Levin, P. Kotanko, M. Lysaght, A Mathematical Model Comparing Solute Kinetics in Low- and High BMI Hemodialysis Patients. Internatl. J. Artificial Organs(30)(11) (2007), 1000-1007.
2. F. Kappel, J. Batzal , M. Bacher , and P.Kotanko A Mathematical Model Comparing Solute Kinetics in Low- and High-BMI Hemodialysis Patients, March 11, (2009).
3. J. J. Batzal and F. Kappel and H. T. Tran and D. Schneditz, Cardiovascular and Respiratory Systems: Modeling, Analysis and Control, Siam, Philadelphia (2006).
4. P. Hartman, A lemma in the Theory of Structural Stability of Differential Equations, Proceedings of the AMS 11(1960) 610-620.
5. P. Howard, Analysis of ODE Models, fall 2009.
6. P. Howard, Modelling with ODE, Available at www.math.tamu.edu~/phoward/M442.html
7. R. Hussain, A. Ali, S.Nasar, Mathematical Model of Dialysis: Stable and Unstable Solution. Mirpur University of Science and Technology. (Submitted paper)
8. R. Hussain, F. Kappel, Fansan Zhu, Nathan W. Levin and Peter. Kotanko, Body Composition and Solute Kinetics in Hemodialysis Patients: A Mathematical Model.
9. S. Beddhu and L. M. Pappas and N. Ramkumar and M. Samore, Effect of Body Size and Body Composition on Survival in Hemodialysis Patients, J Am Soc Nephrol 14 (2003),2366-2372.
10. Schneditz, D. and Daugridas, J. T. (2001). Compartment Effect in Hemodialysis. Semin Dial, Vol. 14, No. 4, pp. (271-7).
11. www.math.tamu.edu~/phoward/M442.html
12. Yon-Ping Chen, 8.Phase Plane Method, NCTU Department of Electrical and Computer Engineering Senior Course <Dynamic System Analysis and Simulation>.
13. Zachary S. Tseng 2008. Phase Plane.
14. Ziolko, M. Pietrzyk, J. A. and Grabska- Chrzastowaska, J. (2000). Accuracy of Hemodialysis Modeling. Kidney Int, Vol. 57, No. 3, pp. (1152-63).

Published By:
Blue Eyes Intelligence Engineering
and Sciences Publication (BEIESP)